

# The NEW METHOD ARITHMETIC

BY  
P. MCINTOSH  
AND  
C. A. NORMAN



PUBLISHED BY  
THE COMMERCIAL TEXT BOOK CO.  
TORONTO, CANADA

Student  
W.B.C.  
L

1063/4  
539 21

RB158,069



*Presented to the*  
LIBRARY *of the*  
UNIVERSITY OF TORONTO  
*by*  
Copp, Clark Pitman Ltd.



S. J. Louch

Lem Wilkin

Harvard

Out

6. *cinerea*





# *The* NEW METHOD ARITHMETIC

BY

*P. MCINTOSH*

AND

*C. A. NORMAN*



PUBLISHED BY

*THE COMMERCIAL TEXT BOOK CO.*

*TORONTO, CANADA*

L

Copyright, Canada, 1908, by P. McIntosh.

Less Drinker

r



## PREFACE

The NEW METHOD ARITHMETIC is what its name implies—a new method of presenting the subject of practical arithmetic. The authors have not been inspired by any desire to add one more to the already large number of practical arithmetics that are on the market, but, rather, to turn out a book that will present the subject in a way that must commend itself to every teacher and student.

We think it will be conceded that in any branch of education the main idea should be to impress principles, and to so arrange details that the principles are not lost sight of in a confusing mass of detail. Who has not met the boy in school who felt that he was strong on profit and loss, but to whom commission and brokerage was always a difficulty. Is this a fault in the boy or in the system under which he has been instructed? We think that the system must shoulder the blame. There is no difference, in a proper arrangement of the study of arithmetic, between profit and loss and commission and brokerage. When we set up these two applications of percentage as separate departments of the work, we are simply transgressing a first principle in properly presenting the subject. The detail of profit and loss and commission and brokerage is made to overshadow the principle of percentage, of which such work is a mere application.

Perhaps no better idea can be obtained of the general plan upon which the whole subject is presented than by a reference to that section of the work which deals with percentage. There, in a systematic order, under the headings of questions of the First, Second, Third, and Fourth Aspects, we have presented all the usual applications of percentage; such as, profit and loss, commission and brokerage, exchange, trade discount, simple interest,

taxes, insurance, and stocks and bonds. These are not presented as so many different departments in the work of arithmetic, but simply as so many applications of the percentage idea.

We have called attention to but one section of the book by way of illustration. A reference to others will show a consistent working out of the same plan. It may be new, but it has been tried and proved.

P. McINTOSH.

C. A. NORMAN.



*Love Walker*

## CONTENTS

### SIMPLE RULES

	Page		Page
Definitions . . . . .	5	Complementary Numbers . . . . .	15
Notation and Numeration . . . . .	5	Multiplication . . . . .	19
Arabic Notation . . . . .	6	Division . . . . .	23
Roman Notation . . . . .	8	Cancellation . . . . .	31
Addition . . . . .	9	Combinations of Processes . . . . .	33
Subtraction . . . . .	14	Methods for Proving Work . . . . .	34

### FACTORS AND MULTIPLES

Factors . . . . .	38	Multiples . . . . .	44
Highest Common Factor . . . . .	41	Lowest Common Multiple . . . . .	44

### COMMON OR VULGAR FRACTIONS

Definitions . . . . .	47	Addition . . . . .	53
Reduction of Fractions . . . . .	48	Subtraction . . . . .	54
Reduction Ascending . . . . .	50	Multiplication . . . . .	56
Reduction Descending . . . . .	51	Division . . . . .	58
Reduction to Common Denominator . . . . .	52	Review of Fractions . . . . .	60
		Sharing . . . . .	66

### DECIMAL FRACTIONS

Definitions . . . . .	72	Multiplication . . . . .	80
Numeration . . . . .	73	Division . . . . .	81
Notation . . . . .	74	Repeating or Circulating Decimals . . . . .	82
Reduction . . . . .	75	Review . . . . .	84
Addition . . . . .	78		
Subtraction . . . . .	79		

### DENOMINATE NUMBERS

Tables of Values, Weights and Measures . . . . .	88	Subtraction . . . . .	103
Reduction . . . . .	97	Multiplication . . . . .	104
Addition . . . . .	101	Division . . . . .	105

### INVOLUTION AND EVOLUTION

Square Root . . . . .	108	Cube Root . . . . .	113
-----------------------	-----	---------------------	-----

### PRACTICAL MENSURATION

Surfaces . . . . .	117	Miscellaneous . . . . .	124
Solids . . . . .	122		

### PRACTICAL MEASUREMENTS

Carpeting . . . . .	134	Roofing . . . . .	141
Papering . . . . .	135	Lathing and Plastering . . . . .	143
Lumber . . . . .	138	Brick and Stone Work . . . . .	144

## PERCENTAGE

PROFIT AND LOSS, TRADE DISCOUNT, COMMISSION AND BROKERAGE, INSURANCE,  
TAXES, DUTIES, EXCHANGE, STOCKS, INTEREST

	Page		Page
Terms Used . . . . .	147	Questions of the Third Aspect .	171
Basis of Calculation . . . . .	153	Questions of the Fourth Aspect	176
Questions of the First Aspect .	154	Review . . . . .	182
Questions of the Second Aspect	166		

## SHORT METHODS

In Multiplication . . . . .	201	In Percentage . . . . .	211
In Division . . . . .	205	Billing . . . . .	218
In Decimals . . . . .	206	Cash Storage Bills . . . . .	233

## APPLICATIONS OF SIMPLE INTEREST

Negotiable Papers . . . . .	236	Averaging Accounts . . . . .	271
Bank Discount . . . . .	245	Averaging Account Sales . . . . .	277
True Discount . . . . .	249	Interest on Partners' Accounts	281
Partial Payments . . . . .	257	Accounts with Banks . . . . .	284
Cash Balance . . . . .	263	Accounts with Stock Brokers .	288
Equation of Payments . . . . .	269		

## COMPOUND INTEREST

Questions of the First Aspect .	292	Questions of the Third Aspect	297
Questions of the Second Aspect	296	Questions of the Fourth Aspect	298

## ANNUITIES

Questions of the First Aspect .	301	Questions of the Third Aspect .	306
Questions of the Second Aspect	304		

## FOREIGN TRADE

Bills of Exchange . . . . .	310	Indirect or Circuitous Exchange	317
Direct Exchange . . . . .	314	Custom House Business . . . . .	318

## AVERAGE AND ITS APPLICATIONS

## ALLIGATION, STORAGE, INVESTMENTS

Questions of the First Aspect .	325	Questions of the Second Aspect	331
---------------------------------	-----	--------------------------------	-----

## PARTNERSHIP

Definitions . . . . .	336	Adjustments . . . . .	337
-----------------------	-----	-----------------------	-----

## APPENDIX

The Metric System . . . . .	347
-----------------------------	-----





# NEW METHOD ARITHMETIC

## SIMPLE RULES

### DEFINITIONS

**Arithmetic** is the science of number. It teaches how numbers may be applied to produce required results.

A **number** is the expression of a quantity by means of characters or figures.

The idea of number is present in every sound human mind. Every time the question, "How many?" is asked concerning a collection of objects of the same kind, a knowledge of number is signified and inferred.

The **unit of a number** is one of the things which it expresses. Thus, in the expression, 15 yards, the unit is one yard; in counting the students in a class, the unit is one student; in selling brick by the thousand, the unit is one thousand brick, etc.

A **concrete number** is a number associated with some particular thing or quantity; as 5 books, 10 men, 7 cords, etc.

An **abstract number** is a number not associated with any particular thing or quantity; as 2, 7, 10, etc.

The **fundamental operations** of Arithmetic are: Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

### NOTATION AND NUMERATION

**Notation** is the art of writing numbers.

**Numeration** is the art of orally expressing numbers when expressed by figures.

Numbers are written in three ways:

1. By words; as, one hundred, twenty-five.
2. By figures, called the Arabic Method; as, 125.
3. By letters, called the Roman Method; as, CXXV.

## ARABIC NOTATION

This system received its name from the fact that it was introduced into Europe by the Arabs. It employs ten figures or characters by which numbers are represented.

<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>0</b>
One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Zero or Naught.

1, 2, 3, 4, 5, 6, 7, 8, 9, are significant figures. They represent value.

The 0 is an insignificant figure. It expresses no value, unless used with another figure.

The first nine numbers are written with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9. The other numbers are expressed by combining two or more figures. Thus, the number seven is written by using the figure 7, while seventeen is expressed by combining 1 and 7—17.

From this illustration, we see that the figure 1, when used with the figure 7 to express 17, really represents ten. If we reverse the order of the figures and write 71, the figure 1 has a value of one, while the 7 has a value of seventy.

The **Simple Value** of a figure is the value which it expresses when standing alone, or in the first place to the right of a whole number.

The **Local Value** of a figure is the value which it has when associated with other figures in expressing a number.

The place which a figure occupies in a number is called its order. Thus, if we set down the figures 1 1 1 1 we know that while the number consists of four 1's there is a significant value given to each figure according to its position. Starting at the right and going to the left we have one unit, one ten, one hundred, one thousand. In reading this number we begin at the left and read to the right as follows: One thousand, one hundred, eleven (ten and one). Even this reading is shortened in business to: Eleven hundred, eleven.

When there are four or more figures in a number it is divided into periods of three figures each, beginning at the right side. The



following table will show the order of the units and the names of the periods, for a reasonable distance up the scale :

NUMERATION TABLE

Periods.	6th			5th			4th			3rd			2nd			1st		
Name	Quadrillions.			Trillions			Billions.			Millions.			Thousands.			Units.		
Orders of units in the Periods.	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
Number.	3	7	4	0	6	5	2	3	0	7	1	3	0	0	3	8	4	

The number written below the table would be read as follows :

37 quadrillions, 406 trillions, 523 billions, 71 millions, 300 thousand, 3 hundred, eighty-four.

SERIES 1

Express by written words, the following numbers :

- |                |                     |                    |
|----------------|---------------------|--------------------|
| 1. 59285.      | 8. 9127438.         | 15. 1900107500.    |
| 2. 1718219.    | 9. 5375816253.      | 16. 400034.        |
| 3. 745364.     | 10. 29285.          | 17. 2900107.       |
| 4. 39219283.   | 11. 92162715208508. | 18. 9800001800070. |
| 5. 65175.      | 12. 4000273001.     | 19. 31768290005.   |
| 6. 8721293592. | 13. 180005107.      | 20. 2831000.       |
| 7. 2195629837. | 14. 8000000026.     | 21. 91000146.      |

SERIES 2

Express, by means of figures, the following numbers :

- Five thousand, two hundred and forty-seven.
- Sixty-five thousand, eight hundred and seventy-one.
- Eight thousand and fifty-three.

4. Twelve thousand and nine.
5. Ninety thousand and thirty-six.
6. Two hundred and eight thousand, and ninety-four.
7. Six million, two hundred and three thousand, and twenty-five.
8. Nine million, thirty-seven thousand, five hundred and forty.
9. Two million, five thousand, seven hundred and six.
10. Five million, two hundred and eight thousand, and thirty-two.
11. Six billion, ninety-five thousand, one hundred and forty-eight.
12. Nine billion, forty-three million, and seven thousand.
13. Ninety-six trillion, thirty-five thousand and nine.
14. Fifteen trillion, seven hundred and thirty-nine billion, fifty million, one hundred and ninety thousand, and seventy-six.
15. Twelve trillion, six hundred and twenty-nine million, two hundred and twelve thousand, five hundred and sixty-one.
16. Ninety-two quadrillion, eight trillion, two hundred and four thousand, one hundred and forty.

### ROMAN NOTATION

The **Roman Method of Notation** uses letters instead of figures to express numbers. It employs seven characters or letters, as follows :

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

As in the Arabic Notation, each character has a definite value when standing alone, as noted, and a varying value when written in certain positions in combination with the other characters. The principles which govern the use of these characters in expressing numbers are as follows :

1. Repeating a letter repeats its value.

Thus, X represents 10, XX represents 20, XXX represents 30.



2. When a letter of less value is placed before one of greater value, the number expressed is the difference between the values of such numbers.

Thus, IX represents 9, XC represents 90.

3. When a letter of less value is placed after one of greater value, the number indicated is the sum of the values of such letters.

Thus, CX represents 110.

4. A bar placed over a letter multiplies the letter by one thousand.

Thus,  $\overline{V}$  represents five thousand ;  $\overline{C}$ , one hundred thousand.

5. A letter should not be repeated more than three times in expressing numbers.

6. A bar is never placed over the letter I.

### COMPARATIVE NOTATION TABLE

Roman.	Words.	Arabic.	Roman.	Words.	Arabic.	Roman.	Words.	Arabic.
I	One	1	XI	Eleven	11	XXX	Thirty	30
II	Two	2	XII	Twelve	12	XL	Forty	40
III	Three	3	XIII	Thirteen	13	L	Fifty	50
IV	Four	4	XIV	Fourteen	14	LX	Sixty	60
V	Five	5	XV	Fifteen	15	LXX	Seventy	70
VI	Six	6	XVI	Sixteen	16	LXXX	Eighty	80
VII	Seven	7	XVII	Seventeen	17	XC	Ninety	90
VIII	Eight	8	XVIII	Eighteen	18	C	One Hundred	100
IX	Nine	9	XIX	Nineteen	19	D	Five Hundred	500
X	Ten	10	XX	Twenty	20	M	One Thousand	1000

### ADDITION

**Addition** is the process of uniting into one number all the units contained in two or more other numbers.

The number obtained by addition is called the *sum*, *amount*, or *aggregate*.

The numbers to be added are called *addends*, or *parts*.

The sign  $+$  signifies *addition*, and is read *plus*.

The sign  $=$  signifies *equality*, and is read *equals*.

Like numbers express the same kind of units ; as, 5 apples and 2 apples.

Unlike numbers express different kinds of units ; as, 5 apples and 2 chairs.

Like numbers only can be added.

If unlike numbers can be made like, they may be added.

For instance, we may add 6 tons and 6 pounds by first finding how many pounds there are in 6 tons.

ILLUSTRATION.—Add 456, 5187, 45, 278, and 7302.

456 5187 45 278 7302 —— 13268	SOLUTION.—First write the numbers, placing units of the same order in the same column, units under units, tens under tens, etc.  Then add the right-hand column, $2 + 8 + 5 + 7 + 6 = 28$ . Place the right-hand figure, 8 (units), under the column added and carry the 2 (tens) or left-hand figure, to the next column. The tens' column added in like manner, $7 + 4 + 8 + 5$ , and the 2 tens from units' column, will produce 26. Write the 6, or right-hand figure, under the tens' column and carry the 2, or left-hand figure, to the next column.
---	--

The hundreds' column,  $3 + 2 + 1 + 4$ , and the 2 hundreds' from the tens' column,  $= 12$ , or 1 thousand and 2 hundreds.

Write the 2 hundreds under the hundreds' column and carry the 1 thousand to the thousands' column.

The thousands' column,  $5 + 7$ , and the 1 thousand from hundreds' column  $= 13$ , which written under the thousands' column, completes the work and gives 13268, the sum.

SUGGESTION.—It will save much valuable time to name results only in adding. Instead of saying 3 and 3 are 6 and 4 are 10, etc., name the sum only, as, 6, 10, etc. In the above example, begin at the bottom of units' column and add upward—10, 15, 22, 28. Write 8 under units and carry 2 to the tens—always adding the carrying figure first.

A very good plan to follow, when the question to be added is a large one, is to set the sum of the various columns down in the manner illustrated below and then add these totals together. Then, the necessity of remembering the carrying figure is removed ; and if one is interrupted while adding



a question, he will be saved the trouble of adding the whole question again to find the carrying figure. Thus, in the question above :

$$\begin{array}{r} 28 \\ 24 \\ 10 \\ 12 \\ \hline 13268 \end{array}$$

RULE

1. Write the numbers, placing units of the same order in the same column.

2. Find the sum of units' column first and place the right-hand figure of this sum under this column and carry the left-hand figure, or figures, to the next column.

3. Act upon the remaining columns in regular order and in a similar manner, always placing the right-hand figure of each sum under the column from which the sum was obtained, and carry the left-hand figure, or figures, if any, to the next column.

4. Under the last column write the last amount.

SERIES 3

Add the following :

1. 4	2. 3	3. 34	4. 46	5. 346	6. 635
2	6	51	63	572	853
3	5	25	38	465	674
1	6	32	57	537	359
3	1	43	82	724	482
5	4	14	74	316	375
—	—	—	—	—	—
7. 4751	8. 8267	9. 3675	10. 6435	11. 78643	12. 34625
6958	5483	4592	7582	85972	87394
8527	3796	8147	4617	43289	51786
3489	5864	6328	3874	76541	46937
5476	4394	7465	6293	63748	84394
3547	8936	3726	7436	62862	68479
4368	7453	4857	3548	26875	75648
5936	2718	8395	6987	49386	49576
—	—	—	—	—	—

13. A grocer's sales were as follows:—Monday, \$67.25; Tuesday, \$79.37; Wednesday, \$58.71; Thursday, \$64.18; Friday, \$86.54; Saturday, \$136.19. What were his total sales for the week?

14. I lost \$1,725 by selling a farm for \$5,465. What did the farm cost me?

15. Eight hogsheds of tobacco weigh respectively 735, 846, 794, 816, 768, 857, 783, and 837 pounds. Find the total weight.

16. A grocer's shop is worth \$1,980, and his goods are worth \$765 more than the shop. What is the value of both?

17. In a certain saw mill 98,368 feet of lumber were cut on Monday; 113,643 feet on Tuesday; 108,581 feet on Wednesday; 86,572 feet on Thursday; 126,416 feet on Friday, and 118,529 feet on Saturday. What was the week's "cut" of the mill?

18. A man bought four farms. For the first he paid \$5,870; for the second, \$4,925; for the third, \$865 more than for the first, and for the fourth, \$1,385 more than for the second. What did the four farms cost him?

19. What is the sum of five hundred and ninety-one dollars, two thousand three hundred and eight dollars, sixty-seven dollars, and nineteen thousand one hundred dollars?

20. Find the total distance around a rectangular field, 1,629 feet long and 1,574 feet wide.

21. A farmer sent to a cheese factory 1,587 pounds of milk in March; 2,316 in April; 2,765 in May; 3,867 in June; 4,634 in July; 4,328 in August; 3,529 in September; 2,714 in October; and 1,052 in November. How much milk did he send to the factory that season?

22. A man bought a lot for \$1,375, paid \$408 to have it graded, \$75 to have it enclosed, and afterward sold it at a profit of \$238. How much did he receive for the lot?

23. In a certain city there are 68,529 Americans, 25,468 Irish, 7,493 Germans, 836 French, and 7,382 of other nationalities. What is the population of the city?

Complete the following tables by showing the totals of the columns, vertically and horizontally. Prove the work by adding the vertical and horizontal totals.



**24. DEPARTMENTAL SALES FOR THE WEEK ENDING  
NOVEMBER 15, 1906**

DAYS	CLOTHING	DRY GOODS	FURNISH- INGS	MILLINERY	HOUSEHOLD UTENSILS	TOTALS
Monday	\$790 50	\$988 40	\$126 50	\$256 85	\$496 80	
Tuesday	640 18	890 50	90 18	420 62	841 62	
Wednesday	960 70	950 40	75 60	398 40	462 50	
Thursday	490 18	960 80	214 90	425 60	521 90	
Friday	930 50	720 50	126 70	396 80	762 80	
Saturday	840 15	989 72	215 20	459 65	925 54	
Totals						

**25. MONTHLY AND YEARLY STATEMENT**

MONTH	1900	1901	1902	1903	1904	TOTALS
January	124832	345325	784395	528349	112233	
February	728941	167832	286948	956873	668734	
March	325768	589435	758586	424632	297865	
April	924876	654321	321476	594657	192843	
May	543768	234567	592763	294632	765345	
June	928328	891234	198725	528647	294763	
July	764732	567895	539876	328943	348694	
August	654924	438927	247632	294742	728643	
September	628328	624932	586529	486532	397544	
October	784623	864743	738564	674384	867347	
November	398472	157635	328924	586432	247689	
December	153287	594765	895855	975683	348789	
Totals						

**26. GRAIN EXPORT OF A CITY FOR ONE WEEK  
(in bushels)**

	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTALS
Corn	28325	15236	35715	29128	75183	46217	
Wheat	35719	41719	50108	32546	59275	81126	
Oats	12136	9237	18265	7268	6950	17230	
Barley	18230	15738	21375	15928	19263	13637	
Rye	5275	6829	7201	11325	7825	13261	
Totals							

## SCHOOL ATTENDANCE

SCHOOL	SEPT.	OCT.	NOV.	DEC.	JAN.	FEB.	MAR.	APR.	MAY	JUNE	TOTAL
1st Ward	1126	1019	987	928	1097	1065	989	1074	994	917	
2nd "	1049	984	946	898	1036	978	943	960	917	889	
3rd "	876	807	758	716	847	827	810	864	793	719	
4th "	948	892	868	807	849	808	796	757	714	658	
5th "	1075	1043	985	924	1019	981	967	1007	947	939	
6th "	829	757	749	717	837	806	795	801	824	783	
7th "	743	704	685	612	728	694	709	716	697	651	
8th "	843	807	784	757	838	816	824	775	786	727	
Totals											

## SUBTRACTION

Subtraction is the process of finding the difference between two numbers.

The **Minuend** is the larger number, the one from which the other is to be taken.

The **Subtrahend** is the smaller number, the number to be subtracted.

The **Remainder** or **Difference** is the result obtained by subtraction.

The sign  $-$  signifies *subtraction*, and is read *minus*, or *less*.

Only like numbers can be subtracted.

**ILLUSTRATION.**—Find the difference between 1235 and 858.

**SOLUTION.**—Write the subtrahend, the number to be subtracted, under the minuend, the number to be diminished, with units of the same order under each other. Commence with ones, and, as 8 ones cannot be taken from 5 ones, take 1 ten from 3 tens and add it to the 5 ones. 1 ten + 5 ones = 15 ones.

8 ones from 15 ones leaves 7 ones. Write down the 7 ones. As 5 tens cannot be taken from 2 tens (3—1) change one hundred to tens and add to the 2 tens. 1 hundred + 2 tens = 12 tens. 5 tens from 12 tens leaves 7 tens. Write down the 7 tens and subtract 8 hundred from 11 hundred,

which leaves 3 hundred. Hence the answer, 3 hundreds, 7 tens, and 7 ones, or 377 ones, or 377.

This process is called "borrowing tens." Having mastered the theory, the ordinary and most convenient method for practice is to leave the minuend in its original form, and, when borrowing is necessary, add 1 to the succeeding subtrahend figure.

#### RULE

1. Write the subtrahend under the minuend, placing units of the same order in the same column. Draw a line beneath.
2. Begin with units and take each figure of the subtrahend from the figure of the minuend just above it, and write the remainder beneath.
3. If any figure of the minuend is smaller than the corresponding figure of the subtrahend, add 10 to that figure of the minuend and then subtract, after which take 1 from the next figure of the minuend and proceed as before.

#### COMPLEMENTARY NUMBERS

**Complementary Numbers** are any two numbers whose sum is equal to a unit of the next higher order.

Thus, 6 and 4, 7 and 3, 8 and 2, etc., are complementary numbers, as the sum of each of these pairs is 10 ; and 36 and 64, 27 and 73, 42 and 58, are complementary, as the sum of each pair is 100.

A number is said to be the complement of another when the sum of the two is a unit of the next higher order ; thus, 7 is the complement of 3, 26 is the complement of 74, and 364 is the complement of 636.

In all complementary numbers of more than one figure, the sum of the unit figures is 10, while that of the other corresponding orders is 9.

EXAMPLE :	4632	247
	5368	753
	<hr/>	<hr/>
	999 <sub>10</sub> = 10000	99 <sub>10</sub> = 1000

By applying the foregoing principle, a little practice will enable one to name, at sight, the complement of any number. The ability to do this is very useful in the business office in making change, etc.



## SERIES 4

1. Write the complements of the following numbers : 36, 26c, 64, 73, 28c, 83, 42, 34, 65c, 48, \$26, 36, 57, \$21, 32, 63, 76, 19, 83, 55, 22, 31, 43, 82, 53.

2. Write the complements of the following : 125, 236, 328, 475, 643, 764, 238, 753, 146, 321, 816, 458, 735, 593, 637.

3. In case a ten-dollar bill is offered in payment, write the amount of change required for each of the following amounts : \$2.75, \$1.45, \$3.56, \$8.27, \$3.42, \$6.29, \$4.53, \$2.35, \$5.75, \$8.21, \$6.15, \$4.38, \$7.23, \$2.91, \$3.54, \$2.74, \$8.75, \$4.18, \$3.37, \$2.85

In each of the following problems, find the change required by subtracting the sum of the items from the sum paid. Where possible, find the result mentally.

ITEMS	PAID	ITEMS	PAID
4. 15c, 12c, 50c,	\$1	11. 75c, 24c, 47c,	\$2
5. 24c, 36c, 18c,	\$1	12. \$1.25, 60c, 22c,	\$5
6. 45c, 15c, 36c,	\$2	13. \$3.25, 50c, \$1.25,	\$5
7. 21c, 18c, 12c,	\$1	14. \$4.60, \$2.25, \$1.20	\$10
8. 45c, 55c, 48c,	\$2	15. \$1.20, \$1.50, \$2,	\$5
9. 23c, 34c, 25c,	\$1	16. \$3.60, \$4, 55c,	\$10
10. 63c, 8c, 4c,	\$1	17. \$4.25, \$5.50, \$8.20,	\$20

## SERIES 5

Subtract the following :

1. 68549	2. 97568	3. 235746	4. 329145
34236	23415	184693	235648
<hr/>	<hr/>	<hr/>	<hr/>
5. 438649	6. 870215	7. 824316	8. 354182
183756	158634	257948	173659
<hr/>	<hr/>	<hr/>	<hr/>
9. 837546	10. 45020000	11. 72006398	12. 32405708
379285	18367459	37587439	16578345
<hr/>	<hr/>	<hr/>	<hr/>

13. If a man receives a salary of \$950 a year and spends \$568, how much will he save per year ?

14. What number must be added to 89,467 to give a result of 113,405 ?

15. From a carload of coal containing 53,247 lbs., 34,865 lbs. have been removed. How much coal remains in the car ?

16. What number must be subtracted from 86,023 to leave 19,362 as remainder ?

17. A man deposited \$19,075 in a bank and afterwards withdrew \$5,129. How much had he remaining in the bank ?

18. The selling price of a business was \$346,238.75 and \$197,354.50 was paid on it. What amount remains unpaid ?

19. I sold a farm for \$8,625, which was \$1,950 more than I paid for it. What did the farm cost ?

20. A lumber company cut 55,326,436 feet of lumber in 1905 and 60,483,379 feet in 1906. By how much was the capacity of the mill increased during the year ?

21. A speculator bought real estate for \$27,368.47 and sold it at a loss of \$697.89. For how much did he sell it ?

22. The area of the United States is 3,602,990 square miles, and the area of Europe is 3,928,252 square miles. How much larger is Europe than the United States ?

23. On July 1st Smith & Brown's bank balance amounted to \$17,683.45. On August 1st their balance was \$21,356.30. What was the amount of the increase in their bank account during July ?

24. In 1890 the unsold government land in one of the Western States was 526,516 acres. In 1900 the unsold land in the same State was 359,879 acres. What number of acres was sold during the decade ?

25. What is my total indebtedness if I owe A \$645, B \$534, C \$465 and D \$238 ?

26. I owe \$1,635 to A, B, C and D. If I owe A \$365, B \$580, and C \$439, how much do I owe D ?

27. Brown, Jones, Todd and Smith enter into partnership. Brown invests \$1,650; Jones invests \$375 more than Brown; Todd as much as Brown and Jones together; and Smith \$1,150 less than Jones and Todd together. What is the total capital of the firm ?

28. A gentleman bought five houses for \$28,350, paying \$3,800 for the first, \$4,500 for the second, \$5,300 for the third, and \$6,975 for the fourth. What did he pay for the fifth house ?

29. A man bought a lot for \$1,925, paid \$425 to have it graded, \$75 to have it enclosed, and then sold it at a loss of \$275. How much did he receive for the lot ?

30. I bought a house for \$4,850, spent \$756 for repairs, and then sold it for \$5,320. How much did I lose ?

31. I lost \$20.15 by selling a cow for \$13.85. What was the real value of the cow ?

32. My bookkeeper's salary is \$1,450 per year. If he requires, for his rent, \$365 ; for personal expenses, \$170 ; and for the support of his family, \$775 ; what amount will he have left at the end of the year ?

33. A bank began business with \$500,000 in cash. During the first year it gained \$25,683.19, but lost \$42,896.36 during the second year. How much did it have at the end of the second year ?

34. The difference between two numbers is 17,896 and the smaller is 46,798. What is the larger ?

35. On November 1, Fay & Brookings' balance at the Mercantile Bank was \$947.60. On November 10, they deposited \$256.75, and on November 27, \$425.83. During the month they withdrew on their cheques \$1,159.25. What was their bank balance on December 1 ?

36. A ranchman being asked how many cattle he had, replied : " If you give me 67 and I buy 189, then I can sell 56 and have 258 left." How many had he ?

37. The remainder is 2,175 and the subtrahend 1,216. What is the minuend ?

38. The cost of my lot was \$1,750. I paid for mason work on my house, \$1,210 ; for carpenter work, \$5,145 ; for plumbing, \$985 ; for decorating, \$1,650 ; for painting, \$625 ; for grading, sodding and fencing grounds, \$590. The interest on outlays to date of sale was \$315. I then sold the property at a gain of \$1,250, receiving cash \$6,000, and a note for the remainder. What was the face of the note ?



39. Which of the two numbers, 78,356 and 26,984 is nearer to 47,215, and how much ?

40. From one hundred and twenty-seven million, six thousand and four, take ninety-nine million, two hundred and eight thousand six hundred and nine.

41. A merchant's receipts for the first half of a year were \$8,275. His receipts in January were \$825 ; in February, \$946 ; in March, \$1,050 ; in April, \$925 ; and in May, \$1,536. What were his receipts in June ?

42. In the year 1907 a gentleman was 89 years old. In what year was he born ?

43. A merchant's income in 1906 was as follows : Wages \$600, rent of house \$325, and interest on funds in a savings bank \$42. His expenditures were : butcher's bill \$130, baker's \$75, grocer's \$264, insurance and taxes \$58, clothing \$130, and other expenses \$70. How much did he save during that year ?

## MULTIPLICATION

**Multiplication** is a short process of finding the sum when the several addends are equal.

The **Multiplicand** is the number to be taken or multiplied. It is one of the equal addends.

The **Multiplier** is the number showing how many times the multiplicand is to be taken or added.

The **Product** is the result obtained by multiplication.

The sign  $\times$  signifies *multiplication*, and is read, *times*, or *multiplied by*.

The multiplicand may be either abstract or concrete, but the multiplier is always abstract.

The product has always the same unit as the multiplicand.

The **Factors of a Number** are the numbers whose product equals the given number ; thus, 7 and 3 are factors of 21, because 7 and 3 multiplied together equal 21.

## MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Call the figures in the column at the left the multipliers, and those arranged horizontally at the top or bottom the multiplicands. The product of any two factors will be found in the line to the right of the multiplier and under or over the figure multiplied. For illustration, the product of 20 multiplied by 20 (400), may be found by following the black lines to the point where they meet.

ILLUSTRATION 1.—Find the product of  $2416 \times 7$ .

SOLUTION.—Write the multiplier 7 below the unit figure of the multiplicand as shown in the margin, and begin at the right to multiply. 7 times 6 units equals 42 units, or 4 tens and 2 units. Write 2 units in the place of units, and reserve 4 tens to add to the product of tens. 7 times 1 ten equals 7 tens, and adding the 4 tens reserved gives 11 tens, or 1 hundred and 1 ten. Write 1 in the place of tens, and reserve 1 to add to the product of hundreds. 7 times 4 hundreds equals 28 hundreds, and adding the 1 hundred reserved gives 29 hundreds, or 2 thousands and 9 hundreds. Write 9 in the place of hundreds, and reserve 2 to add to the product of thousands. 7 times 2 thousands equals 14 thousands, and adding the 2 thousands

reserved gives 16 thousands. Write this entire sum to the left of the figures already written, thus completing the multiplication and obtaining as a result 16,912.

**ILLUSTRATION 2.**—Find the product of  $417 \times 356$ .

**SOLUTION.**—Write the multiplier below the multiplicand in the same unit order from the right. The multiplier is composed of 6 units, 5 tens, and 3 hundreds, or  $6 + 50 + 300$ . Multiplying the multiplicand by 6, 50, and 300 respectively, and adding the products, the results are as follows :

(a) FULL PARTIAL PRODUCTS.      (b) ABBREVIATED PARTIAL PRODUCTS.

417		417
356		356
<hr/>		
2502	Partial Product by 6	2502
20850	Partial Product by 50	2085
125100	Partial Product by 300	1251
<hr/>		
148452	Complete Product.	148452

The ciphers at the right of the partial products are of no value in finding the complete product, and they should be omitted in practice as shown in (b). Observe that the first right-hand figure of each partial product is then always directly under the figure of the multiplier used.

**RULE**

1. *Write the multiplicand with the multiplier under it, placing units under units, tens under tens, etc.*
2. *Multiply each figure of the multiplicand by each figure of the multiplier successively, beginning with units, always putting the right-hand figure of each partial product under that figure of the multiplier used in producing it.*
3. *The sum of the partial products thus obtained will be the required product.*

**SERIES 6**

Multiply

- |                 |                      |
|-----------------|----------------------|
| 1. 34658 by 2.  | 10. 59638 by 47.     |
| 2. 74639 by 3.  | 11. 38279 by 79.     |
| 3. 259376 by 4. | 12. 4856 by 378.     |
| 4. 938647 by 5. | 13. 8579 by 695.     |
| 5. 385916 by 6. | 14. 36847 by 836.    |
| 6. 486725 by 7. | 15. 783685 by 7984.  |
| 7. 835497 by 8. | 16. 453179 by 63597. |
| 8. 675382 by 9. | 17. 349628 by 74865. |
| 9. 4637 by 32.  | 18. 936754 by 87596. |



19. The net cost of making a certain book was found to be 59 cents per copy. What was the net cost of 24 such books ?

20. Find the value of 267 tons of coal at \$4.75 per ton.

21. In a cornfield there are 469 rows and 528 hills in each row. How many hills of corn are there in the field ?

22. There are 320 rods in one mile ; how many rods are there in 129 miles ?

23. Scott & Co., contractors, employ 75 men at wages averaging \$1.75 per day. What is the amount of the pay-roll for one month of 26 working days ?

24. An orchard consists of 27 rows of 35 apple trees each. If the trees bear on an average 18 bushels each, how many bushels will the orchard yield ?

25. A bookkeeper's salary is \$95 per month, and his expenses are \$46 per month. How much will he save in 4 years ?

26. A man bought a span of horses for \$275. He hired them out at \$12 per week, but paid \$5.25 per week for their keep. At the end of 7 weeks he sold them for \$250. How much did he gain or lose on them ?

27. Find the cost of building 329 miles of railway at \$34,586 per mile.

28. A man bought 475 acres of land at \$48 per acre, sold 160 acres at \$55 per acre, 225 acres at \$42 per acre, and the remainder at \$62 per acre. What was his total gain ?

29. What will 87 crates containing 105 dozen oranges each cost at 3 cents each ?

30. The cost of the Atlantic Telegraph Cable as originally made was as follows : 2,500 miles at \$485 per mile, 10 miles deep sea cable at \$1,450 per mile, and 25 miles shore ends at \$1,250 per mile. What was the total cost ?

31. A farmer exchanged 754 bushels of wheat at \$1.25 per bushel, for 78 barrels of flour at \$2 per barrel, and received the balance in money. How much money did he receive ?

32. A merchant bought 240 barrels of flour for \$1,920, and sold it at \$10.50 per barrel. What did he gain ?

33. A certain building contains 74 windows, each window containing 8 panes of glass. Find the cost of the glass at 14 cents per pane.

34. A man deposited in a bank \$8,752; he drew out at one time \$4,234; at another time, \$1,700; at another, \$862; at another, \$49. How much had he remaining in the bank?

35. A dealer gained \$75.90 on 55 tons of baled hay, which he bought at \$7.75 per ton. For what sum did he sell it?

## DIVISION

**Division** is the process of finding how many times one number is contained in another number, or how many times it must be taken as an addend to produce the number.

One of the numbers is called the *dividend* and the other the *divisor*.

The **Dividend** is the number divided.

The **Divisor** is the number by which the dividend is divided.

The **Quotient** is the number which shows how many times the dividend contains the divisor; or,

The number obtained by dividing; or,

The number showing how many times the divisor must be taken as an addend to produce the number.

The **Remainder** is the part remaining when the division is not exact.

The **Sign of Division** is the character  $\div$ ; it indicates that the number placed before it is to be divided by the number after it. Thus,  $24 \div 3 = 8$ , is read, 24 divided by 3 equals 8.

Division is sometimes indicated by writing the divisor under the dividend and drawing a horizontal line between them; as  $\frac{24}{3}$ ; or, by placing the divisor on the left of the dividend with a curved line between them; thus  $3)24$ . These ways of indicating division are read alike and require the same operation to be performed,

A number is said to be divisible by another number when the latter will divide the former without a remainder.

An **Even Number** is a number of which 2 is an exact divisor.

An **Odd Number** is a number of which 2 is not an exact divisor.

### SHORT DIVISION

In **Short Division** the operations are performed mentally and are restricted to those divisions in which the divisor consists of one figure, or is a number coming within one's thorough knowledge of the multiplication table.

ILLUSTRATION 1.—Divide 8462 by 2.

2)8462 —— 4231	SOLUTION.—Write the divisor at the left of the dividend, separating them by a line ; next draw a line below the dividend and then divide each figure of the dividend by the divisor, writing the quotient below the figure divided. Thus, 2 is contained in 8 thousands, 4 thousand times ; write the 4 below the 8 in thousands' column ; next, 2 is contained in 4 hundreds, 2 hundred times ; place the 2 below the 4 in hundreds' column ; 2 is contained in 6 tens, 3 tens times ; write the quotient in tens' column ; 2 is contained in 2 ones, 1 time, or once ; write one in ones' place, thus completing the division, and obtaining 4231 as a quotient.
----------------------	--

ILLUSTRATION 2.—Divide 936 by 6.

6)936 —— 156	SOLUTION.—9 (hundreds) $\div$ 6 = 1 (hundred) and 3 (hundred) over. Write this partial quotient (1) under the order divided, carry the remainder (3 hundreds or 30 tens) to the next order of the dividend (3 tens), and divide the result (33 tens) by 6, obtaining 5 (tens) as the next quotient figure, and 3 (tens) as the next remainder. Carry the last remainder (3 tens or 30 units) to the next order of the dividend (6 units), and divide the result (36 units) by 6, obtaining 6 units as the last quotient figure.
--------------------	---

### LONG DIVISION

When the divisor is a number larger than can be treated mentally the following method, called Long Division, is employed :



ILLUSTRATION.—Divide 81449 by 37.

37)81449(2201 $\frac{12}{37}$

74

74

74

49

37

—

12 Remainder.

SOLUTION.—Write the terms as in short division, and place a line after the dividend to separate it from the quotient, which is now to be written at the right. Then divide the first two figures of the dividend, 81, by the divisor, 37, and obtain 2 as the first figure of the quotient; then subtract from 81 the product of  $2 \times 37$ , or 74, obtaining 7 as a remainder; to this remainder annex 4, the succeeding figure of the dividend; which gives 74 as the next partial dividend; the divisor is contained in this dividend twice,

or 2 times, giving 2 as the next or second quotient figure; subtracting the product of  $2 \times 37$  from 74, nothing remains; then bring down 4, the next figure of the dividend, and as it is less than the divisor, place a 0 in the quotient; next bring down 9, the remaining figure of the dividend, which gives 49 as the last partial dividend; the divisor is contained in this dividend once, or 1 time; writing this 1 in the quotient and subtracting the last partial product from the last partial dividend, 12 remains; write this remainder in the form of a fraction at the right of the other figures in the quotient, and  $2201\frac{12}{37}$  is the result of dividing 81449 by 37.

### Italian Method

By the **Italian Method** of dividing, the divisor is placed on the right of the dividend, and the quotient immediately beneath it. The advantage in this is that the numbers concerned in the operation are closer together.

To illustrate      28260(36  
                         30   785  
                         18

### SERIES 7

- |                             |                               |
|-----------------------------|-------------------------------|
| 1. Divide 1978 by 7.        | 11. $21786 \div 19 = ?$       |
| 2. Divide 8976 by 6.        | 12. $87463 \div 28 = ?$       |
| 3. Divide 10274 by 8.       | 13. $10271086 \div 146 = ?$   |
| 4. Divide 7861247 by 4.     | 14. $94207658 \div 67 = ?$    |
| 5. Divide 20761201 by 5.    | 15. $560217563 \div 496 = ?$  |
| 6. Divide 12172946 by 9.    | 16. $85205617 \div 649 = ?$   |
| 7. Divide \$8673.57 by 3.   | 17. $417601924 \div 4567 = ?$ |
| 8. Divide \$407232.45 by 9. | 18. $93681596 \div 837 = ?$   |
| 9. Divide 9706421 by 11.    | 19. $170215862 \div 299 = ?$  |
| 10. Divide 201634596 by 12. | 20. $37021675 \div 7019 = ?$  |

21. If a train moves at an average rate of 24 miles per hour, how long will it require to go 336 miles ?

22. By a rise of 25 cents per bushel in the market price, the value of a farmer's crop is increased \$198.75. How many bushels did he raise ?

23. The product of two numbers is 40,548. One of the numbers is 327 ; what is the other ?

24. How long will a steam pump with a capacity of 575 gallons a minute take to fill a tank holding 32,775 gallons ?

25. A tract of land containing 2,529 acres was sold for \$42,360.75 ; what was the price per acre ?

26. If 267 horses are worth \$25,031.25, what is the value of 375 horses at the same average price ?

27. If \$350 be paid for 56 barrels of flour, how many barrels can be bought for \$256.25, at the same price per barrel ?

28. I bought a farm for \$6,737.50 and sold it for \$7,218.75, gaining \$2.75 per acre. How many acres did the farm contain ?

29. I bought 24 yards of goods for \$8.40, and a friend bought the remainder of the piece at the same price per yard, paying \$6.30. How many yards did the piece contain ?

30. A, B and C joined in buying a farm containing 420 acres for \$7,665. How much ought each to pay if A took 75 acres, B 130 acres, and C the remainder, the land being of uniform value ?

31. If 37 sheep and 45 pigs are worth \$783.75, and 28 sheep are worth \$210, what are 68 pigs worth ?

32. A grocer mixed 245 pounds of Mocha coffee costing 25 cents per pound with 180 pounds of Java coffee costing 35 cents per pound. At what price per pound must he sell it to gain \$11.75 ?

33. A farmer sold an equal number of horses, cows and calves, receiving \$3,540 for the whole. Valuing a horse at \$69, a cow at \$37, and a calf at \$12, find the number of each.

34. Ten cents will buy 3 oranges, 4 lemons, or 5 apples ; how many apples are worth as much as 5 dozen oranges and 7 dozen lemons ?

35. The difference in weight of two chests of tea is 25 lbs. ; the value of both at 65 cents per lb. is \$113.75. How many pounds are there in each chest ?

36. A fruit merchant bought a quantity of apples for \$144 ; he sold half of them for \$82.80, thereby gaining 12 cents per bushel on what he sold. What did the apples cost him per bushel ?

37. A bushel of wheat weighs 60 lbs. and a barrel of flour weighs 196 lbs. If 3 lbs. of wheat make 2 lbs. of flour, how many barrels of flour can be made from 343 bushels of wheat ?

38. A farmer employs a number of men and 8 boys ; he pays the boys 65 cents and the men \$1.10 per day. The amount that he paid to all was as much as if each received 92 cents per day ; how many men were employed ?

39. By selling my cloth at \$1.26 per yard I gain 11 cents more than I lose by selling it at \$1.05 per yard ; what would I gain by selling 800 yards at \$1.40 per yard ?

40. If a garrison of 1,000 men have provisions for 12 months, how long will the provisions last if at the end of 2 months they be reinforced by 250 men ?

41. A horse worth \$170, and 3 cows worth \$36 each, were exchanged for 14 calves and \$82. Find the value of a calf.

42. How much water must be added to 90 gallons of brandy worth \$4.60 per gallon in order that the mixture may be worth only \$3.60 per gallon ?

43. Bought oranges at the rate of 10 cents per dozen and sold them at the rate of 5 oranges for 11 cents. How much did I gain on 11 boxes each containing 20 dozen ?

44. The difference between 82,610 and the product of two numbers is 70,291,360. One of the numbers is 9,402 ; find the other.

45. A dealer bought 8 carloads of lumber, each containing 9,870 feet, at \$13.50 per M. He retailed it at \$1.45 for 100 feet. Find his gain on the whole lot.

46. What is the least number that must be added to five millions to make it exactly divisible by seven thousand and nineteen ?

47. A drover bought a number of cattle for \$4,375, and sold a certain number of them for \$43 per head for the total sum of \$3,655, gaining \$680; for how much per head must he sell the remainder so as to gain \$400 on them?

48. A merchant bought a number of barrels of flour for \$4,600 and sold them for \$5,200, thereby gaining 75 cents per barrel; how many barrels did he buy, and what did they cost him per barrel?

49. A person, after paying out of his income for a year a tax of 4 cents in the dollar, has \$7,200 left. Find his income for a year.

50. A residence property rents for \$35 per month. The annual taxes average \$75 and the repairs \$50. The owner pays the water rate of \$1.75 per month; what is the net income from the property during a period of five years?

51. A committee has a certain sum with which to purchase a carpet for a lodge room. They find that if they purchase carpeting at \$1.10 per yard, they will lack \$15.60 of having enough money to pay for it, but, if they purchase a grade at 95 cents per yard, they will have \$8.40 of their fund unexpended. How many yards are required?

52. Walter Bright bought a 30-acre wood lot at \$26 per acre. He paid \$275 for fencing the lot, sold from it 160 cords of wood at \$1.50 per cord, and then sold the lot for \$950. What did he gain on the speculation?

53. An orchard contains 26 rows of trees, with 42 trees in each row. The apple crop from the orchard was sold for \$4,095. What was the value of the yield per tree?

54. A farmer has \$180.50 in cash, and by selling his crop of apples at 75 cents per bushel, 3 calves at \$8 each, 7 lambs at \$4.25, and borrowing \$72 from a neighbor, he is able to pay off a mortgage upon his farm amounting to \$550. How many bushels of apples did he sell?

55. A grocer mixed 350 pounds of Rio coffee costing 12 cents per pound with 175 pounds costing 16 cents per pound. At what price per pound must he sell the mixture to gain \$8.75?



56. A contractor expected to complete a contract in 38 days by employing 75 men ; but after working 28 days, 25 of these men were discharged. In how many days can the remainder of the men complete the contract ?

57. A dealer having 750 bushels of corn, bought 830 bushels additional, then sold 235 bushels, then bought 340 bushels, then sold 465 bushels, then bought 190 bushels, and then sold enough to reduce his stock of corn to 620 bushels. What was the amount of his last sale at 68 cents per bushel ?

58. If a man buys 230 acres of land for \$6,210, sells 50 acres at \$29 per acre, 76 acres at \$28.50 per acre, and 41 acres at \$31.25 per acre, at what price per acre must he sell the remainder to make a total profit of \$1,349 ?

59. A farmer sold to a merchant 15 pounds of butter at 28 cents per pound, 25 dozen eggs at 14 cents per dozen, 5 dozen chickens at \$2.20 per dozen, and in payment received 4 gallons of molasses at 55 cents per gallon, 9 pounds of coffee at 18 cents per pound, 1 barrel of flour worth \$6.40, and the remainder in sugar at 8 cents per pounds. How many pounds of sugar did the farmer receive ?

60. A contractor engages to build a house for \$2,875, provided he uses lumber costing \$18.50 per thousand feet ; but if lumber worth \$20 per thousand is used, he will ask \$2,945.50. If the cheaper grade is selected, what will be the amount of the lumber bill ?

61. On a field of wheat containing 75 acres a farmer harvested 1,397 bushels ; on a second field of 64 acres he harvested 1,123 bushels ; and on a third field of 57 acres he harvested 1,008 bushels. What was the average yield per acre for the whole crop ?

62. Five men bought a paper mill for \$47,500. They operated the mill for one year at a total outlay of \$48,560.75. During the time they sold paper to the amount of \$69,744.25. They then sold the mill for \$50,000. What was the amount of each man's share of the gain ?

63. A cyclist rode 4 hours and 20 minutes at the rate of 15 miles an hour. He then rode back over the same road in just

5 hours. At what average rate per hour did he ride on his return trip.

64. A man bought 16,480 pounds of wool @ 19c., and sold it @ 26c. He then invested the gains of this speculation in pork at 8c. a pound, which he afterwards sold at 11c. a pound. What did he gain on the pork ?

65. Walter's expense account while attending Dalhousie College from Jan. 1 to June 1, 1900, was as follows : Board and room, \$75.50 ; tuition and books, \$62.85 ; clothing and laundry, \$41.15 ; incidentals, \$7.95. What were his average monthly expenses ?

66. The items of cost for an edition of 6,000 books are as follows : paper, \$318.25 ; printing and binding, \$1,345.80 ; packing, \$15.95. If the books are sold at 50c. each, what is the gain on each book ?

67. A speculator bought a piece of city property for \$2,650, sold the property at an advance of \$206 over the purchase price, and invested the money in wheat at 68c. a bushel. He afterwards sold the wheat at 74c. a bushel, paying commission and other charges to the amount of \$62.75. How much was his original capital increased ?

68. A buyer for a department store is allowed a certain sum with which to purchase bicycles. He bought 55 bicycles at \$27 each, and had \$15 remaining. Supposing he had bought bicycles at \$32 each, how many could he have purchased, and what sum would remain unexpended ?

69. Four boys desire to know the average age of their fathers. John's father is 61 ; Henry's 54 ; David's 48 ; James, having made the calculation, says that his father's age will make the average of the four just 52 years. What is the age of James's father ?

70. If to a certain number 485 be added, the sum multiplied by 96, 4,962 be subtracted from the product, and the remainder divided by 323, the result will be 234. What is the number ?

71. How long will it take 120 men to perform what 180 men were engaged to do in 80 days ?

72. A young man graduated from college when he was 25 years old. His mother was born in 1835, and his father is 4 years older than his mother. If the father was 32 years old when the son was born, in what year did he graduate ?

73. What is the least number that must be subtracted from one million to make it exactly divisible by 359 ?

74. A manufacturer buys a mixed car of packing lumber at \$12 per M., f.o.b. cars at mill, and shipped same into Toronto, paying freight at the rate of 19 cents per cwt. The lumber on being unloaded and measured is found to be as follows : 2 M. feet basswood, 3 M. feet red pine, and 4 M. feet ash. Estimated weight of basswood is 2,700 lbs. per M. feet, pine 3,000 lbs. per M., and ash 3,500 lbs. per M. Find cost delivered per M. feet of each kind of lumber.

75. A Toronto merchant wishes to purchase a car of hard coal and receives two quotations ; one naming price of \$5.25 per short ton f.o.b. cars Toronto, and the other naming \$4.60 per long ton f.o.b. cars Suspension Bridge, New York. On latter there would be no duty but there would be freight to Toronto of 60 cents per ton, to be paid by purchaser. Which quotation is cheaper, and by how much per short ton ?

76. A Toronto merchant wishes to ship 11,150 lbs. of merchandise to Brantford, Ontario, by the cheapest route. He has two options : 1st, to ship in the ordinary way and pay freight at rate of 25 cents per cwt., plus  $1\frac{1}{2}$  cents per cwt. cartage at Toronto and the same at Brantford ; 2nd, he can ship in a car at rate of 11 cents per cwt. on a minimum weight of 20,000 lbs., and pay a flat rate of \$5 for cartage at Toronto and \$4 at Brantford. Show cost of shipping goods by each method.

## CANCELLATION

**Cancellation** is the process of shortening the operation of division, or the combined operations of multiplication and division, by omitting or striking out equal factors from the dividend and divisor.

ILLUSTRATION.—What is the quotient of  $25 \times 6 \times 8 \times 9 \times 27 \times 5$  divided by  $4 \times 3 \times 3 \times 5 \times 4 \times 2 \times 9 \times 6$ ?

$$\frac{25 \times 6 \times 8 \times 9 \times 27 \times 5}{4 \times 3 \times 3 \times 5 \times 4 \times 2 \times 9 \times 6} = \frac{75}{4}, \text{ or } 18\frac{3}{4}$$

SOLUTION:—Place the dividend above and the divisor below a horizontal line ; cancel all factors common to both dividend and divisor. The pro-

duct of the numbers above will be the new dividend, and the product of the numbers below will be the new divisor. If the dividend is larger than the divisor, divide the dividend by the divisor, and the remainder, if any, may be written above the divisor.

SERIES 8

1. Divide the product of  $21 \times 15 \times 9$  by the product of  $5 \times 7 \times 3$ .
2. What is the quotient when  $24 \times 108 \times 12 \times 7 \times 5$  is divided by  $18 \times 4 \times 8 \times 14$ ?
3. Simplify  $81 \times 25 \times 34 \times 30 \div$  by  $21 \times 5 \times 6 \times 17$ .
4. Divide  $72 \times 210 \times 95 \times 60 \times 42$  by  $21 \times 19 \times 12 \times 10 \times 15 \times 7$ .
5. Find the quotient on dividing  $51 \times 7 \times 9 \times 27 \times 40 \times 56$  by  $63 \times 17 \times 9 \times 175$ .
6. How many pounds of beef at 18 cents a pound should be given in exchange for 45 bushels of wheat worth 80 cents a bushel?
7. How many pieces of cloth, each containing 48 yards, at 10 cents a yard, should be received for 6 loads of potatoes, of 60 bushels each, at 35 cents a bushel?
8. A merchant exchanged 35 pieces of bunting, averaging 52 yards to the piece and worth 18 cents per yard, for a number of pieces of calico, averaging 42 yards to the piece and worth 5 cents per yard. How many pieces of calico did he receive?
9. A farmer brought to market 10 loads of rye, averaging 16 bags of 2 bushels each. The merchant paid for the rye with 12 barrels of sugar of 225 pounds each at 5 cents per pound. What price per bushel was allowed for the rye?
10. If 8 loads of corn, each containing 15 bags, and each bag 2 bushels, were given in exchange for 8 barrels of sugar, averaging



250 pounds per barrel, and worth 9 cents per pound, what was the corn worth per bushel ?

### COMBINATIONS OF ARITHMETICAL PROCESSES

It is often desirable to express in one formula, or statement, several arithmetical operations ; also to indicate the order in which these operations are to be performed.

This is accomplished by means of the four arithmetical signs  $+$ ,  $-$ ,  $\times$ , and  $\div$ , used in connection with the comma, the parenthesis ( ), and the vinculum         .

Thus, the expression  $6 + 7, \times 9$  indicates that 6 and 7 are to be added, and the sum multiplied by nine, the result being 117 ; while the expression  $6, + 7 \times 9$  indicates that the product of  $7 \times 9$ , or 63, is to be added to 6, the result being 69.

The parenthesis is used to inclose one or more indicated operations, the result of which is to be considered as one quantity. Thus,  $5 + (12 \div 3) + 8 - (2 \times 3)$  indicates that the quotient of  $12 \div 3$ , or 4, is to be added to 5, that 8 is to be added to this, and that from this result, 17, the product of  $2 \times 3$ , or 6, is to be subtracted, giving the result, 11.

The vinculum may be used instead of, or in connection with, the parenthesis. Thus,  $27 - \frac{24}{12 - 8}$  indicates that the whole quantity, 24 divided by the quantity  $12 - 8$ , is to be subtracted from 27.  $27 - \frac{24}{12 - 8}$

### SERIES 9

Solve the following :

1.  $\frac{17 - 8}{\phantom{00}} \div 3.$
2.  $51 \div 17 +, - (2 \times 4).$
3.  $34 \div, 15 + \frac{(3 \times 2)}{\phantom{00}} \div 3).$
4.  $\frac{(75 \div 15) + 9.}{2}$
5.  $(98 - 7) \div \frac{(8 - 4)}{\phantom{00}} \times \frac{(28 - 9)}{\phantom{00}}.$
6.  $54 \div (12 + \frac{78}{\phantom{00}} \div 13) + 15 - (8 - 4).$
7.  $\frac{(25 - 6) \times (52 \div 13).}{(3 \times 13) - 37.}$

## METHODS FOR PROVING WORK

Addition may be verified by reversing the order of adding. Thus, if the various columns in a problem have been added from the bottom to the top, reverse the process and add them from the top to the bottom. If the two answers are identical, the work may fairly be assumed to be correct.

Subtraction may be verified by adding the remainder to the subtrahend. If the sum is identical with the minuend, the work is correct.

Multiplication may be verified in two ways :

(1) By interchanging the multiplier and multiplicand and remultiplying. If the results obtained by both operations are the same, the work is assumed to be correct.

(2) By dividing the product by the multiplier or multiplicand. If the result obtained is the multiplicand or multiplier, respectively, the work is correct.

Division may be verified by multiplying the quotient by the divisor and adding the remainder. If the result is identical with the dividend, the work is correct.

Each of these processes may also be verified by means of a "check figure," or by "casting out" a certain figure. There are a great many check figures in common use, such as 9, 11, 13, 17, 19, etc.

We will illustrate the use of these check figures by showing how the four simple rules may be verified by "casting out the 9's."

**To cast the nines out of any number** is to find the remainder in dividing the number by 9. To do this, add together the digits of the given number, omitting any nines there may be among the digits, then add together the digits of that sum, again omitting all nines, and so continue until a number of one digit is obtained. This last number, if it be less than 9, will be the remainder in dividing the given number by 9; if it be 9 the remainder will be zero.

ILLUSTRATION 1.—Cast the nines out of 73856942.

SOLUTION

$$7 + 3 + 8 + 5 + 6 + 4 + 2 = 35, 3 + 5 = 8, \text{ remainder.}$$

Instead of adding all the digits together and casting the nines out of the sum, the nines may be cast out of the partial sums as fast as they rise above 8.

8. Adopting this method, the preceding example would appear :

$$7 + 3 = 10, (1 + 0 = 1), 1 + 8 = 9; 5 + 6 = 11, (1 + 1 = 2), 2 + 4 + 2 = 8.$$

Wording : ten, *one*, nine, eleven, *two*, six, eight.

ILLUSTRATION 2.—Cast the nines out of 3587968594.

SOLUTION

$$8, 16, (7), 14, (5), 11, (2), 10, (1), 6, 10, 1 \text{ remainder.}$$

To prove addition by casting out the nines.

ILLUSTRATION.—Add 375, 425, 623, and 412. Prove the work by casting out the nines.

$\begin{array}{r} 375 = 6 \\ 425 = 2 \\ 623 = 2 \\ 412 = 7 \\ \hline 1835 = 17 = 8 \end{array}$	$\left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} 8$	<p>SOLUTION.—The excess of nines in 375 is 6 ; in 425 is 2 ; in 623 is 2 ; in 412 is 7. The excess of nines in the sum of 6, 2, 2, and 7 is 8. The excess of nines in 1835 is also 8. Since the excess of nines in all the numbers is equal to the excess of nines in the sum of the numbers, the work is assumed to be correct.</p>
---	--	--

To prove subtraction by casting out the nines.

The excess of nines in the minuend, minus the excess of nines in the subtrahend, should equal the excess of nines in the remainder ; or, the excess of nines in the subtrahend, plus the excess of nines in the remainder, should equal the excess of nines in the minuend.

To prove multiplication by casting out the nines.

ILLUSTRATION.—Find the product of  $512 \times 324$  and verify the result by casting out the nines.

$\begin{array}{r} 512 = 8 \\ 324 = 0 \\ \hline 165888 = 0 \end{array}$	$\left. \begin{array}{l} \\ \\ \end{array} \right\} 0$	<p>SOLUTION.—The excess of nines in 512 is 8 ; in 324, 0. <math>8 \times 0 = 0</math>. The excess of nines in the completed product is 0. Since the excess of nines in the multiplicand, multiplied by the excess of nines in the multiplier, is equal to the excess of nines in the product, the work is assumed to be correct.</p>
--	--	--

### To prove division by casting out the nines.

Examples in division may be proved by multiplying the excess of nines in the divisor by the excess of nines in the quotient. If the work is correct, the result should equal the excess of nines in the dividend, or the dividend minus the remainder when there is a remainder.

The process of proving the simple rules by the use of any other check figure is exactly similar to that involved in the case of the nine, except that the method of casting out the check figure is different.

Thus, to cast out the elevens from a number we begin with the units digit and add to it every alternate digit. Then we begin with the tens digit and add to it every alternate digit. Then, if the sum of the digits in the odd places is greater than the sum of the digits in the even places, the result is the difference between these sums. If the sum of the digits in the even places is the larger, the result is eleven less the difference between these sums. In case either of the sums referred to is greater than eleven, we subtract from it the largest multiple of eleven it contains and proceed as stated above.

ILLUSTRATION 1.—Cast the elevens out of 5182619274.

#### SOLUTION

The sum of the digits in the odd places  $= 4 + 2 + 1 + 2 + 1 = 10$ .

The sum of the digits in the even places  $= 7 + 9 + 6 + 8 + 5 = 35$ .

35 being greater than 11, subtract from it 33, which is the largest multiple of 11 it contains, and 2 is left.

10, the sum of the digits in the odd places, is now greater than 2, so the result is  $10 - 2 = 8$ .

ILLUSTRATION 2.—Cast the elevens out of 5160459674.

#### SOLUTION

The sum of the digits in the odd places  $= 4 + 6 + 5 + 0 + 1 = 16$ .

The sum of the digits in the even places  $= 7 + 9 + 4 + 6 + 5 = 31$ .

From 16 subtract 11 and 5 are left.

From 31 subtract 22 and 9 are left.

5, which comes from the sum of the digits in the odd places, is now less than 9, which comes from the sum of the digits in the even places, so the result is  $11 - (9 - 5) = 7$ .



There is no short method of casting out the thirteens, seventeens, or nineteens from a number. To do this, it is necessary to actually divide the numbers by these various check figures. But a little practice will enable the student to divide by any one of these numbers mentally.

Probably the best check figure to be used is 13, although the adherents of each one will advance various reasons why their particular choice is the best. There are numbers of errors that 9 and 11 will not detect. On the other hand, the process of casting out nines or elevens is much easier than that of casting out thirteens, seventeens, or nineteens. Of course, the smaller the divisor the easier the division is effected. So that, all things considered, probably 13 is the best one to be used.

Many bookkeepers and accountants use a check figure in doing their posting and in taking off a Trial Balance. In fact, some will make no calculations that they do not verify in this manner. The check figure 9, however, should for obvious reasons be avoided by the bookkeeper.

#### SERIES 10

1. Find the product of 8467 and 359 and verify the work by casting out the nines.
2. Divide 367452 by 627 and prove the correctness of the work by casting out the elevens.
3. Add 3685, 2736, 4985, 283, 34756, 23754, and 9136, and verify the work with the check figure 13.
4. Subtract 458367 from 923645 and verify by casting out the nines.
5. Multiply 37546 by 963 and verify by division.
6. Divide 7436254 by 397 and verify without the use of a check figure.
7. Find the difference between 3782654 and 4253263 and verify without use of a check figure.

# FACTORS AND MULTIPLES

## FACTORS

A **Factor** of a number is any exact divisor of that number, except itself and one.

Thus, 3 and 2 will each divide 6 exactly, so they are both said to be factors of 6.

Every factor of a number has its co-factor, which, when multiplied by it, produces the number.

Thus, 3 and 2 are co-factors in relation to the number 6.

A **Prime Number** is one that has no factors ; as 2, 5, 7, 11, 13, etc.

A **Composite Number** is one that can be resolved into factors ; as 8, 10, 24, etc.

TABLE OF PRIME NUMBERS FROM 1 TO 1000

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

A **Prime Factor** is a factor which is a prime number.

Thus, 3, 5, and 7 are prime factors of 105.

A **Composite Factor** is a factor which is a composite number. Thus, 15, 21, and 35 are composite factors of 105.

**Tests of Exact Divisibility.**—The following tests of exact divisibility are often useful in a search for the factors of a number.

(1) A number is exactly divisible by 2 if its right-hand figure is zero or a number exactly divisible by 2.

(2) A number is exactly divisible by 4 if its two right-hand figures are zeros or express a number exactly divisible by 4.

**EXAMPLES.**—173528 is exactly divisible by 4, for 28 is exactly divisible by 4; but 319378 is not a multiple of 4, for 78 is not exactly divisible by 4.

(3) A number is exactly divisible by 8 if its three right-hand figures are zeros or express a number exactly divisible by 8.

**EXAMPLES.**—536 is a multiple of 8, therefore 1397536 is exactly divisible by 8; but 356 is not a multiple of 8, consequently 4679356 is not exactly divisible by 8.

(4) A number is exactly divisible by 5, 25, 125, . . . . . if the number expressed by the right-hand figure or the two, three, . . . . . right-hand figures is exactly divisible by 5, 25, 125, . . . . .

(5) A number is exactly divisible by 3 if the sum of its digits is exactly divisible by 3.

(6) A number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9.

**EXAMPLES.**—Test whether 18637569 and 7385621 are divisible by 9.

$1 + 8 + 6 + 3 + 7 + 5 + 6 + 9 = 45 = 9 \times 5$ ,  $\therefore$  18637569 is exactly divisible by 9.

$7 + 3 + 8 + 5 + 6 + 2 + 1 = 32 = 9 \times 3 + 5$ ,  $\therefore$  7385621 is not exactly divisible by 9.

(7) A number is exactly divisible by 6 if it is exactly divisible by both 2 and 3.

(8) A number is exactly divisible by 10 if its right-hand figure is zero.

(9) A number is exactly divisible by 12 if it is exactly divisible by both 4 and 3.

(10) A number is exactly divisible by 11 if the difference between the sum of its 1st, 3rd, 5th, 7th, etc., figures and the sum of its 2nd,

4th, 6th, 8th, etc., figures is zero or a number exactly divisible by 11.

EXAMPLES.—Test whether 729583624 and 457983621 are exactly divisible by 11.

$4 + 6 + 8 + 9 + 7 = 34$ ;  $2 + 3 + 5 + 2 = 12$ ;  $34 - 12 = 22 = 11 \times 2$ ;  $\therefore$  729583624 is exactly divisible by 11.

$1 + 6 + 8 + 7 + 4 = 26$ ;  $2 + 3 + 9 + 5 = 19$ ;  $26 - 19 = 7$ ;  $\therefore$  457983621 is not exactly divisible by 11.

### SERIES II

Make a list of the following numbers that are exactly divisible by (a) 2, (b) 3, (c) 4, (d) 5, (e) 6, (f) 8, (g) 9, (h) 10, (i) 11, (j) 12, without actually dividing :

- |             |                 |
|-------------|-----------------|
| 1. 345672.  | 6. 4362416.     |
| 2. 2143560. | 7. 3758362.     |
| 3. 3657247. | 8. 43621545.    |
| 4. 2803652. | 9. 213654829.   |
| 5. 5329170. | 10. 3267037021. |

**Factoring** is the process of separating a number into its prime factors.

ILLUSTRATION.—Factor the number 1260.

2	1260		SOLUTION.—Since the number is even,
2	630, first quotient.		divide it by 2. $1260 \div 2 = 630$ . The first
3	315, second quotient.		quotient being even, divide again by 2. $630$
3	105, third quotient.		$\div 2 = 315$ . The sum of the digits in 315,
5	35, fourth quotient.		$(3 + 1 + 5)$ , being 9, which is divisible by
	7, last quotient.		3, we divide by 3 and get the third quotient,
			105. For the reason last given, divide by 3
			again and we get 35. Now divide by 5,
			since 35 ends with 5. Dividing thus we get
			7, the last quotient, which completes the
			work, the prime factors being 2, 2, 3, 3, 5,
			and 7.

### RULE

*Divide the given number by its least prime factor; similarly divide the resulting quotient; and so continue until a prime quotient is obtained. The several divisors and the last quotient are the prime factors.*



## PROOF

*The continued product of all the prime factors should equal the given number.*

## SERIES 12

Resolve the following numbers into their prime factors :

1. 64.	6. 1575	11. 15625.
2. 144.	7. 2376.	12. 18216.
3. 512.	8. 2744.	13. 21659.
4. 1050.	9. 4367.	14. 25785.
5. 1527.	10. 6435.	15. 38577.

A **Common Factor**, or **Divisor**, of two or more numbers is a factor which belongs to each of them.

Thus, since the prime factors of 6 are 3 and 2, and of 10 are 5 and 2, it will be seen that 2 is a common factor of 6 and 10.

The **Highest Common Factor** (or, as it is sometimes called, the **Greatest Common Measure** or **Greatest Common Divisor**) of two or more numbers is the largest factor common to the numbers.

Thus, all the exact divisors of 12 are 2, 3, 4, and 6 ; of 18, are 2, 3, 6, and 9 ; and of 30, are 2, 3, 5, 6, 10, and 15. The divisors common to the three numbers are 2, 3, and 6. The **Greatest Common Divisor** or **Highest Common Factor** of the three numbers is, therefore, 6.

Numbers are prime to each other when they have no common factor.

Thus, the prime factors of 9 are 3 and 3, and of 22 are 2 and 11. 9 and 22 are, therefore, prime to each other.

**ILLUSTRATION 1.**—What is the G.C.D. or the H.C.F. of 30, 45, 60, and 75 ?

$$30 = 2 \times 3 \times 5$$

$$45 = 3 \times 3 \times 5$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$75 = 3 \times 5 \times 5$$

$$\text{H.C.F.} = 3 \times 5 = 15$$

**SOLUTION 1.**—Factor each number and pick out all the common factors. The product of all the common factors will be the H.C.F.

**SOLUTION 2.**—The G.C.D. of these numbers cannot be greater than 30 ; if 30 is an exact divisor of the rest of the numbers it is the G.C.D. By inspection it is found not to be a divisor of all the numbers. Dropping the smallest factor, 2, out of 30, and using the other one, 15, as a divisor, we find by

inspection that 15 is an exact divisor of all the numbers ; therefore, it is the G.C.D. of 30, 45, 60, and 75.

If the numbers are large and the factoring, therefore, difficult, the following method is commonly used :

ILLUSTRATION 2.—Find the G.C.M. of 1645 and 1833.

SOLUTION.—First Method :

$$\begin{array}{r}
 1645 \overline{)1833}(1 \\
 \underline{1645} \\
 188 \\
 188 \overline{)1645}(8 \\
 \underline{1504} \\
 141 \\
 141 \overline{)188}(1 \\
 \underline{141} \\
 47 \\
 47 \overline{)141}(3 \\
 \underline{141} \\
 0
 \end{array}$$

$$\text{G.C.M.} = 47$$

### RULE

*Divide the greater number by the less, and the last divisor by the last remainder, and so continue to divide until a quotient is obtained without a remainder. The last divisor obtained will be the greatest common divisor.*

*Where more than two numbers are given, first find the G.C.D. of any two of them ; then of the G.C.D. thus found and the third number, etc.*

The following arrangement of the work necessitates the placing of fewer figures on the paper :

SOLUTION.—Second Method :

$$\begin{array}{r|l}
 1833 \\
 1645 \overline{)1} 1645 \\
 1504 \overline{)8} \underline{\hspace{1cm}} \\
 141 \overline{)1} 141 \\
 141 \overline{)3} \underline{\hspace{1cm}} \\
 0 \hspace{1.5cm} 47
 \end{array}$$

$$\text{G.C.M.} = 47$$

## SERIES 13

Find the G.C.D. of the following numbers :

Mentally.	By Division.
1. 12 and 18.	11. 964 and 1272.
2. 15 and 25.	12. 729 and 1701.
3. 18 and 24.	13. 513 and 1368.
4. 20 and 36.	14. 4125 and 6750.
5. 24 and 72.	15. 8778 and 9702.
By Factoring.	16. 4853 and 5697.
6. 48 and 84.	17. 1177 and 1498.
7. 60 and 90.	18. 2827 and 10537.
8. 75 and 125.	19. 1027, 2133, and 2449.
9. 84, 96, and 120.	20. 1157, 2047, and 2937.
10. 48, 72, and 360.	21. 1268, 1472, and 1684.

22. A man has four large lots, containing respectively 675, 1125, 1575, and 1800 sq. ft. He wishes to divide them all into smaller lots of the largest possible size to contain an equal number of sq. ft. How many sq. ft. will each of the smaller lots contain ?

23. How many ft. in circumference is the largest locomotive drive-wheel that will make an exact number of revolutions in going a distance of 513, 608, and 1368 ft. respectively ?

24. Find the largest number that will divide 49, 73, 97, and 121 and leave a remainder of 1 each time.

25. What is the largest number that will divide 179 and 301 and leave remainders of 17 and 13 respectively ?

26. At the intersection of three avenues is a triangular piece of ground whose separate sides are 64, 80, and 112 ft. What is the length of the longest boards that can be used to build a fence around it, if the boards are of equal length and no allowance is made for waste ?

27. A carpenter is directed to make the widest possible sidewalk, no wastage, out of the following planks each 1 foot in width and 2 inches in thickness : 9 planks 20 feet long, 8 planks 16 feet long, and 6 planks 12 feet long. What will be the length and width of the walk ? (Planks to run crosswise of the walk.)

## MULTIPLES

A **Multiple** of a number is a number which will contain the given number exactly.

Thus, 12 is a multiple of 4, of 3, of 6, or of 2.

A **Common Multiple** of two or more numbers is a number which will contain each number exactly.

Thus, 12 is a multiple of 3, and 12 is also a multiple of 4. It is, therefore, a common multiple of 3 and 4.

The **Lowest Common Multiple** of two or more numbers is the lowest, or smallest, number that will contain each of them exactly.

Thus, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, and 120 are the first twelve multiples of 10; 12, 24, 36, 48, 60, 72, 84, 96, 108, and 120 are the first ten multiples of 12; and 15, 30, 45, 60, 75, 90, 105, and 120 are the first eight multiples of 15. Examining these various groups of multiples we find that 60 and 120 are common multiples of 10, 12, and 15. 60 is, therefore, the L.C.M. of the three numbers.

**ILLUSTRATION 1.**—What is the L.C.M. of 5, 6, 10, 12, 18, and 24?

**SOLUTION.**—Since 24 is the L.C.M. of itself, if it is an exact dividend of the other numbers it is the L.C.M. of the numbers. By inspection it is found to be an exact dividend of but two numbers, 6 and 12. By comparison it is found that 5 is not an exact divisor of 24, and since it is a prime factor no part of it can be found in 24; consequently, 5 as a factor must be used,  $5 \times 24$  is 120. The next number, 10, is found to be an exact divisor of 120. The last number, 18, is not an exact divisor; its factors are 3, 3, and 2.  $9$  ( $3 \times 3$ ) is not an exact divisor, but 6 ( $2 \times 3$ ) is; consequently, 120 should be increased by 3 as a factor.  $3 \times 120$  is 360, the required number, the L.C.M.

## SERIES 14

Find the L.C.M. of the following:

- |                            |                           |
|----------------------------|---------------------------|
| 1. 3, 4, 5, 6, and 8.      | 5. 24, 32, 28, and 18.    |
| 2. 2, 6, 8, 12, and 24.    | 6. 6, 8, 12, 24, and 32.  |
| 3. 3, 9, 12, 6, and 18.    | 7. 4, 8, 16, 32, and 128. |
| 4. 12, 18, 24, 36, and 48. | 8. 4, 6, 14, 21, and 42.  |



ILLUSTRATION 2.—What is the L.C.M. of 75, 120, 150, and 180 ?

$$\begin{aligned} 75 &= 3 \times 5 \times 5 \\ 120 &= 2 \times 2 \times 2 \times 3 \times 5 \\ 150 &= 2 \times 3 \times 5 \times 5 \\ 180 &= 2 \times 2 \times 3 \times 3 \times 5 \\ 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 &= 1800 \\ 1800 &\text{ is the L.C.M.} \end{aligned}$$

and two 5's, or it must be 1800.

SOLUTION 1.—Set the numbers down in a column, and opposite each one set its prime factors. Scanning this list of factors, we find that they consist of 2's, 3's, and 5's. The most 2's we find in any number is 3; the most 3's is 2; and the most 5's, 2. The L.C.M. of these numbers, then, must contain three 2's, two 3's,

SHORT METHOD SOLUTION.

2) 75.	120.	150.	180
2) 60	75	90	
3) 30	75	45	
5) 10	25	15	
2	5	3	

SOLUTION 2.—Set the numbers down in a row as for short division. Strike 75 out of the list, as it is contained exactly in 150, and we know that any number that will contain 150 will contain 75. Beginning with 2, the lowest prime number (except 1), we see that it will divide all three numbers, so we divide each number by it, setting the quotients immediately below the numbers.

Again, we see that 2 will divide 60 and 90, so we divide them by it as before. 2 will not divide 75 exactly, so we simply set it down among the quotients. Now, we see that 2 will not divide *two* of the numbers, so we try 3, which will divide all of them. Then, since 3 will not divide two of the numbers again, we try 5, the next prime number, and see that it will divide all three of them. After dividing by 5, we see that there is no number that will divide any of the two quotients. The L.C.M. is the product of the final quotients and the various divisors,  $2 \times 2 \times 3 \times 5 \times 2 \times 5 \times 3 = 1800$ . It will be noted that these factors are the same as those in the previous solution, with different arrangement. If at any stage of the work we had found any number, such as 75, which was exactly contained in any of the other numbers in the same line, we would have struck it out as we did the 75.

### SERIES 15

Find the L.C.M. of

- |                |                        |                      |
|----------------|------------------------|----------------------|
| 1. 32 and 88.  | 6. 12, 44, 60.         | 11. 144, 1728, 60.   |
| 2. 34 and 60.  | 7. 44, 88, 100.        | 12. 56, 512, 128.    |
| 3. 57 and 95.  | 8. 3, 5, 8, 9, 10, 15. | 13. 11, 33, 66, 165. |
| 4. 81 and 117. | 9. 5, 6, 10, 12, 15.   | 14. 9, 18, 45, 63.   |
| 5. 36 and 150. | 10. 4, 8, 12, 16, 50.  | 15. 14, 49, 77, 22.  |

**16.** A boy has four boxes full of money. The first contains 3-cent pieces ; the second, 5-cent pieces ; the third, 10-cent pieces ; and the fourth, 25-cent pieces. What is the smallest bill that he can pay and take the money from any one of the boxes ?

**17.** A committee appointed to decorate a hall found that a pretty effect could be secured by stretching wires whose lengths are respectively 12, 15, 18, and 30 ft. Before deciding which length to adopt, they sent for the wire. What length of wire must be bought that no loss be sustained in the adoption of either length ?

**18.** John can walk around a circular path in 14 min., James in 18 min., and Frank in 21 min. If they start directly opposite a point on the inner side of the path and at the same time, how long will it be until they occupy the same relative position ?

**19.** What is the least number that can be divided by 30, 34, 51, and 85 and leave a remainder of 7 each time ?

**20.** Four travelling salesmen meet in Toronto ; they visit the city regularly every 2, 4, 6, and 8 weeks respectively. How long will it be until they meet again ?

# COMMON OR VULGAR FRACTIONS

## WHAT IS A FRACTION?

A Fraction is simply a part.

Cut an ordinary twelve-inch ruler in two at the six-inch mark, and you have two equal parts. Pick up one part and you call it one-half of the original ruler, or you might just as readily say that you hold but a fraction of the ruler. Of course, in Arithmetic you do not represent this part by words "one-half," any more than you would use the word "one" to represent 1. We use figures to represent fractions just as we use figures to represent whole quantities. One-half is written in figures as  $\frac{1}{2}$ , or as  $\frac{1}{2}$ .

Next, suppose you cut each of these parts again in two. Your original ruler is now in four pieces, and if you pick up one out of the four you might say that you hold one-quarter, or, expressed in figures, it would be  $\frac{1}{4}$ . If you pick up three pieces you have three-quarters, or  $\frac{3}{4}$ , of the ruler. So we see that we can have different fractions just as often as we can find parts into which to divide anything. Cut your ruler into eight equal parts and you have eighths— $\frac{1}{8}$ ,  $\frac{2}{8}$ ,  $\frac{3}{8}$ , and so on. Cut it into nine equal parts and you have ninths— $\frac{1}{9}$ ,  $\frac{2}{9}$ ,  $\frac{3}{9}$ , and so on. The bottom part of the fraction always tells the number of parts into which the thing is divided, and is called the *denominator*, while the top part tells the number of these parts that are held in mind, and is called the *numerator*, thus :

$$\frac{3 \text{ numerator.}}{4 \text{ denominator.}}$$

If we understand how fractions are made—by dividing things into parts—we should, to a certain extent, be able to handle fractional numbers as easily as whole numbers. We know, for instance, that a fifty-cent piece represents one-half of a dollar. Suppose, then, that we place a row of fifty-cent pieces before us in this fashion :

50c	50c	50c	50c	50c	50c
$\$ \frac{1}{2}$	$\$ \frac{1}{2}$	$\$ \frac{1}{2}$	$\$ \frac{1}{2}$	$\$ \frac{1}{2}$	$\$ \frac{1}{2}$

If we stop to pick them up and as we go along to call out the sums, we should say as follows : 50c, \$1.00, \$1.50, \$2.00, \$2.50, and so on ; or again, we might speak of the different amounts as follows :  $\$ \frac{1}{2}$ , \$1,  $\$ 1\frac{1}{2}$ , \$2,  $\$ 2\frac{1}{2}$ , and so on.

It should not be much harder if we introduced some twenty-five-cent pieces :

25c	25c	50c	25c	50c
$\$ \frac{1}{4}$	$\$ \frac{1}{4}$	$\$ \frac{1}{2}$	$\$ \frac{1}{4}$	$\$ \frac{1}{2}$

Picking them up in order and calling out the sums by fractions we have  $\$ \frac{1}{4}$ ,  $\$ \frac{1}{2}$ ,  $\$ 1$ ,  $\$ 1 \frac{1}{4}$ ,  $\$ 1 \frac{3}{4}$ . To see that you have thus far grasped the idea of fractions, we would like you to consider the following exercises, giving in each case the sum total of the fractions which are indicated.

### SERIES 16

Find the complete amount of

- $\frac{1}{3}$  bus. and  $\frac{2}{3}$  bus. and  $\frac{2}{3}$  bus. and  $\frac{1}{3}$  bus. and  $\frac{1}{3}$  bus. and  $\frac{1}{3}$  bus.
- $\frac{1}{8}$  yd. and  $\frac{2}{8}$  yd. and  $\frac{5}{8}$  yd. and  $\frac{4}{8}$  yd. and  $\frac{7}{8}$  yd. and  $\frac{3}{8}$  yd.
- $\frac{1}{5}$  rod and  $\frac{2}{5}$  rod and  $\frac{4}{5}$  rod and  $\frac{3}{5}$  rod and  $\frac{1}{5}$  rod and  $\frac{2}{5}$  rod.
- $\frac{1}{7}$  ton and  $\frac{2}{7}$  ton and  $\frac{4}{7}$  ton and  $\frac{5}{7}$  ton and  $\frac{3}{7}$  ton and  $\frac{1}{7}$  ton.
- $\frac{1}{4}$  and  $\frac{1}{4}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{4}$  and  $\frac{1}{2}$ .
- $\frac{1}{3}$  bus. and  $\frac{2}{3}$  bus. and  $\frac{1}{6}$  bus. and  $\frac{2}{6}$  bus. and  $\frac{1}{3}$  bus. and  $\frac{1}{6}$  bus. and  $\frac{2}{3}$  bus.
- $\frac{1}{8}$  yd. and  $\frac{1}{4}$  yd. and  $\frac{3}{8}$  yd. and  $\frac{3}{4}$  yd. and  $\frac{5}{8}$  yd. and  $\frac{1}{8}$  yd. and  $\frac{3}{4}$  yd.
- $\frac{2}{5}$  rod and  $\frac{2}{10}$  rod and  $\frac{1}{10}$  rod and  $\frac{5}{10}$  rod and  $\frac{1}{10}$  rod and  $\frac{3}{5}$  rod.

### REDUCTION OF FRACTIONS

Cut an apple in two equal parts, and each of the parts is known as one-half of the apple. Put the two parts together, and we may speak of them as the two-halves of the apple, or in figures it would be written  $\frac{2}{2}$ .

In our first lesson we learned that a fraction is a part. One-half is, therefore, properly speaking, a fraction. Two-halves is not a fraction, for it represents a complete amount.

$\frac{1}{2}$  is a Proper Fraction.

$\frac{2}{2}$  is an Improper Fraction.



A **Proper Fraction** represents a part less than a whole.

An **Improper Fraction** represents either a whole amount, or more than a whole amount.

If we cut several apples of the same size each into halves, we know that every time we take up two of these parts we have one whole apple. This we could represent as 1, or as  $\frac{2}{2}$ . Suppose we pick up 3 pieces. We would then have three-halves, or we could speak of them as one apple and one-half apple.

Expressing this in figures,  $\frac{3}{2} = 1\frac{1}{2}$ .

In the last expression we have a combination of a fraction and a whole number, or we speak of it as a **Mixed Number**.

Now that we know what a mixed number is, we should be able to tell what improper fraction it is equal to. We have seen that  $\frac{3}{2}$  and  $1\frac{1}{2}$  represent the same value. In other words, the mixed number  $1\frac{1}{2}$  is equal to the improper fraction  $\frac{3}{2}$ . We explain it as follows: The whole number 1 is equal to  $\frac{2}{2}$ , and this along with the  $\frac{1}{2}$  makes  $\frac{3}{2}$ .

Again,  $3\frac{3}{4} = \frac{15}{4}$ . The whole number 3 is equal to  $\frac{12}{4}$ , and this along with  $\frac{3}{4}$  makes up  $\frac{15}{4}$ .

Our rule may therefore be stated as follows:

*Find how many parts of the same kind as indicated by the fraction are in the whole number, and add the parts expressed by the fraction to them.*

Of course, it should be quite as easy for us to reverse this work. That is, we should be able to tell that  $\frac{15}{4} = 3\frac{3}{4}$ . This may be illustrated by putting before us 15 twenty-five cent pieces, or quarter-dollars as we sometimes call them. We know that as often as we can get four quarters we have \$1.00. Now we can pick up 4 quarters three times out of 15 quarters, and still there will be 3 quarters left, or, in other words,  $\frac{15}{4}$  are equal to  $3\frac{3}{4}$ ; it is just a matter of division—4 into 15 goes 3 times and 3 over. In the same way  $\frac{17}{3}$  equal  $5\frac{2}{3}$ , and so on.

This work of thus changing the form of an expression is sometimes called **Reduction of Fractions**.

## SERIES 17

EXERCISE 1.—Divide a sheet of paper by ruling into three columns, heading the first column “Proper Fractions,” the second “Improper Fractions,” and the third “Mixed Numbers.” Sort out the following expressions by placing each one in the column to which it belongs. Put the expressions one beneath the other in straight lines down the columns :

$\frac{1}{2}, \frac{3}{2}, 4\frac{1}{2}, \frac{3}{8}, \frac{15}{4}, 2\frac{3}{4}, \frac{5}{8}, \frac{12}{18}, 6\frac{1}{8}, 4\frac{1}{3}, \frac{2}{3}, \frac{12}{3}, 6\frac{1}{5}, \frac{17}{5}, \frac{3}{5}, 5\frac{1}{6}, \frac{16}{6}, \frac{4}{8}, 2\frac{1}{6},$   
 $14\frac{2}{7}, \frac{8}{7}, \frac{15}{7}, 11\frac{1}{9}, \frac{5}{9}, \frac{14}{9}, \frac{21}{9}, \frac{2}{9}, \frac{12}{10}, \frac{1}{10}, 5\frac{1}{10}, \frac{23}{10}, 7\frac{1}{11}, \frac{2}{11}, \frac{15}{11}, 8\frac{1}{15},$   
 $\frac{17}{15}, 23\frac{1}{3}, 7\frac{2}{9}, 4\frac{8}{11}, \frac{15}{27}, 9\frac{7}{30}, 2\frac{8}{17}, \frac{15}{22}, 9\frac{8}{25}, \frac{47}{125}, 29\frac{7}{50}, 10\frac{12}{225}, \frac{17}{864},$   
 $29\frac{1}{8}, \frac{74}{11}, \frac{86}{115}.$

EXERCISE 2.—Go down your list of Improper Fractions, and set opposite each one the Mixed Number or the Whole Number to which it is equivalent.

EXERCISE 3.—Go down your column of Mixed Numbers and set opposite each one the Improper Fraction to which it is equivalent.

## REDUCTION ASCENDING

If we cut an apple in two equal parts, we have in each part one-half of the apple. Should we take one-half of the apple and cut it in two equal parts, we have in each half two-quarters. Now the two-quarters are just exactly equal to the one-half. There is a difference in form, but not in value. If we continue cutting, and give each of the two-quarters a cut into two equal parts, we would have, out of our one-half apple,  $\frac{4}{8}$ . From this simple example we gather that  $\frac{1}{2}, \frac{2}{4}$ , and  $\frac{4}{8}$  are the same in value. This shows us that there is another way in which we may change or reduce fractions. We may change the form without changing the value. This is of great use to us in such operations as addition and subtraction. We cannot add 16 ounces and 1 ton, if we leave them as they are. If we consider that 16 ounces equals 1 pound, and that 1 ton equals 2,000 pounds, we may say that the total is 2,001 pounds. In the

same way we add  $\frac{1}{2}$  and  $\frac{1}{4}$  by figuring that  $\frac{1}{2}$  is equal to  $\frac{2}{4}$ . The sum of  $\frac{2}{4}$  and  $\frac{1}{4}$  is  $\frac{3}{4}$ .

ILLUSTRATION.—Reduce  $\frac{2}{3}$  to sixths.

SOLUTION.—Dividing 3 into 6 we get 2, which tells us that there are always twice as many sixths in any amount as there are thirds. If we have  $\frac{2}{3}$ , we must have twice 2 or four sixths.

### RULE

*Divide the required denominator by the denominator of the given fraction.*

*Multiply the numerator of the given fraction by the quotient thus obtained and write the product over the required denominator.*

*The result is the fraction in higher terms.*

### SERIES 18

Change

- |                             |                              |                                |
|-----------------------------|------------------------------|--------------------------------|
| 1. $\frac{2}{3}$ to 15ths.  | 6. $\frac{1}{3}$ to 36ths.   | 11. $\frac{7}{18}$ to 52nds.   |
| 2. $\frac{1}{5}$ to 15ths.  | 7. $\frac{5}{9}$ to 36ths.   | 12. $\frac{2}{23}$ to 115ths.  |
| 3. $\frac{2}{7}$ to 21sts.  | 8. $\frac{3}{9}$ to 27ths.   | 13. $\frac{3}{16}$ to 128ths.  |
| 4. $\frac{2}{3}$ to 21sts.  | 9. $\frac{3}{40}$ to 120ths. | 14. $\frac{5}{24}$ to 192nds.  |
| 5. $\frac{5}{12}$ to 36ths. | 10. $\frac{5}{8}$ to 88ths.  | 15. $\frac{11}{21}$ to 147ths. |

### REDUCTION DESCENDING

We have seen that it is possible to change the form of a fraction without changing its value. This we did by multiplying both parts of the fraction by the same number. By reversing the process we should be able to change a fraction of a higher denomination into an equivalent one of a lower denomination.

ILLUSTRATION.—Reduce  $\frac{8}{16}$  to its lowest terms.

SOLUTION.—To get the fraction in its lowest terms  $\frac{8}{16} \div \frac{8}{8} = \frac{1}{2}$  we must know the largest number that will divide both 8 and 16. In a simple case of this kind we can tell by inspection that this number is 8. Dividing both 8 and 16 by 8 we produce the fraction  $\frac{1}{2}$ , which is an expression of  $\frac{8}{16}$  in its lowest terms.

ILLUSTRATION.—Reduce  $\frac{192}{330}$  to its lowest terms.

SOLUTION.—Where the fraction is large, we find  
 $\frac{192}{330} \div \frac{6}{6} = \frac{32}{55}$  the largest number that will divide both terms by  
 finding the highest common factor. The H.C.F. of 192  
 and 330 is 6. Divide 6 into each of the numbers and we get the fraction  $\frac{32}{55}$ .

### RULE

*Cancel all factors common to both numerator and denominator ;  
 or divide both terms by their greatest common divisor.*

### SERIES 19

Reduce to their lowest terms

1. $\frac{16}{24}$	6. $\frac{39}{143}$	11. $\frac{549}{671}$	16. $\frac{1623}{3787}$
2. $\frac{25}{60}$	7. $\frac{504}{792}$	12. $\frac{936}{1368}$	17. $\frac{2695}{3080}$
3. $\frac{78}{192}$	8. $\frac{396}{432}$	13. $\frac{485}{679}$	18. $\frac{5608}{7711}$
4. $\frac{50}{625}$	9. $\frac{385}{539}$	14. $\frac{1680}{1920}$	19. $\frac{3178}{5221}$
5. $\frac{112}{280}$	10. $\frac{498}{850}$	15. $\frac{5184}{6912}$	20. $\frac{35211}{37248}$

### REDUCTION TO A COMMON DENOMINATOR

To reduce a series of fractions to the same common denominator.

ILLUSTRATION.—Reduce  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$  to 12ths.

SOLUTION.—The number expressing the denomination  
 $\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$  to which we may reduce halves, thirds, and quarters must  
 $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$  be a number which will contain 2, 3, 4 exactly. This, in  
 $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$  other words, must be the L.C.M. of 2, 3, 4. The L.C.M. of  
 these three numbers is 12.  $\frac{1}{2}$  reduced to twelfths gives us  
 $\frac{6}{12}$ ;  $\frac{1}{3}$  reduced to twelfths gives us  $\frac{4}{12}$ ; and  $\frac{1}{4}$  reduced to twelfths gives us  $\frac{3}{12}$ .

### RULE

1. Find the L.C.M. of the denominators.
2. Divide this L.C.M. by the denominator of the fraction that is being reduced and multiply the terms of that fraction by the quotient thus obtained.



3. *Continue with the fractions in regular order until all have been reduced.*

Before beginning the above operation, each fraction should be in its lowest terms, and mixed numbers should be reduced to improper fractions.

Whole numbers are written in the form of a fraction by writing 1 for the denominator.

### SERIES 20

Reduce to equivalent fractions having the L.C.D.

- |   |  |
|---|--|
| 1. $\frac{1}{2}$ and $\frac{2}{3}$                    | 7. $\frac{17}{36}, \frac{35}{48}, \frac{11}{12}$ , and $\frac{43}{60}$ |
| 2. $\frac{1}{2}$ and $\frac{1}{4}$                    | 8. $\frac{13}{35}, \frac{31}{45}, \frac{15}{63}$ , and $\frac{59}{84}$ |
| 3. $\frac{3}{5}$ and $\frac{7}{10}$                   | 9. $\frac{9}{10}, \frac{11}{15}, \frac{17}{20}$ , and $\frac{21}{25}$  |
| 4. $\frac{5}{6}, \frac{7}{12}$ , and $\frac{11}{18}$  | 10. $\frac{6}{18}, \frac{8}{15}, \frac{9}{30}$ , and $\frac{23}{60}$   |
| 5. $\frac{2}{9}, \frac{4}{27}$ , and $\frac{19}{54}$  | 11. $\frac{5}{9}, \frac{11}{30}, \frac{43}{60}$ , and $\frac{14}{90}$  |
| 6. $\frac{7}{10}, \frac{9}{25}$ , and $\frac{11}{75}$ | 12. $\frac{7}{12}, \frac{11}{18}, \frac{13}{24}$ , and $\frac{23}{36}$ |

### ADDITION OF FRACTIONS

ILLUSTRATION 1.—Find the sum of  $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$ .

SOLUTION.—First Step—

$$\begin{array}{ll} \frac{2}{3} \times \frac{4}{4} = \frac{8}{12} & \text{Change these fractions to equivalent fractions having} \\ \frac{3}{4} \times \frac{3}{3} = \frac{9}{12} & \text{the same common denominator by the process already} \\ \frac{5}{6} \times \frac{2}{2} = \frac{10}{12} & \text{illustrated. As a result we have } \frac{8}{12}, \frac{9}{12}, \frac{10}{12}. \end{array}$$

SOLUTION.—Second Step—

$\frac{8}{12}$  may be expressed, 8 twelfths.  
 $\frac{9}{12}$  may be expressed, 9 twelfths.  
 $\frac{10}{12}$  may be expressed, 10 twelfths.  
 The sum of these is 27 twelfths.

$$\begin{array}{l} 27 \text{ twelfths} = \frac{27}{12} \\ \frac{27}{12} = 2\frac{3}{12} \text{ or } 2\frac{1}{4} \end{array}$$

Having once changed the fractions to the same common denominator the addition may be just as easily carried out as the addition of eight pounds, nine pounds, and twelve pounds.

Note that it is the numerator or the part which shows how many that enters into the addition. After we have found that there are 27 twelfths in the sum of these three

fractions, we reduce this improper fraction to a mixed number,  $2\frac{3}{12}$ , which in turn we reduce to its lowest terms,  $2\frac{1}{4}$ .

ILLUSTRATION 2.—Find the sum of  $3\frac{1}{4}$ ,  $1\frac{2}{3}$ ,  $4\frac{1}{2}$ .

$$3 + 1 + 4 = 8$$

$$\frac{1}{4} = \frac{3}{12}$$

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{17}{12} = 1\frac{5}{12}$$

$$9\frac{5}{12}$$

SOLUTION.—Find first the sum of the whole numbers, which is 8. Next find the sum of the fractions, by the method already illustrated. They amount to  $1\frac{7}{12}$ , or  $1\frac{5}{12}$ . Add 8 and  $1\frac{5}{12}$  and we have the complete sum,  $9\frac{5}{12}$ .

### RULE

*Reduce to equivalent fractions of a common denominator; add the resulting numerators; and place the sum over the common denominator.*

NOTE 1.—To find the sum of two or more mixed numbers: *add the fractions and integers separately, and combine the two results.*

NOTE 2.—When necessary, all results, if proper fractions, should be reduced to lowest terms; and if improper fractions, to equivalent mixed numbers.

### SERIES 21

Add:

- |   |   |
|---|---|
| 1. $\frac{3}{4}$ , $\frac{2}{3}$ and $\frac{4}{5}$                                      | 2. $\frac{4}{5}$ , $\frac{7}{8}$ , $\frac{9}{10}$ and $\frac{7}{20}$                            |
| 3. $4\frac{1}{2}$ , $2\frac{1}{3}$ , $5\frac{3}{4}$ and $6\frac{7}{8}$                  | 4. $\frac{7}{8}$ , $\frac{3}{4}$ , $\frac{5}{6}$ , $\frac{7}{16}$ and $1\frac{1}{24}$           |
| 5. $\frac{5}{9}$ , $\frac{3}{5}$ , $\frac{7}{15}$ , $\frac{3}{10}$ and $\frac{3}{20}$   | 6. $\frac{1}{2}$ , $\frac{2}{3}$ , $\frac{3}{4}$ , $\frac{4}{5}$ and $\frac{5}{6}$              |
| 7. $1\frac{1}{2}$ , $2\frac{2}{3}$ , $3\frac{3}{4}$ and $4\frac{4}{5}$                  | 8. $4\frac{1}{2}$ , $\frac{3}{16}$ , $5\frac{3}{8}$ and $\frac{1}{4}$                           |
| 9. $7$ , $8\frac{3}{4}$ , $9\frac{1}{7}$ , $6\frac{5}{28}$ and $8$                      | 10. $3\frac{1}{2}$ , $9\frac{5}{12}$ , $8$ , $7\frac{5}{56}$ and $6\frac{4}{7}$                 |
| 11. $\frac{3}{4}$ , $\frac{7}{8}$ , $\frac{9}{16}$ , $\frac{17}{28}$ and $\frac{9}{56}$ | 12. $5\frac{11}{14}$ , $9\frac{11}{12}$ , $\frac{85}{56}$ , $1\frac{7}{16}$ and $1\frac{5}{14}$ |
| 13. $2\frac{5}{11}$ , $3\frac{2}{7}$ , $4\frac{7}{55}$ and $8\frac{4}{135}$             | 14. $\frac{1}{3}$ , $\frac{1}{5}$ , $\frac{1}{7}$ , $\frac{1}{9}$ and $\frac{1}{11}$            |
| 15. $364\frac{7}{8}$ , $243\frac{5}{6}$ , $327\frac{7}{4}$ and $162\frac{9}{16}$        |   |
| 16. $123\frac{5}{16}$ , $247\frac{2}{3}$ , $842\frac{11}{48}$ and $375\frac{7}{24}$     |   |
| 17. $325\frac{5}{8}$ , $426\frac{5}{12}$ , $342\frac{5}{24}$ and $136\frac{5}{48}$      |   |
| 18. $243\frac{7}{16}$ , $327\frac{7}{24}$ , $436\frac{7}{48}$ and $244\frac{7}{96}$     |   |

### SUBTRACTION OF FRACTIONS

ILLUSTRATION 1.—Find the difference between  $\frac{5}{8}$  and  $\frac{2}{3}$ .

$$\frac{2}{3} = \frac{16}{24}$$

$$\frac{5}{8} = \frac{15}{24}$$

$$\frac{16}{24} - \frac{15}{24} = \frac{1}{24}$$

SOLUTION.—First Step—As in the case of addition, we first change our fractions to equivalent fractions having the same common denominator. By this we find that  $\frac{2}{3}$  is equal to  $\frac{16}{24}$ , while  $\frac{5}{8}$  is equal to  $\frac{15}{24}$ .

SOLUTION.—Second Step—Subtracting  $\frac{15}{24}$  from  $\frac{16}{24}$  we have a remainder of  $\frac{1}{24}$ .

ILLUSTRATION 2.—Find the difference between  $8\frac{2}{3}$  and  $3\frac{5}{8}$ .

SOLUTION.—As in the previous case, we change the fractions to equivalent fractions having the same common denominator. Having done this, we compare fractions and find the difference to be  $\frac{1}{24}$ , while a comparison of the whole numbers shows a difference of 5. The complete result is therefore  $5\frac{1}{24}$ .

ILLUSTRATION 3.—Find the difference between  $7\frac{5}{8}$  and  $3\frac{3}{8}$ .

SOLUTION.—As in the previous cases, we change our fractions to equivalent fractions having the same denominator. Next we proceed to compare the fractions with the idea of subtracting  $\frac{1}{4}$  from  $\frac{5}{8}$ . This, we see, is impossible unless we borrow one unit which is equal to  $\frac{2}{4}$ . Adding this  $\frac{2}{4}$  to  $\frac{5}{8}$  we get  $\frac{3}{4}$ .

Now we subtract  $\frac{1}{4}$  from  $\frac{3}{4}$ , getting a difference of  $\frac{2}{4}$ .

When we start to compare the whole numbers with the idea of getting their difference, we must remember the one that we borrowed, so that we are really subtracting 3 from 6, which leaves us 3. The whole answer is, therefore,  $3\frac{2}{4}$ .

### RULE

1. TO SUBTRACT FRACTIONS.—*When necessary, reduce the fractions to their least common denominator. Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference over the common denominator.*

2. TO SUBTRACT MIXED NUMBERS.—*Reduce the fractions, if necessary, to a common denominator, and if the fraction in the subtrahend is smaller than that in the minuend, subtract one fraction from the other, and the smaller whole number from the larger whole number. But if the fraction in the subtrahend is larger than that in the minuend, borrow one from the whole number. After changing it to the same denominator as the fraction, add it to the fraction in the minuend. Then subtract as before.*

### SERIES 22

Find the difference between

1.  $\frac{2}{3}$  and  $\frac{3}{4}$

6.  $\frac{2}{3}$  and  $\frac{1}{2}$

11.  $3\frac{1}{2}$  and  $2\frac{3}{4}$

2.  $\frac{8}{9}$  and  $\frac{5}{21}$

7.  $\frac{2}{5}$  and  $\frac{7}{15}$

12.  $5\frac{2}{3}$  and  $1\frac{4}{5}$

3.  $\frac{3}{4}$  and  $\frac{7}{12}$

8.  $\frac{5}{6}$  and  $\frac{7}{9}$

13.  $8\frac{5}{6}$  and  $9\frac{3}{4}$

4.  $\frac{9}{16}$  and  $\frac{11}{24}$

9.  $\frac{3}{8}$  and  $\frac{5}{7}$

14.  $6\frac{4}{5}$  and  $4\frac{3}{8}$

5.  $\frac{5}{8}$  and  $\frac{1}{6}$

10.  $\frac{3}{4}$  and  $\frac{4}{9}$

15.  $5\frac{4}{9}$  and  $7\frac{1}{4}$

From

$$16. 25\frac{3}{4} \text{ take } 16\frac{1}{2}$$

$$17. 83\frac{1}{2} \text{ take } 19\frac{7}{8}$$

$$18. 150 \text{ take } 13\frac{1}{3}$$

$$19. 144\frac{1}{8} \text{ take } 50\frac{3}{4}$$

$$20. 25\frac{17}{20} \text{ take } 15\frac{1}{3}$$

$$21. 51\frac{1}{3} \text{ take } 18\frac{3}{4}$$

$$22. 75 \text{ take } 11\frac{5}{8}$$

$$23. 62\frac{11}{15} \text{ take } 8\frac{1}{9}$$

$$24. 72\frac{9}{11} \text{ take } 16\frac{8}{11}$$

$$25. 195 \text{ take } 101\frac{4}{7}$$

## MULTIPLICATION OF FRACTIONS

If we attempt to take 3 times 2 pounds the operation would be expressed something as follows :

2 pounds.

3

—

6 pounds.

The figures alone concern us. 3 times 2 are 6, whether it is 2 pounds, or 2 gallons, or 2 boys.

If we take 3 times 2 quarters it will be just the same.

2 quarters.

3

—

6 quarters.

Instead of using the word “quarters” we represent the fraction in figures, as  $\frac{2}{4}$ . Then  $3 \times \frac{2}{4} = \frac{6}{4}$ , or what is the same thing,  $\frac{2}{4} \times 3 = \frac{6}{4}$ . There is no difference between 3 times 2 quarters, and  $\frac{2}{4}$  of 3, as far as the work of finding the result is concerned.

Suppose again that instead of multiplying  $\frac{2}{4}$  by 3, we have  $\frac{2}{4}$  to be multiplied by  $\frac{3}{8}$ . The answer is not  $\frac{6}{4}$  as before, because our quarters are each to be divided into 8 equal parts. When quarters are thus divided into 8 equal parts, we get thirty-seconds. Our work would be represented as follows :

$$\frac{2}{4} \times \frac{3}{8} = \frac{6}{32}.$$



**Multiplication of Fractions** is therefore a matter of multiplying the two numerators for a numerator, and the two denominators for a new denominator.

If we have to deal with mixed numbers we can bring them under the same rule.

ILLUSTRATION.—Multiply  $2\frac{1}{2}$  by  $3\frac{1}{3}$ .

$$\begin{aligned} \text{SOLUTION.}— \quad 2\frac{1}{2} &= \frac{5}{2}, \quad 3\frac{1}{3} = \frac{10}{3} \\ \frac{5}{2} \times \frac{10}{3} &= \frac{50}{6} = 8\frac{2}{3} = 8\frac{1}{3}. \end{aligned}$$

**Cancellation.**—In all work of this kind, we can often shorten the operation by cancelling the factors that are common to both numerator and denominator, before proceeding with the multiplication.

ILLUSTRATION.—MULTIPLY  $\frac{2}{4}$  by  $\frac{3}{8}$ .

$\frac{1}{4} \times \frac{3}{8} = \frac{3}{16}$  SOLUTION.—It will be noted that 2 is a common factor of 2 and 8. We, therefore, divide 2 into 2, placing the result above the original figure after putting a cancellation stroke through it. In the same way we divide 2 into 8, placing the result, 4, underneath, after cancelling the 8. We next multiply the two figures of the numerator, 3 and 1, producing the new numerator, 3. We also multiply the two denominators, 4 and 4, thus producing the new denominator, 16.

A **Compound Fraction** is a fractional part of a whole number or mixed number, or another fraction; as  $\frac{1}{8} \times \frac{1}{9}$ ,  $\frac{1}{6} \times 2\frac{1}{2}$ ,  $\frac{2}{5}$  of  $\frac{9}{10}$ .

Compound fractions may be reduced to simple fractions by the process of multiplication.

Thus—

- (1)  $\frac{1}{8} \times \frac{1}{9} = \frac{1}{72}$
- (2)  $\frac{1}{6} \times 2\frac{1}{2} = \frac{1}{6} \times \frac{5}{2} = \frac{5}{12}$
- (3)  $\frac{2}{5}$  of  $\frac{9}{10} = \frac{2}{5} \times \frac{9}{10} = \frac{9}{25}$

## RULE

1. Change all whole or mixed numbers to fractions.
2. Eject all factors common to both the numerator and denominator.
3. Multiply the numerators together for a new numerator; multiply the denominators together for a new denominator.

4. If the new numerator is equal to, or greater than the new denominator, reduce the fraction to a whole or mixed number.

NOTE 1.—If one of the factors is an integer, change it to the form of a fraction by writing 1 under it for a denominator, or by imagining 1 to be thus written, and then apply the rule.

NOTE 2.—The word *of* written between fractions, or between fractions and integers, indicates that they are to be multiplied. Fractions thus connected are called *compound fractions*.

### SERIES 23

Find the value of

- |  |   |                               |
|--|---|-------------------------------|
| 1. $18 \times \frac{5}{8}$   | 8. $93 \times 3\frac{3}{4}$   | 15. $\frac{4}{15} \times 128$ |
| 2. $23 \times \frac{2}{3}$   | 9. $125 \times 4\frac{2}{3}$  | 16. $\frac{5}{16} \times 35$  |
| 3. $125 \times \frac{3}{4}$  | 10. $625 \times 3\frac{7}{10}$  | 17. $\frac{7}{32} \times 36$  |
| 4. $240 \times \frac{9}{16}$   | 11. $1825 \times 2\frac{2}{3}$  | 18. $\frac{19}{64} \times 84$ |
| 5. $84 \times \frac{3}{25}$  | 12. $124 \times 7\frac{5}{16}$  | 19. $\frac{18}{5} \times 36$  |
| 6. $25 \times \frac{7}{8}$   | 13. $\frac{7}{8} \times 21$   | 20. $2\frac{1}{2} \times 47$  |
| 7. $36 \times \frac{11}{5}$  | 14. $\frac{3}{16} \times 125$   | 21. $3\frac{1}{3} \times 64$  |
| 22. $\frac{3}{4} \times \frac{7}{8} \times \frac{9}{16} \times \frac{8}{9} \times \frac{5}{6}$ | 26. $5\frac{1}{2} \times 5\frac{1}{2} \times 7\frac{1}{2} \times \frac{5}{6}$ |                               |
| 23. $2\frac{1}{2} \times 4\frac{1}{2} \times 6\frac{3}{4} \times \frac{2}{3}$                  | 27. $3\frac{7}{8} \times 4\frac{1}{2} \times 18\frac{3}{4}$                   |                               |
| 24. $16\frac{2}{3} \times 9 \times 24\frac{3}{4} \times 12\frac{1}{2}$                         | 28. $88\frac{1}{2} \times 9\frac{3}{4} \times 7\frac{1}{2}$                   |                               |
| 25. $12\frac{1}{2} \times 12\frac{1}{2} \times 8 \times 4\frac{3}{4}$                          | 29. $16\frac{2}{3} \times 30\frac{7}{8} \times 1\frac{15}{16}$                |                               |

### DIVISION OF FRACTIONS

When we know how to multiply fractions, there is little to learn in mastering the division of fractions. Suppose we wish to divide  $\frac{8}{9}$  by  $\frac{4}{7}$ , or as it is indicated,  $\frac{8}{9} \div \frac{4}{7}$ . We know that  $\frac{4}{7}$  means  $\frac{1}{7}$  of 4. Let us make two operations of the work.

First, we will divide  $\frac{8}{9}$  by 4, or what amounts to the same thing, we will take  $\frac{1}{4}$  of  $\frac{8}{9}$ , which by the process of multiplication already illustrated, will give us  $\frac{2}{9}$ .

Thus far we have just divided by 4, while our real divisor is  $\frac{4}{7}$  or  $\frac{1}{7}$  of 4. The divisor we have used is therefore 7 times too large, or the quotient we have obtained is 7 times too small. Our second operation therefore will be to take our quotient, thus far obtained, and multiply it by 7.

$$\frac{2}{9} \times 7 = \frac{14}{9}$$

If this work is all put into one operation it means that we must multiply  $\frac{2}{9}$  by 7 and divide it by 4, or expressed as a multiplication question, it appears as follows :

$$\frac{2}{9} \times \frac{7}{4} = \frac{14}{9} = 1\frac{5}{9}.$$

Our rule for division may, therefore, be shortly stated as follows :  
*Invert the divisor and proceed as in multiplication.*

ILLUSTRATION 1.—Divide  $\frac{2}{8}$  by  $\frac{3}{4}$ .

$\frac{2}{8} \times \frac{4}{3} = \frac{1}{3}$  SOLUTION.—Following the rule, we set down the figure to be divided,  $\frac{2}{8}$ , followed by a multiplication sign, and after this the divisor, in an inverted position. The balance of the work is straight multiplication.

ILLUSTRATION 2.—Divide  $2\frac{1}{2}$  by  $3\frac{1}{4}$ .

$$\text{SOLUTION.} \quad 2\frac{1}{2} = \frac{5}{2}, \quad 3\frac{1}{4} = \frac{13}{4}, \quad \frac{5}{2} \div \frac{13}{4} = \frac{5}{2} \times \frac{4}{13} = \frac{10}{13}.$$

A **Complex Fraction** is a fraction having one or both of its terms fractional ; as,  $\frac{\frac{3}{4}}{9}$ ,  $\frac{7}{\frac{3}{4}}$ ,  $\frac{\frac{3}{4}}{\frac{7}{8}}$ ,  $\frac{2\frac{1}{2}}{\frac{9}{10}}$ .

Complex fractions may be reduced to simple fractions by the process of division.

$$\text{Thus,} \quad (1) \quad \frac{\frac{3}{4}}{9} = \frac{3}{4} \div 9 = \frac{3}{4} \times \frac{1}{9} = \frac{1}{12}$$

$$(2) \quad \frac{7}{\frac{3}{4}} = 7 \div \frac{3}{4} = 7 \times \frac{4}{3} = \frac{28}{3}$$

$$(3) \quad \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$$

$$(4) \quad \frac{2\frac{1}{2}}{\frac{9}{10}} = \frac{5}{2} \div \frac{9}{10} = \frac{5}{2} \times \frac{10}{9} = \frac{25}{9}$$

## SERIES 24

Divide :

- |                                    |  |  |
|------------------------------------|--|--|
| 1. $\frac{6}{7}$ by 3.             | 9. 6 by $\frac{5}{8}$ .                                    | 17. $16\frac{2}{3}$ by $\frac{5}{7}$ .       |
| 2. $\frac{10}{17}$ by 5.           | 10. $\frac{5}{6}$ by $\frac{3}{4}$ .                       | 18. $21\frac{7}{8}$ by 6.                    |
| 3. $\frac{3\frac{2}{5}}$ by 8.     | 11. $1\frac{1}{2}$ by $\frac{2}{3}$ .                      | 19. $2\frac{1}{2}$ by $2\frac{1}{3}$ .       |
| 4. $\frac{7}{9}$ by 6.             | 12. $\frac{1\frac{5}{6}}$ by $\frac{5}{6}$ .               | 20. $8\frac{1}{3}$ by $2\frac{1}{2}$ .       |
| 5. $\frac{8}{1\frac{5}{6}}$ by 16. | 13. $\frac{3}{1\frac{8}{9}}$ by $\frac{5}{6}$ .            | 21. $18\frac{6}{7}$ by $4\frac{10}{11}$ .    |
| 6. $\frac{2\frac{1}{8}}$ by 11.    | 14. $\frac{7}{1\frac{2}{3}}$ by $\frac{5}{8}$ .            | 22. $283\frac{5}{6}$ by $32\frac{3}{4}$ .    |
| 7. 8 by $\frac{4}{5}$ .            | 15. $\frac{1}{1\frac{6}{7}}$ by $\frac{1}{1\frac{2}{3}}$ . | 23. $202\frac{1}{4}$ by 125.                 |
| 8. 12 by $\frac{7}{9}$ .           | 16. $3\frac{1}{3}$ by $\frac{5}{6}$ .                      | 24. $3614\frac{5}{12}$ by $144\frac{1}{4}$ . |

Find the value of

25.  $(\frac{2}{3} \text{ of } \frac{6}{7} \times \frac{3}{8}) \div (\frac{1}{9} \text{ of } \frac{3}{8} \text{ of } \frac{2}{11})$ .
26.  $(3\frac{4}{7} \times \frac{5}{6} \text{ of } 71\frac{2}{3}) \div 4\frac{1}{2} \times \frac{7}{8} \text{ of } 16$ .
27.  $7\frac{8}{11} \div \frac{1}{4} \text{ of } \frac{2}{3} \text{ of } \frac{8}{9}$ .
28.  $2\frac{3}{4} \text{ of } 5\frac{1}{2} \div 6\frac{2}{3} \text{ of } \frac{9}{10}$ .
29.  $(\frac{3}{4} \div \frac{5}{3}) \div (\frac{9}{8} \div 2\frac{9}{11})$ .
30.  $(\frac{7}{8} \text{ of } 2\frac{1}{4} \times \frac{1\frac{5}{8}}{2}) \div \frac{4\frac{7}{4}}{7}$ .
31.  $(\frac{1}{9} \text{ of } \frac{5}{8}) \times (\frac{4}{5} \div \frac{7}{9})$ .
32.  $\frac{7}{8} \text{ of } \frac{5}{16} \text{ of } 1\frac{2}{7} \div \frac{1}{9} \text{ of } \frac{7}{11} \text{ of } 3\frac{1}{2} \text{ of } 9$ .
33.  $11 \div (\frac{2}{3} \times 5\frac{1}{2} \times 7)$ .
34.  $(2\frac{1}{2} \div \frac{3}{4}) \times \frac{2}{3} \text{ of } \frac{4}{5} \text{ of } \frac{11}{13}$ .
35.  $\frac{3\frac{1}{4} + 27\frac{5}{6} - 6\frac{1}{2} \times 3\frac{1}{4}}{(\frac{3}{5} \text{ of } 25\frac{1}{4}) + (\frac{3}{4} \text{ of } 27)}$
36.  $\frac{75\frac{1}{3} + 8\frac{1}{6} + 7\frac{3}{4} - 1\frac{7}{8}}{31\frac{1}{4} \div 6\frac{1}{4}}$ .

## REVIEW OF FRACTIONS

## SERIES 25

1. A grocer packed in a box for shipment  $15\frac{5}{8}$  lbs. sugar,  $19\frac{3}{4}$  lbs. coffee,  $2\frac{5}{16}$  lbs. tea,  $17\frac{3}{8}$  lbs. ham, and  $25\frac{1}{2}$  lbs. bacon. What was the total weight of the contents of the box ?

2. If  $18\frac{4}{5}$  gallons were sold from a barrel of molasses containing  $45\frac{5}{8}$  gallons, how many gallons remained in the barrel ?



3. The average yield per acre of a field of wheat was  $18\frac{3}{4}$  bushels. What was the total yield if the field contained  $13\frac{5}{8}$  acres ?

4. How many pounds of cheese at  $18\frac{2}{3}$  cents per pound can be bought for  $\$1.23\frac{1}{5}$  ?

5. If  $\frac{2}{5}$  of a bin holds 165 bushels of wheat, how many bushels would  $\frac{3}{11}$  of the bin hold ?

6. A merchant sold 75 pounds of butter at  $24\frac{3}{5}$  cents per pound, 93 dozen eggs at  $16\frac{2}{3}$  cents per dozen, and  $48\frac{5}{8}$  gallons of milk at 25 cents per gallon. What was the total amount of the sales ?

7. If  $3\frac{1}{8}$  yards of cloth cost  $\$12.50$ , what will  $23\frac{7}{8}$  yards cost ?

8. If  $\$2.06\frac{1}{4}$  buy  $7\frac{1}{2}$  lbs. of tea, how many lbs. can be bought for  $\$3.71\frac{1}{4}$  ?

9. What fraction of a short ton is  $\frac{5}{7}$  of a long ton ?

10. A pole was broken off 12 feet from the top, leaving 28 feet still standing. What part of the pole remained standing ?

11. The distance by rail from Baltimore to Washington is  $39\frac{5}{8}$  miles. If a way-station on the road is  $18\frac{5}{12}$  miles from Baltimore, how far is it from Washington ?

12. I sold  $\frac{5}{8}$  of a piece of goods containing  $39\frac{3}{4}$  yards. How many yards remained in the piece ?

13. A person owns  $\frac{3}{8}$  of a ship and sells  $\frac{2}{3}$  of his share for  $\pounds 1,260$ . What is the value of the ship ?

14. A man invested  $\frac{2}{5}$  of his capital in bank stock,  $\frac{3}{4}$  of the remainder in real estate, and had still  $\$6,000$  left. Find his capital.

15. After taking out of a purse  $\frac{2}{5}$  of its contents,  $\frac{2}{3}$  of the remainder was found to be 13s.  $5\frac{1}{2}$ d. What part of  $\pounds 3$  was the whole sum ?

16. What fraction of  $\pounds 58$  5s. 6d. is  $\frac{3}{7}$  of  $\pounds 17$  2s. 3d ?

17. If a pint contains  $34\frac{2}{3}$  cubic inches, how many gallons of water will fill a cistern 4 ft. 4 in. long, 2 ft. 8 in. broad, and 1 ft.  $1\frac{1}{2}$  in. deep ?

18. A receives  $\frac{3\frac{1}{8}}{7\frac{1}{2}}$  of an estate and B  $\frac{4}{9}$  of  $\frac{1}{3}\frac{8}{5}$  of the remainder.

C gets what is left and finds that his share is worth \$872 more than A's. What is the value of the estate ?

19. The divisor is  $3\frac{3}{8} + 3\frac{6}{7}$  and the quotient is  $\frac{\frac{3}{4} - \frac{1}{6}}{\frac{4}{9}}$  of  $\frac{1}{16}$ . Find the dividend.

20. How many boxes, each holding  $\frac{3}{4}$  of a quart, will be required to hold 12 bus. 3 pk. 1 gal. 2 qt. of strawberries ?

21. A man earns \$280 in  $2\frac{1}{3}$  months. If he spend in  $4\frac{1}{8}$  months what he earns in  $3\frac{1}{2}$  months, how much will he save in a year ?

22. A produce merchant exchanged  $48\frac{3}{5}$  bushels of oats at  $39\frac{3}{4}$  cents per bushel, and  $13\frac{1}{2}$  barrels of apples at \$3.85 per barrel, for butter at  $37\frac{1}{2}$  cents per pound. How many pounds of butter did he receive ?

23. What quantity taken from  $159\frac{1}{7}$  will make it exactly divisible by  $12\frac{5}{8}$  ?

24. What must be the length of a plot of ground, if the breadth is  $15\frac{3}{4}$  feet, that its area may contain 46 square yards ?

25. A merchant bought a number of barrels of flour for \$1,800 ; he used 20 barrels, and sold  $\frac{4}{5}$  of the remainder for \$1,568, which was \$224 more than cost. How many barrels did he buy ?

26. A, B, and C own a vessel, each having equal shares. They sell respectively  $\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{1}{5}$  of their shares to D, who dies and leaves his share equally among them. If B's and C's interests in the ship be now worth \$37,300, what is the value of A's share ?

27. A merchant sold 20 barrels of flour for  $\$127\frac{1}{2}$ , which was  $\frac{5}{8}\frac{1}{6}$  of what he received for what he had left and which he sold at  $\$6\frac{1}{4}$  a barrel. How many barrels in all did he sell ?

28. After spending \$10 less than  $\frac{3}{5}$  of my money, I had \$15 more than  $\frac{3}{10}$  of it left. How much had I at first ?

29. I had a sum of money of which I paid away  $\frac{1}{5}$ , then  $\frac{1}{2}$  of the remainder, then  $\frac{2}{3}$  of what was still left, and found that I had still left half a dollar less than  $\frac{1}{5}$  of  $\frac{3}{4}$  of the whole. What sum had I at first ?

30. A, B, and C counted their money and found that A had  $\frac{2}{5}$  of the total amount, B  $\frac{1}{4}$ , and C the remainder. How much had B and C if A had \$30 ?

31. A pedestrian travelled  $32\frac{3}{8}$  miles on Monday,  $23\frac{5}{8}$  miles on Tuesday,  $37\frac{3}{4}$  miles on Wednesday,  $19\frac{2}{3}$  miles on Thursday,  $24\frac{4}{5}$  miles on Friday, and  $16\frac{1}{2}$  miles on Saturday. What was the average distance travelled per day ?

32. If coffee loses  $\frac{1}{15}$  of its weight in roasting, how much green coffee will be needed to make 252 pounds of roasted coffee ?

33. What is (a) the H.C.F. and (b) the L.C.M. of  $\frac{20}{63}$  and  $\frac{25}{108}$  ?

34. Divide the L.C.M. of  $4\frac{1}{5}$  and  $9\frac{1}{3}$  by their H.C.F.

35. How many times does the sum of  $12\frac{4}{5}$  and  $8\frac{7}{8}$  contain their difference ?

36. A certain number divided by  $8\frac{4}{17}$ , the quotient increased by  $2\frac{3}{4}$ , the sum multiplied by  $\frac{2\frac{1}{2}}{3}$ , and the result diminished by  $\frac{1}{2}$  of  $\frac{3}{7}$  of  $14\frac{4}{9}$ , gives  $2\frac{2}{3}$ . What is the number ?

37. One-fourth of  $\frac{2\frac{1}{3}}{7}$  of the length of a pole is in the mud, two-thirds of the remainder is in the water, and there are  $5\frac{1}{2}$  feet in the air. What is the length of the pole ?

38.  $\frac{4}{5}$  of A's stock was destroyed by fire,  $\frac{7}{8}$  of the remainder was injured by water and smoke ; he sold the uninjured goods at cost price, and the injured goods at  $\frac{1}{3}$  of cost price. He realized \$1,155. What did he lose by the fire ?

39. A can do a piece of work in 35 days ; B can do it in 40 days ; C can do it in 45 days. In what time will they do it, all working together ?

40. A and B can reap a field of wheat in 3 days ; A and C in  $3\frac{1}{2}$  days ; and B and C in 4 days. In what time would each working alone do the work ?

41. A can do a piece of work in 27 days, and B in 15 days ; A works at it alone for 12 days, B then works alone for 5 days, and then C finishes the work in 4 days. In what time could C have done the work by himself ?

42. A can do a piece of work in  $\frac{2}{3}$  of a day and B in  $\frac{1}{2}$  of a day. In what time can both together do it? If \$1.40 be paid for the work, how much should A receive?

43. A cistern is filled by two pipes in 18 and 20 minutes respectively, and emptied by a tap in 40 minutes. What part of it will be filled in 10 minutes when all are opened at the same time?

44. I bought  $48\frac{2}{3}$  pounds of tea at  $63\frac{3}{4}$  cents per pound. How many pounds more would have been received, if the price had been  $7\frac{1}{2}$  cents per pound less, for the same money?

45. A farmer exchanged  $8\frac{3}{4}$  tons of hay at \$9 $\frac{3}{4}$  per ton for coal at \$5 $\frac{1}{4}$  per ton. How many tons of coal did he receive?

46. A dealer bought 46 bales of cotton, averaging 435 pounds per bale, at  $11\frac{1\frac{7}{8}}{2}$  cents per pound, and sold the entire quantity at  $12\frac{3}{4}$  cents per pound. If his expenses for freight, insurance, commission, etc., amounted to \$95.30, what was his total net profit?

47. If a fruit vendor buy lemons at the rate of 5 for 3 cents, how many must he sell at the rate of 6 for 5 cents to gain 14 cents?

48. A and B being  $135\frac{3}{4}$  miles apart started toward each other and met in 4 days, A travelling at the rate of  $3\frac{1}{2}$  miles per hour and  $6\frac{3}{4}$  hours per day, and B at the average rate of  $2\frac{3}{4}$  miles per hour. How many hours per day did B travel?

49. A barrel has two faucets of different sizes. If only the larger faucet is opened, the barrel will be emptied in 8 minutes; and if both are opened, it will be emptied in 5 minutes. In what time will the barrel be emptied if only the smaller faucet be opened?

50. A owes B \$30.28 $\frac{1}{8}$ , and pays it in  $32\frac{1}{4}$  yards of dress goods at  $82\frac{1}{2}$  cents per yard, and the remainder in calico at  $5\frac{1}{4}$  cents per yard. How many yards of calico should B receive?

51. A laborer withdrew  $\frac{5}{12}$  of his deposit from a savings bank, and with  $\frac{2}{5}$  of the money thus withdrawn paid his rent, which was \$12. How much money has he still on deposit?



52. A speculator invested  $\frac{1}{4}$  of his money and \$200 in bonds,  $\frac{1}{3}$  of his money and \$500 in railway stocks,  $\frac{1}{6}$  of his money and \$600 in real estate, and the balance, which was \$1,200, he deposited in a bank. How much was he worth ?

53. I sold 5 jars of butter weighing  $35\frac{1}{2}$ ,  $32\frac{3}{4}$ ,  $31\frac{7}{8}$ ,  $29\frac{5}{8}$ , and  $36\frac{1}{4}$  pounds respectively, at  $18\frac{3}{4}$  cents per pound. The jars weighed  $5\frac{1}{2}$ ,  $6\frac{1}{4}$ ,  $5\frac{5}{16}$ ,  $6\frac{1}{4}$ , and  $7\frac{7}{8}$  pounds respectively. I received in payment 20 pounds of coffee at  $33\frac{1}{3}$  cents per pound, 5 pounds of tea at  $62\frac{1}{2}$  cents per pound, and the balance in sugar at 4 cents per pound. How many pounds of sugar did I receive ?

54. An estate was divided among two brothers and a sister ; the elder brother received  $\frac{4}{7}$  of the estate, the younger  $\frac{1}{3}$ , and the sister the remainder, which was \$5,740 less than the elder brother received. What was the value of the estate ?

55. A tank whose capacity is 126 gallons, is  $\frac{5}{8}$  full ; if  $11\frac{3}{8}$  gallons more be poured into it, what part of the tank is full ?

56. How many suits of clothes containing  $7\frac{3}{8}$  yards each can be cut from  $110\frac{5}{8}$  yards ?

57. How much ironstone ore must be raised from a mine, so that on losing  $\frac{17}{40}$  in roasting, and  $\frac{8}{19}$  of the remainder in smelting, there may result 506 tons of pure metal ?

58. Reduce to its lowest terms the fraction  $\frac{95469}{359784}$ .

59. Two wine glasses are filled with a mixture of spirit and water, one containing 3 parts of water and 1 of spirit, and the other 4 parts of water and 3 parts of spirit. When the contents of the two glasses are mixed in a tumbler, find how many parts of the whole mixture are spirit and water.

60. Find the total number of bushels that can be put in five bins based upon the following :

$\frac{2}{3}$  of 1st bin equals 150 bushels.

$\frac{17}{20}$  of 2nd bin equals 850 bushels.

$\frac{5}{19}$  of 3rd bin equals 95 bushels.

$\frac{3}{13}$  of 4th bin equals 120 bushels.

$\frac{13}{35}$  of 5th bin equals 260 bushels.

## SHARING

## SERIES 26

1. Two farms together cost \$12,250. If one cost \$750 more than the other, find the cost of each.

2. A merchant's profits for two successive years were \$3,765. If he gained \$486 more the second year than he did the first, find his gain for each year.

3. Two farms together contain 537 acres. If one contains 43 acres 60 square rods more than the other, find the area of each.

4. The sum of two numbers is 3509 and their difference is 335. Find the numbers.

5. The sum of two numbers is 15472 and their difference is 3818. Find the numbers.

6. Divide \$2,367 among A, B, and C, giving A \$522 more than either B or C.

7. If \$3,529 were divided among Smith, Brown and Jones, so that Smith and Brown each received \$398 more than Jones, how much did each receive ?

8. Three farms cost \$14,355. The first cost \$320 more than the second, but \$590 less than the third. Find the cost of each.

9. The total length of three railroads is 3551 miles. The first is 148 miles shorter than the second, which is 294 miles shorter than the third. Find the length of each.

10. A, B, and C have together \$19,505. A has \$5,885 more than B, who has \$2,745 less than C. How much has each ?

11. The sum of three numbers is 17950. The first is 4529 less than the second but only 2083 less than the third. Find the numbers.

12. Divide \$868 between A and B, giving A \$4 for every \$3 given B. How much does each receive ?

13. The sum of two numbers is 4764 and one is three times the other. Find the numbers.

14. The sum of two numbers is 4875 and one is  $\frac{6}{7}$  of the other. Find the numbers.

15. Two men together have \$4,635. If one has  $\frac{2}{7}$  as much as the other, how much has each ?

16. A ship and her cargo are worth \$23,750. If  $\frac{3}{4}$  of the value of the ship equals  $\frac{5}{6}$  of the value of the cargo, find the value of each.

17. The sum of two numbers is 7293. If  $\frac{2}{5}$  of one equals  $\frac{4}{7}$  of the other, find the numbers.

18. The sum of two numbers is 2079. If the numbers bear to each other the ratio of 3 to 8, find them.

19. A mixture of three grades of sugar is made by putting 3 lbs. of the first grade and 4lbs. of the second to every 6 lbs, of the third. How many pounds of each grade are there in 2,470 pounds of the mixture.

20. A mixture of three grades of sugar is made by putting 2 lbs. of the first grade with every 3 of the second and 4 of the second to every 5 of the third. How many pounds of each grade are there in 1,645 lbs. of the mixture ?

21. A receives  $\frac{3\frac{1}{8}}{7\frac{1}{2}}$  of an estate and B  $\frac{4}{9}$  of  $\frac{1\frac{8}{9}}{\frac{3}{5}}$  of the remainder. C gets what is left and finds that his share is worth \$872 more than A's. What is the value of the estate ?

22. In a factory there are 264 men, women and boys. Find the number of each if there are  $\frac{1}{3}$  more women than boys and  $\frac{1}{4}$  more men than women.

23. The weekly wages of 4 men and 5 boys are \$58.50. If a man earns twice as much as a boy, what are the daily wages of each ?

24. A farmer bought 17 sheep and 15 lambs for \$137.50, paying \$1.50 more for a sheep than for a lamb. How much did he pay for a sheep and a lamb ?

25. Divide \$4,117 among four boys and three men, giving each boy \$2 when you give each man \$5.

26. Divide \$234.85 among three men, giving the first four times as much as the second and the third twice as much as the second.

27. A has 6 cows on pasture for 3 weeks, B 5 cows for 4 weeks, and C 13 cows for 2 weeks. The total cost of the pasture is \$12.80. How much should each pay ?

28. A boy rubbed a fraction off the blackboard and said when he subtracted the numerator from the denominator he got 42, but when he added the numerator to the denominator he got 104. What fraction did he rub off the blackboard ?

29. The cost of a quantity of silk at \$3.25 per yard and tweed at \$2.50 per yard was \$409.75, the whole cost of the tweed being 25 cents more than that of the silk. Find the number of yards of each.

30. Two loads of wheat at 65 cents a bushel cost \$81.25. One load weighed 780 lbs. more than the other. Find the number of bushels in each load.

31. A had 4 pints of berries and B 5 pints. C joined them and they ate all the berries. C handed over 18 cents to pay for his share. How much should A get of the 18 cents ?

32. A and B enter into partnership for a year, and A receives  $\frac{3}{8}$  of the profits and B the remainder. A invests \$2,000 additional for the second year's partnership and at the end of the year receives  $\frac{4}{9}$  of the profits. How much did each invest the first year ?

33. Three persons, A, B, and C, trade together, having a joint capital of \$4,700. A's money is in the business 6 months, B's for 8 months, and C's for 10 months. Each receives \$600 as his share of the profits. How much of the capital did each contribute ?

34. The outfit of a livery stable is worth \$3,000. One-seventh the value of the horses is equal to one-fifth the value of the vehicles, harness, etc. Find the value of the horses.

35. A can do a piece of work in  $\frac{2}{3}$  of a day and B in  $\frac{1}{2}$  a day. If both work together and \$1.40 be paid for the job, how should the money be divided ?

36. A can do as much work in 8 days as B can in 11 days, but A works only  $\frac{3}{4}$  of the time. How should \$97.50 be divided between them for work ?

37. A and B work respectively 7 and 8 hours a day, and receive equal wages. On the second day they work 9 and 10 hours respectively. They receive for their whole work \$12.50. How much should each get ?

38. A bankrupt's assets are only half of his liabilities, but



one-third of the assets are only worth 25 cents on the dollar. How many cents on the dollar can he pay ?

39. A grocer bought sugar and coffee for \$96.30, the cost of the sugar being .3794 of the cost of the coffee. What did he pay for each ?

40. The loss on a property was \$8,000. If the property was insured in three different companies for \$3,000, \$2,500 and \$4,000 respectively, what amount should each company pay ?

41. A, B, and C formed a partnership. A invested \$7,000 for 10 months, B \$6,000 for 8 months, and C \$10,000 for 12 months. The gain for the year being \$12,000, how should it be divided ?

42. Johnson and Harris engage in business Jan. 1, 1906, Johnson investing \$5,000 and Harris \$8,000. On May 1 Johnson invested \$2,000 additional ; and on June 1 Harris withdrew \$1,000. On Aug. 1 they took in Cook as a third partner, who invested \$7,000. On Jan. 1, 1907, their net gain was \$5,000. What was the share of each partner ?

43. A, B, and C began business. On Jan. 1, A invested \$7,500, and on July 1 \$2,500 additional ; on Jan. 1 B invested \$12,000, and on May 1 withdrew \$4,000 ; on Jan. 1 C invested \$10,000, and on August 1 \$7,000. The net profit for the year being \$8,500, what was each man's share, the gain being divided according to average investment ?

44. Messrs. White & Todd agree to divide their travelling expenses so that White shall pay at the rate of \$7 to every \$5 Todd pays. Now White has paid \$86.25 and Todd \$37.35. How much must Todd pay White to settle the account ?

45. The weekly wages at a mill amounted to \$874.80. In the mill there were six times as many women and twice as many men as there were boys. A man's wages were \$2.40 per day, a woman's \$1.10 per day, and a boy's 75 cents per day. How many women were there in the mill ?

46. A cash register contains \$42.05 in nickels, dimes, quarters and half-dollars. The number of quarters is 19 more than the number of halves, and 8 more than the number of dimes ; the number of nickels is twice the number of dimes, less one. Find the number of each kind of coin.

47. Divide \$105 among A, B, and C, so that B's share may be half as much again as A's, and C's a third as much again as A's and B's together.

48. A, B, and C caught a certain number of fish. When A's fish and B's fish are put together they make 110; B's and C's 130; A's and C's 120. How many did each catch?

49. A farmer shared his farm among his three sons. To the youngest he gave 80 acres, to the eldest  $\frac{4}{5}$  of the whole, and to the second  $\frac{3}{4}$  as much as to both the others. How many acres did the farm contain?

50. A man divided \$17,940 among his three sons, whose ages are 16, 18, and 26 years, in proportion to their ages. Three years afterwards he similarly divided an equal sum, and again after three years more an equal sum. How much did each son receive altogether?

51. The sum of \$1,416 is to be divided among 15 men, 20 women and 30 children, in such a manner that a man and a child shall together receive as much as two women, and all the women together shall receive \$480. Find the amount received by each man, woman, and child respectively.

52. If 13 men, 19 women, and 25 boys earn \$15,190.44 in a year (309 working days), and if a woman earns  $\frac{3}{5}$  of what a man earns, and a boy  $\frac{5}{7}$  of what a woman earns, what is the daily earning of each?

53. A, B, and C do a piece of work and are paid \$73.50 for it. The money is divided according to their efficiency and the time each worked. A's efficiency is to B's as 2 to 3 and B's to C's as 4 to 5; A worked 6 days, B 7 days, and C 8 days. How should the money be divided?

54. What does each man, woman and child get when \$178.92 is divided among 6 men, 8 women, and 10 children, so that 2 men may get as much as 3 women, and 2 women as much as 3 children?

55. Divide \$171.50 into parts proportional to  $\frac{1}{6}$ ,  $\frac{1}{4}$ ,  $\frac{2}{5}$ .

56. A debt of \$528 is paid in \$5 bills, \$2 bills and \$1 bills, the number of each denomination being proportional to 4, 7, and 10. How many of each were there?

57. A and B entered into partnership, their capitals being in the ratio of 7 to 9. After 3 months A withdrew part of his capital, so that the ratio was 2 to 3. At the end of the year A's share of the gain was \$1,500; what was B's gain?

58. A farm is divided into two parts, whose areas are as 9 to 13; the area of the larger part exceeds that of the smaller by  $18\frac{2}{11}$  acres. Find the number of acres in the farm.

59. A sum of money was divided among A, B, and C. A received  $\frac{2}{5}$  of the sum; B \$20 less than  $\frac{2}{3}$  of what was left; and the remainder, which was  $\frac{3}{4}$  of A's share, was given to C. Find the sum divided.

60. A and B have together 210 acres of land, and  $\frac{3}{4}$  of A's share is equal to  $\frac{6}{7}$  of B's. B paid \$1,470 for his land; for how much per acre must he sell it to gain \$20 per acre?

61. A flagstaff 120 feet high was broken off by the wind, and it was found that .76 of the longer part was  $\frac{2}{15}$  of  $9\frac{1}{2}$  times the shorter part. Find the length of each part.

62. A and B were candidates for election in a constituency of 2,700 voters. The votes polled by A were to those polled by B as 23 to 25, and B was elected by a majority of 100. How many persons did not vote?

63. Gunpowder is composed of nitre, charcoal, and sulphur, in the proportion of 15, 3, and 2. A certain quantity of gunpowder is known to contain 20 cwt. of charcoal; find its weight, and also the weight of nitre and of sulphur it contains.

64. Divide \$1,200 among A, B, and C, so that A may have \$70 more than B and twice as much as C.

65. A house and lot are together worth \$2,100, and  $\frac{1}{4}$  of the value of the house is equal to  $\frac{1}{3}$  of the value of the lot. Find the value of each.

66. A legacy of \$9,500 is to be divided among A, B, and C, so that A will get  $\frac{5}{19}$  of the whole and B will get  $\frac{3}{4}$  as much as C. Find the shares of each.

67. A grocer has 630 lbs. of a mixture containing chicory and coffee in the proportion of 3 to 4. What amount of coffee must be added to the mixture to make the proportion 7 to 10?

# DECIMAL FRACTIONS

## DEFINITIONS

Ours is a decimal system of notation. The unit values of the digits in every number increase in a tenfold ratio from right to left. For instance, the figure 1 standing alone denotes 1 unit. Put two 1's side by side, thus 11, and they denote 1 ten and 1 unit. Put three 1's, thus, 111, and we have 1 hundred, 1 ten, and 1 unit.

If this plan works in expressing whole numbers, why not use it in expressing fractions? If values go up in tenfold steps as we go to the left from the unit's place, we may consider them as going down in tenfold steps as we go to the right from the unit's place. All we need is a plan of marking the unit's figure. This we do by placing a dot or period after it. This is called the *Decimal Point*. Then :

$$1. = 1 \text{ unit.}$$

$$11.1 = 1 \text{ ten, 1 unit, 1 tenth, or eleven and one tenth.}$$

$$111.11. = 1 \text{ hundred, 1 ten, 1 unit, 1 tenth, 1 hundredth, or one hundred, eleven and eleven hundredths.}$$

Again :

$$1. = 1 \text{ unit.}$$

$$10.1. = 1 \text{ ten, 0 units, 1 tenth, or ten and one tenth.}$$

$$100.01. = 1 \text{ hundred, 0 tens, 0 units, 0 tenths, 1 hundredths, or one hundred and one hundredth.}$$

From these illustrations we gather that, while figures to the left of the decimal point represent whole numbers, figures to the right of the decimal point represent fractional numbers.

A **Decimal Fraction** is a fraction whose denominator is ten or some power of ten.

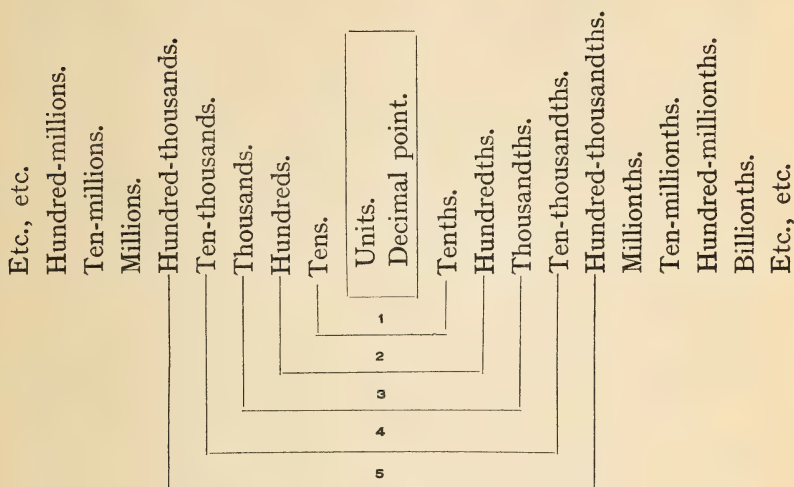
A **Pure Decimal** corresponds to a Proper Fraction, and has a value less than 1 ; as, .5, .26, .2794.

A **Mixed Decimal** corresponds to an Improper Fraction, and has a value more than 1 ; as, 7.4, 29.678.

A **Complex Decimal** corresponds to a Complex Fraction, and has a common fraction in its right-hand place ; as,  $.66\frac{2}{3}$ .



DECIMAL NOTATION SCALE



Note that the units figure occupies the centre. One place to the left and we have tens. One place to the right and we have tenths. Two places to the left and we have hundreds. Two places to the right and we have hundredths, and so on.

NUMERATION OF DECIMALS

QUESTION.—Express, orally, the decimal .0127.

METHOD.—Note that there are four decimal places occupied by figures. Starting at the decimal point, four places will carry us through tenths, hundredths, thousandths, and, lastly, ten thousandths. Read the significant figures as in the case of the whole numbers (one hundred twenty-seven). As a name for this amount use the name of the last decimal place (one hundred twenty-seven ten thousandths).

QUESTION.—Express, orally, the mixed decimal 475.175.

ANSWER.—Four hundred seventy-five, and one hundred seventy-five thousandths.

NOTE.—Never use the word “and” except to denote the break between the whole number and the decimal.

Hence the following rule :

*Numerate from the decimal point to determine the denominator.*

*Read the decimal as a whole number, and give to it the denomination of the right-hand figure.*

### SERIES 27

Express, orally, the following decimals :

1. .75	9. .00625	17. .0017834.
2. .075	10. .937	18. 5.083291.
3. .325	11. .0937.	19. 7.0056832.
4. .0325.	12. .00937	20. 8.0010005.
5. .215	13. .11625	21. 6.10070082.
6. .0215.	14. .83715.	22. 1.00015008.
7. .625	15. .517382	23. 6.005832 $\frac{3}{4}$ .
8. .0625.	16. .8267513.	24. 4.10075008 $\frac{3}{8}$ .

### NOTATION OF DECIMALS

QUESTION.—Write, decimally, seventy-five hundred-thousandths.

METHOD.—Hundred-thousandths occupy the fifth place from the decimal point. To express seventy-five alone, we need only two figures (75), therefore we complete the five places needed by placing three ciphers after the decimal point, and follow with the two figures 75 ; thus, .00075.

### RULE

*Write the decimal the same as a whole number, prefixing ciphers when necessary to give each figure its true local value. Place the decimal point before the left-hand figure of the decimal.*

### SERIES 28

Write the following as decimals :

1. Seven tenths.
2. Twenty-seven hundredths.
3. Seven hundredths.
4. Three-hundred-twenty-seven thousandths.
5. Twenty-seven thousandths.
6. Seven thousandths.
7. Three-thousand-two-hundred-twenty-five ten-thousandths.

8. Six-hundred-five ten-thousandths. 9. Seventy-five ten-thousandths. 10. Eight ten-thousandths. 11. Eighty-thousand-eight hundred-thousandths. 12. Eight-thousand-eight hundred-thousandths. 13. Nine-hundred-nine hundred-thousandths. 14. Seventy-five hundred-thousandths. 15. Eight hundred-thousandths. 16. 625 ten-millionths. 17. 324 hundred-thousandths. 18. 4165 millionths. 19. 34 ten-thousandths. 20. Seven, and seven tenths. 21. Eight, and 24 hundredths. 22. 9, and 175 thousandths. 23. Twenty-five, and 3125 ten-thousandths. 24. 225, and 324 millionths. 25. Two-hundred-twenty-six millionths. 26. Two-hundred, and twenty-six millionths. 27. Six-thousand-forty-five hundred-thousandths. 28. Six thousand, and forty-five hundred-thousandths. 29. Five hundred-thousandths. 30. Five-hundred thousandths. 31. Twenty-five tenths. 32. Two-hundred-six hundredths.

33. $\frac{5}{10}$	34. $\frac{125}{1000}$	35. $\frac{204}{1000000}$	36. $\frac{13}{1000}$	37. $\frac{2400}{1000000}$
38. $\frac{24}{100}$	39. $\frac{524}{10000}$	40. $\frac{25}{10000}$	41. $\frac{25}{10000}$	42. $\frac{6}{100}$
43. $\frac{4035}{1000}$	44. $\frac{1200065}{10000}$	45. $\frac{87}{1000000}$	46. $\frac{5005}{1000}$	47. $\frac{120021}{100}$

## REDUCTION OF DECIMALS

To reduce a decimal to a vulgar fraction.

ILLUSTRATION 1.—Reduce .045 to a vulgar fraction.

SOLUTION.—The decimal .045 is read forty-five thousandths. This in vulgar fraction form is  $\frac{45}{1000}$ . This fraction, brought to its lowest terms, is equal to  $\frac{9}{200}$ .

ILLUSTRATION 2.—Reduce  $.66\frac{2}{3}$  to a vulgar fraction.

SOLUTION.—

$$.66\frac{2}{3} = \frac{66\frac{2}{3}}{100} = \frac{\frac{200}{3}}{\frac{100}{1}} = \frac{200}{3} \times \frac{1}{100} = \frac{200}{300} = \frac{2}{3}$$

## SERIES 29

Reduce to common fractions in their lowest terms :

1. .5.	9. .008.	17. 3.00128.
2. .8.	10. .5856.	18. 5.34375.
3. .25.	11. 16.75.	19. 50.0004.
4. .68.	12. 5.064.	20. .12 $\frac{1}{2}$ .
5. .06.	13. .00625.	21. .6 $\frac{1}{4}$ .
6. .375.	14. 5.00125.	22. .33 $\frac{1}{8}$ .
7. .625.	15. 18.03125.	23. .0 $\frac{5}{8}$ .
8. .075.	16. 7.31625.	24. 25.16 $\frac{2}{3}$ .

To reduce a vulgar fraction to a decimal.

ILLUSTRATION 1.—Reduce  $\frac{7}{3125}$  to an equivalent decimal.

```

3125)7000(.00224
    6250
    -----
     7500
     6250
     -----
    12500
    12500
    -----

```

SOLUTION.—Divide the denominator into the numerator, adding ciphers as required to complete the division. In the case given, 5 ciphers need to be added. Now, adding a cipher to a number multiplies it by 10, so that in adding 5 ciphers to the numerator we have really multiplied the numerator by 100000. Now, multiplying the dividend by 100000 would have the effect of multiplying the quotient by 100000. So that the quotient, to be

made correct, must be divided by 100000.  $224 \div 100000 = \frac{224}{100000} = 224$  hundred-thousandths = .00224. It will be noted that in .00224 there are five figures to the right of the decimal point, which is the number of ciphers added in the division.

From this and similar cases we derive the following rule :

## RULE

*Annex ciphers to the numerator and divide by the denominator.*

*Point off as many decimal places in the quotient as there are ciphers annexed.*

*If the quotient does not contain the required number of figures, add ciphers to the left of the quotient to complete the number.*

NOTE 1.—If the division is not exact, when a sufficient number of decimal places has been obtained, the sign + may be annexed to show



that the division is not complete, or it may be expressed as a complex decimal.

NOTE 2.—A fraction in its lowest terms can be reduced to a pure decimal only when its denominator contains no prime factors but 2 and 5. If the denominator contains any other prime factor the division will not end. The decimals thus produced are called *Repeating Decimals*, and the figures repeated, *Repetends*.

ILLUSTRATION 2.—Reduce  $\frac{1}{3}$  to an equivalent decimal.

$$\begin{array}{r} 3 \overline{)1000} \\ 333 + \end{array}$$

$$\begin{array}{l} \frac{1}{3} = .333 + \\ = .\dot{3} \end{array}$$

SOLUTION.—The division in this case would never end. One-third, therefore, produces a repeating or circulating decimal. This is expressed, as illustrated, by placing a period above the three—thus,  $\dot{3}$ .

ILLUSTRATION 3.—Reduce  $\frac{5}{6}$  to an equivalent decimal.

$$\begin{array}{r} 6 \overline{)50000} \\ 8333 + \end{array}$$

$$\begin{array}{l} \frac{5}{6} = .8333 + \\ = .8\dot{3} \end{array}$$

SOLUTION.—In this case we have a mixed circulating decimal, which does not begin to repeat until after the first figure.

### SERIES 30

Reduce to equivalent decimals. Do not extend beyond four significant figures.

1.  $\frac{1}{2}$ .

2.  $\frac{3}{4}$ .

3.  $\frac{4}{5}$ .

4.  $\frac{5}{8}$ .

5.  $\frac{7}{8}$ .

6.  $\frac{5}{9}$ .

7.  $\frac{5}{16}$ .

8.  $\frac{1}{3}$ .

9.  $\frac{2}{3}$ .

10.  $\frac{1}{16}$ .

11.  $\frac{5}{12}$ .

12.  $\frac{113}{800}$ .

13.  $\frac{19}{25}$ .

14.  $\frac{1}{64}$ .

15.  $\frac{3}{12500}$ .

Reduce the following to decimals of

2 dec. places. | 3 dec. places. | 4 dec. places. | 5 dec. places.

16.  $19\frac{2}{8}$ .

18.  $75\frac{7}{12}$ .

20.  $278\frac{3}{4}$ .

22.  $8\frac{7}{128}$ .

17.  $415\frac{5}{8}$ .

19.  $91\frac{9}{32}$ .

21.  $67\frac{5}{9}$ .

23.  $15\frac{6}{11}$ .

5 dec. places.

6 dec. places.

7 dec. places.

24.  $46.342\frac{7}{8}$ .

26.  $812.9312\frac{13}{16}$ .

28.  $2.91\frac{11}{64}$ .

25.  $75.19\frac{17}{32}$ .

27.  $73.41\frac{5}{6}$ .

29.  $18.273\frac{2}{3}$ .

## ADDITION OF DECIMALS

Since integers and decimals increase and decrease regularly by tenfold steps, it is clear that decimals may be added, subtracted, multiplied, or divided in the same manner as integers.

ILLUSTRATION 1.—Add 7.41, 247.516, 17.2179, 6.003.

$$\begin{array}{r}
 7.41 \\
 247.516 \\
 17.2179 \\
 6.003 \\
 \hline
 278.2069
 \end{array}$$

SOLUTION.—Write the numbers so that the decimal points are in the same vertical line. This must bring tenths under tenths, hundredths under hundredths, and so on. Add as in the case of integers. Place the decimal point in the sum directly beneath the points in the addends.

ILLUSTRATION 2.—Add  $37.64$ ,  $.48\frac{3}{4}$ ,  $.12\frac{1}{2}$ , and  $.6\frac{2}{3}$ .

$$\begin{array}{r}
 37.64 \\
 .4875 \\
 .125 \\
 .6666+ \\
 \hline
 38.9191+
 \end{array}$$

SOLUTION.—Write the  $37.64$ . The complex decimal,  $.48\frac{3}{4}$ , made pure, equals  $.4875$ . It is terminate. Write it as shown in the operation.  $.12\frac{1}{2}$  equals  $.125$ . It is terminate also. Write it.  $.6\frac{2}{3}$  being interminate, it is necessary to extend it to four places, since one of the other decimals has four places. Place the sign + after  $.6666$  to show that a small fraction must be added to it to make it exact. Add and point as directed.

### SERIES 31

1. Add  $.125$ ,  $9.75$ ,  $31.5625$ ,  $3.6$  and  $75.0004$ .
2. Add  $.325$ ,  $300.025$ ,  $.9375$ ,  $32.125462$  and  $7.5$ .
3. Add  $49.327$ ,  $.458$ ,  $8317.05$ ,  $341.875$  and  $32.4962$ .
4. Add  $560.379$ ,  $.45687$ ,  $350.0036$ ,  $7.074$  and  $52.257$ .

5. Add three, and seven-tenths ; twenty-five, and one hundred twenty-five thousandths ; ninety-five millionths ; two hundred two ten-thousandths ; two hundred, and two ten-thousandths.

6. What is the sum of  $25$ , and  $75$  millionths ;  $4$ , and  $24$  hundredths ;  $700$ , and  $25$  thousandths ;  $925$  thousandths ;  $7$ , and  $8$  hundredths ;  $125$  hundred-millionths ;  $327$  ten-thousandths ; and  $1000$ , and  $625$  thousandths ?

7. Add  $16\frac{3}{4}$ ,  $25\frac{1}{2}$ ,  $13\frac{7}{8}$ ,  $4\frac{7}{16}$ ,  $5\frac{1}{32}$ , and  $3\frac{5}{8}$ , expressing the result decimally.

8. What is the sum of  $\frac{1}{3}$ ,  $\frac{7}{8}$ ,  $\frac{5}{6}$ ,  $\frac{1}{12}$ ,  $\frac{3}{4}$ ,  $\frac{2}{3}$ ,  $\frac{2}{7}$ , and  $\frac{8}{9}$ , correct to five decimal places?

9. What is the sum of .125, .75, .625, .5625, .9375, .0025, .075, .00875, and .00005, expressing the result as a mixed number?

10. Find the sum of 30.5, 28.25, 40.125,  $52\frac{7}{8}$ ,  $174\frac{31}{32}$ ,  $5\frac{1}{320}$ ,  $49\frac{3}{16}$ ,  $245.01\frac{3}{4}$ , and .625.

11. Add  $4.7\frac{1}{2}$ ,  $21.3\frac{3}{4}$ ,  $8.9\frac{3}{8}$ ,  $11.0\frac{5}{16}$ , and 9.

12. Find the sum, correct to six decimal places, of  $.48\frac{2}{7}$ ,  $.006\frac{5}{8}$ ,  $46.38\frac{1}{3}$ ,  $.0008\frac{2}{9}$ , 9.26, and  $432.56\frac{2}{7}$ .

## SUBTRACTION OF DECIMALS

ILLUSTRATION.—What is the difference between 4.295 and 2.49167?

SOLUTION.—Arrange, as in the case of subtracting whole numbers, so that the larger number is above and the smaller one is beneath it. Remember the size of the number is gauged by the whole number, not by the number of figures it may contain in the decimal part. If there are not as many figures in the decimal part of the minuend as there are in the subtrahend, supply 0's. Subtract as in the case of whole numbers. Place the decimal point in the answer directly beneath the position it occupies in the minuend.

### SERIES 32

	1.	2.	3.	4.
From	18.5	2.8706	.50376	.36
Take	2.3476	.49	.065	.12704

From

5.	1.869	take	.0374.	9.	204.1	take	36.002.
6.	.0061	take	.00089.	10.	1000	take	999.99.
7.	6.723	take	2.7981.	11.	2	take	1.3678.
8.	9.305	take	7.9.	12.	17.36	take	9.0184.

## MULTIPLICATION OF DECIMALS

ILLUSTRATION.—Find the product of .497 and .31.

$$\begin{array}{r} .497 \\ .31 \\ \hline 497 \\ 1491 \\ \hline .15407 \end{array}$$

$$.497 = \frac{497}{1000}$$

and

$$.31 = \frac{31}{100}$$

SOLUTION.—Applying the rule for the multiplication of common fractions :

$$\frac{497}{1000} \times \frac{31}{100} = \frac{15407}{100000} = .15407.$$

Note that the denominator of the first fraction is 1, followed by three ciphers, and that the denominator of the second fraction is 1, followed by two ciphers. The denominator of the product is therefore 1, followed by three plus two, or five ciphers.

## RULE

*Multiply as with whole numbers ; and from the right of the product point off as many decimal places as are contained in both factors.*

NOTE 1.—If the product contains fewer figures than are to be pointed off, supply the deficiency by prefixing ciphers.

NOTE 2.—Terminate Complex Decimals are usually expanded to simple decimals before applying the rule ; and Interminate Complex Decimals are multiplied as convenience best dictates, either by reducing to equivalent common fractions or by expanding the decimal places sufficiently far for accuracy.

Thus,  $31.87\frac{1}{2} \times .97\frac{1}{4}$  would be handled as  $31.875 \times .9725$ .

Again,  $.66\frac{2}{3} \times .14\frac{2}{7}$  would be handled as  $\frac{66\frac{2}{3}}{100} \times \frac{14\frac{2}{7}}{100} = \frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$ .

NOTE 3.—Decimals are multiplied by 10, 100, 1000, etc., by moving the decimal point of the multiplicand as many places to the right as there are ciphers in the multiplier.

Multiply

SERIES 33

- |                    |                                       |
|--------------------|---------------------------------------|
| 1. .75 by .8.      | 9. .0023 by .014.                     |
| 2. .17 by .9.      | 10. .4362 by 28.                      |
| 3. .62 by .12.     | 11. .562 by .00074.                   |
| 4. 6.2 by .39.     | 12. .0354 by 6.005.                   |
| 5. 9.26 by .0025.  | 13. 6.354 by .34 $\frac{3}{4}$ .      |
| 6. .493 by .647.   | 14. 7.59 by .75 $\frac{2}{3}$ .       |
| 7. 35.24 by .005.  | 15. 30.03 $\times$ .5 $\times$ .0007. |
| 8. 43.86 by 17.45. | 16. 25 $\times$ .125 $\times$ .3.     |



## DIVISION OF DECIMALS

Division of decimals is a process similar to division of whole numbers. The placing of the decimal point is the only thing that may occasion any difficulty, and even this point is made clear by the application of two principles that should already be in hand.

## ILLUSTRATION 1.—Divide .25 by 5.

$$\begin{array}{r} 5 \overline{) .25} \\ \underline{.05} \end{array}$$

SOLUTION.—Look at the decimal as a vulgar fraction,  $.25 = \frac{25}{100}$ . Then  $\frac{25}{100} \div 5 = \frac{25}{100} \times \frac{1}{5} = \frac{5}{100} = .05$ .

Note that the quotient must be the same, in denomination, as the dividend. If we had to divide .025 by 5, it would be  $\frac{25}{1000} \div 5$ , or  $\frac{25}{1000} \times \frac{1}{5} = \frac{5}{1000} = .005$ . Note the application of the first principle. If the divisor is a whole number, the quotient will be of the same denomination as the dividend. 25 hundredths divided by 5 gives as a result 5 hundredths, just as 25 tens divided by 5 gives as a result 5 tens, and so on.

## ILLUSTRATION 2.—Divide 47.1346 by 7.35.

$$7.35 \overline{) 47.1346}$$

$$735 \overline{) 4713.46} (6.41$$

$$\begin{array}{r} 4410 \\ 3034 \\ 2940 \\ 946 \\ 735 \\ \hline 11 \end{array}$$

SOLUTION.—Multiply both divisor and dividend by 100. Since the divisor is not a whole number, we make it a whole number by multiplying it by 100. To keep the equality we also multiply the dividend by 100 (second principle). With the divisor a whole number we proceed as in ordinary long division. In pointing off the places we proceed, as in the first example, to give the quotient the same number of places as the dividend contains (two). If it were required to carry the division out to any further number of places, it may be done by affixing ciphers to the remainder

until the required number of divisions can be performed.

## RULE

1. Reduce the divisor to a whole number by moving the decimal point the required number of places to the right.
2. Move the decimal point in the dividend the same number of places to the right, annexing ciphers if necessary.
3. The quotient to the decimal point is a whole number, that to the right is a decimal.

Divide

SERIES 34

- |                      |  |
|----------------------|--|
| 1. .5625 by 25.      | 13. 29 by .06 (4 places).                          |
| 2. .1291136 by 262.  | 14. 82.97 by 71 (3 places).                        |
| 3. .96351 by 346.    | 15. 2.3783 by 43 (4 places).                       |
| 4. .83094 by 75.     | 16. $75\frac{2}{3}$ by 27 (5 places).              |
| 5. .84375 by .25.    | 17. $15\frac{7}{8}$ by $14\frac{1}{3}$ (4 places). |
| 6. .1291136 by .262. | 18. $\frac{5}{9}$ by 27 (3 places).                |
| 7. .763 by .125.     | 19. $\frac{5}{8}$ by 30 (4 places).                |
| 8. .843 by 2.25.     | 20. $1.008\frac{3}{4}$ by 9 (4 places).            |
| 9. 375 by .25.       | 21. $14\frac{2}{7}$ by $8\frac{1}{3}$ (2 places).  |
| 10. 8.45 by .075.    | 22. 35 by 21 (4 places).                           |
| 11. 94.3 by .0004.   | 23. 378.45 by 93.4 (6 places).                     |
| 12. .635 by .000005. | 24. $63.71\frac{7}{18}$ by 25 (3 places).          |

## REPEATING OR CIRCULATING DECIMALS

In reducing common fractions to decimals we find that sometimes the division does not terminate. Even in such fractions, if the division is carried far enough, a remainder would be obtained which had occurred before and a set of figures would be repeated in the same order in a never-ending succession. Such a decimal is called a *repeating*, or *circulating*, or *interminate decimal*, or simply a *repetend*.

Thus,  $\frac{1}{3} = .3333+$  or  $\frac{1}{3} = .\dot{3}$ . Again,  $\frac{5}{7} = .714285714285+$  or  $\frac{5}{7} = .\dot{7}1428\dot{5}$ .

Note that when the repetend consists of one figure it is indicated by placing a point over it. If it consists of more than one figure, a point is placed over the first and last figures of the repeating part.

A **Mixed Circulating Decimal** is one in which all of the figures do not repeat.

Thus,  $\frac{1}{12} = .08333+$ , or  $\frac{1}{12} = .08\dot{3}$ .

To reduce a pure repetend to an equivalent vulgar fraction.

ILLUSTRATION.—Express  $.2\dot{5}$  as a common fraction.

$$.2\dot{5} = \frac{25}{99}$$

SOLUTION.—

$$.2\dot{5} = .252525 +$$

$$\therefore 100 \text{ times } .2\dot{5} = 25.252525 +$$

$$\text{And } 1 \text{ times } .2\dot{5} = .252525 +$$

$$\text{Subtracting, } .99 \text{ times } .2\dot{5} = 25$$

$$\therefore .2\dot{5} = \frac{25}{99}$$

### RULE

*Put the period for numerator, and as many nines as there are places in the period for denominator.*

To reduce a mixed repetend to an equivalent vulgar fraction.

ILLUSTRATION.—Express  $.24\dot{5}\dot{6}$  as a common fraction.

SOLUTION.—

$$2456$$

$$24$$

$$2432 \quad \frac{2432}{9900} \text{ Ans.}$$

$$.24\dot{5}\dot{6} = .2456565656 +$$

$$\therefore 10000 \text{ times } .24\dot{5}\dot{6} = 2456.565656 +$$

$$\text{And } 100 \text{ times } .24\dot{5}\dot{6} = 24.565656 +$$

$$\text{Subtracting, } 9900 \text{ times } .24\dot{5}\dot{6} = 2432 = 2456 - 24$$

$$\therefore .24\dot{5}\dot{6} = \frac{2456 - 24}{9900}$$

### RULE

*Subtract the finite part from the whole and set down the difference for the numerator.*

*For denominator put as many 9's as there are places in the "infinite" part, followed by as many 0's as there are places in the "finite" part.*

### SERIES 35

Express as circulating decimals :

$$1. \quad \frac{1}{3}.$$

$$2. \quad \frac{2}{3}.$$

$$3. \quad \frac{1}{6}.$$

$$4. \quad \frac{5}{6}.$$

$$5. \quad \frac{3}{7}.$$

$$6. \quad \frac{6}{7}.$$

$$7. \quad \frac{5}{12}.$$

$$8. \quad \frac{9}{11}.$$

$$9. \quad \frac{5}{14}.$$

$$10. \quad \frac{13}{28}.$$

$$11. \quad \frac{13}{30}.$$

$$12. \quad \frac{11}{21}.$$

$$13. \quad \frac{5}{18}.$$

$$14. \quad \frac{19}{80}.$$

$$15. \quad \frac{7}{13}.$$

Express as fractions in their lowest terms :

16. $\dot{.5}$ .	21. $\dot{.263\dot{5}}$ .	26. 7.2011.
17. $\dot{.7\dot{2}}$ .	22. $\dot{.312\dot{3}}$ .	27. 2.5306.
18. $\dot{.46\dot{8}}$ .	23. $\dot{.42\dot{5}}$ .	28. $\dot{.251\dot{6}}$ .
19. $\dot{.15\dot{3}}$ .	24. 4.253.	29. $\dot{.03\dot{5}\dot{7}}$ .
20. $\dot{.36\dot{9}}$ .	25. $\dot{.0472\dot{6}}$ .	30. $\dot{.0024\dot{5}}$ .

## REVIEW OF DECIMALS

### SERIES 36

1. If a pound of tea be worth  $\$.62\frac{1}{2}$ , what is .8 of a pound worth ?
2. Which is the greater, .0025 of a mile or .79 of a rod ? How much greater ?
3. Reduce  $\frac{31}{85}$  of a long ton (2240 lbs.) to the decimal of a short ton.
4. Bought a cord of wood for \$4.625, a cheese for \$7.56 $\frac{1}{4}$ , and 14 $\frac{9}{16}$  lbs. of butter at 25 cents per lb. What was the cost of the whole ?
5. How many pairs of trousers can be made from 48.6 yards of cloth, allowing 1.8 yards per pair ?
6. A certain number multiplied by 2.5 and the product divided by 5.2 produces 1. What is the number ?
7. A man who owned .1875 of a mine sold .17 of his share. What fractional part had he left ?
8. Divide the sum of .075 and .0075 by the difference between 7.5 and .75.
9. Express 3.74976 minutes as the decimal of a week (4 significant figures).
10. What decimal must be taken from the sum of 69 $\frac{1}{8}$ , 8.2, 5.445, .065, and 20 $\frac{1}{12}$  so that it will contain 6.05 an exact number of times ?
11. Multiply 350.4 by .0105 and divide the product by .0000219.
12. The weight of a cubic foot of water is 62 $\frac{1}{2}$  lbs., and an imperial gallon contains 277.274 cubic inches. Find the weight in ounces of a pint of water,



13. Express  $.72\dot{5}6\dot{7}$  as a vulgar fraction in its lowest terms.
14. The average of seven numbers is 26.98. The average of the first two is 34.5 and of the next three is 19.3. Find the average of the remaining two.
15. If the price of 1.875 lb. of tea is 1.3749 shillings, how much can be bought for £15 8s. ?
16. The average of four quantities is  $18\frac{35}{297}$ ; the first is  $26.2\dot{0}\dot{7}$ , the second  $3.59\dot{2}$ , and the third 38.06. Find the fourth.
17. At \$1.75 per rod what will it cost to fence a piece of land 63.5 rods long and 27.75 rods wide ?
18. A drover lost .065 of his flock by wolves, .105 by disease, and .27 by theft; he then sold .75 of what remained, and has 280 sheep left. Find the number in his original flock.
19. A man left .375 of his estate to his wife, .4 of the remainder to his daughter, and the remainder to his son, who received \$7,678.50. How much did his wife and daughter each receive ?
20. A speculator has 640 acres of land; he sells .25 of it at \$43 per acre, .16 of the remainder at \$30 per acre, as many acres as he sold the first and second times at  $\$37\frac{1}{2}$  per acre, and the balance at \$24 per acre. How much did he gain if he paid \$25 per acre for the land ?
21. If .375 of a barrel of sugar cost \$4.77, what will .875 of a barrel cost ?
22. A man has a flock of 720 sheep and sells .625 of his flock. What is the value of the remainder of the flock at  $\$3\frac{1}{4}$  per head ?
23. 427 is .875 of what number ?
24. From a farm of  $125\frac{3}{10}$  acres there were sold at one time 27.63 acres and at another  $34\frac{3}{8}$  acres. How many acres remained ?
25. By dividing 11.4575 by a certain divisor we obtain the quotient 2.5. What would be the quotient were we to divide .192486 by the same divisor ?
26. From an oil tank containing 375.087 gallons there leaked out each day  $2\frac{5}{8}$  gallons. How many gallons remained in the tank at the end of 27 days ?

27. A certain decimal multiplied by 100 equals .65. What is .2 of the decimal ?

28. I sold .355 of a cask of vinegar at \$.35 a gallon and received \$4.97 for it. How many gallons did the cask contain ?

29. How many tons of coal in five carloads, weighing respectively 22.815 tons, 21.86 tons, 20.185 tons, 19.998 tons, and 18.125 tons ?

30. I lack 15.035 bushels of having 24.08 bushels of wheat. How many bushels of wheat have I ?

31. The sum of five addends is 375.045. If four of the addends are 72.05, 31.009, .4158, and 8.3, what is the fifth ?

32. A man bequeathed .125 of his property to an orphan asylum,  $\frac{5}{16}$  to his son, and the remaining \$24,120 to his wife. What was the value of his property ?

33. A and B at first had farms of equal size, but after A added 75.003 acres to his farm and B sold 15.043 acres from his, it was found that they had 360.12 acres. How many acres had each at first ?

34. Multiply  $\frac{3}{7}$  of .175 by .285714 and divide the result by .00425.

35. Assuming that £1 = \$4.86 $\frac{2}{3}$ , find the value of \$2,300 in English currency.

36. A farmer bought 48.125 tons of hay ; for 20.25 tons of it he paid \$16 per ton, and for the rest \$18.2625 per ton ; he sold the whole at an average price of \$.945 per cwt. ; how much did he gain or lose ?

37. The weight of a cubic inch of water is 253.17 grains, and that of a cubic inch of air is .310017 grains. How many cubic inches of air are equal in weight to one cubic foot of water ?

38. What decimal multiplied by 125 will give the sum of  $\frac{5}{8}$ ,  $\frac{7}{16}$ ,  $\frac{3}{4}$ , .09375, and 2.46 ?

39. The product of three vulgar fractions is  $\frac{4}{7}$  ; two of them are expressed by the decimals .6 $\frac{2}{3}$  and .13 $\frac{1}{6}$  ; by what fraction will the third one be expressed ?

40. If a business produces an annual return of \$6,000, and of three partners one has .475 and another .38 of the profits, how much money falls to the share of the third partner ?

41. A man paid \$120 for a horse ; for a buggy  $\$36\frac{5}{18}$  more than  $\frac{3}{5}$  of the cost of the horse ; for harness  $.18\dot{5}$  of the cost of horse and buggy. Find his entire outlay.

42. The metre is 39.371 inches in length. Express the length of 25 metres as a fraction of a mile.

43. If a pound of sugar cost .0093125 of \$8, find the value of .0625 of 16 barrels of 200 pounds each.

44. A man who owned  $\frac{3}{5}$  of a steamboat sold  $\frac{7}{10}$  of his share for \$1,400 ; what decimal part of the boat does he still own, and what was the boat worth ?

# DENOMINATE NUMBERS

## TABLES OF VALUES, WEIGHTS AND MEASURES

### Canadian Money

TABLE

10 Mills . . . . .	= 1 Cent . . . . .	<i>ct.</i> or <i>c.</i>
100 Cents . . . . .	= 1 Dollar . . . . .	<i>dol.</i> or <i>\$.</i>

The mill is defined by statute, but is not recognized in ordinary commercial transactions. Its use is practically confined to stating rates of local taxation, which are generally described as so many mills on the dollar of assessed value ; thus, a rate of .015 is described as 15 mills on the dollar.

The dollar is defined by statute to be of such value that four dollars and eighty-six cents and two-thirds of a cent shall be equal in value to one pound sterling ( $\$4.86\frac{2}{3} = \pounds 1$ ).

The silver coins are the fifty-cent piece, the twenty-five-cent piece, the twenty-cent. piece, the ten-cent piece, and the five-cent piece.

The Copper coin is the cent.

There are no Canadian gold coins ; those of England and the United States are a legal tender.

### United States Money

U. S. Money is the legal currency of the United States, and is often called Federal Money. Its denominations are Eagles, Dollars, Dimes, Cents, and Mills.

The Gold coins are the double eagle, eagle, half-eagle, quarter-eagle.

The Silver coins are the dollar, half-dollar, quarter-dollar, and dime.

The Bronze and Nickel coins are the one-cent and five-cent pieces.

TABLE

10 mills . . . . .	= 1 Cent . . . . .	<i>ct.</i>
10 Cents . . . . .	= 1 Dime . . . . .	<i>d.</i>
10 Dimes or 100 Cents . . . . .	= 1 Dollar . . . . .	<i>dol.</i> or <i>\$.</i>
10 Dollars . . . . .	= 1 Eagle . . . . .	<i>E.</i>

### British Money

British or Sterling money is the currency of Great Britain.

The unit is the **Pound Sterling**, which is represented by a gold sovereign, and is equal in value to  $\$4.86\frac{2}{3}$ .



TABLE

4 Farthings ( <i>qr.</i> or <i>far.</i> ).	=	1 Penny	. . . .	<i>d.</i>
12 Pence . . . . .	=	1 Shilling	. . . .	<i>s.</i>
20 Shillings . . . . .	=	1 Pound or Sovereign	<i>£.</i>	
21 Shillings . . . . .	=	1 Guinea.		

The Gold coins are the sovereign and the half-sovereign.

The Silver coins are the crown (= 5s.), the half-crown (2s. 6d.), the shilling, and the sixpenny piece.

The Copper coins are the penny, halfpenny, and farthing.

The standard purity of the gold coins of Great Britain is 22 carats fine ; that is,  $\frac{11}{12}$  pure gold and  $\frac{1}{12}$  alloy. That of silver coins is  $\frac{37}{40}$  pure silver and  $\frac{3}{40}$  alloy.

### Avoirdupois Weight

Avoirdupois Weight is used for all the ordinary purposes of weighing.

The **measuring unit** is the pound, which is equal to 7,000 grains.

TABLE

7000 Grains = 16 Ounces ( <i>oz.</i> )	=	1 Pound	. . . .	<i>lb.</i>
100 Pounds . . . . .	=	1 Hundredweight	<i>cwt.</i>	
2000 Pounds, or 20 <i>cwt.</i>	=	1 Ton	. . . .	<i>T.</i>

The Dominion Weights and Measures Act declares that "all articles sold by weight shall be sold by Avoirdupois weight, except that gold, silver, platinum, and precious stones, and articles made thereof, may be sold by the ounce Troy or by any decimal part of such ounce."

In Great Britain and Ireland the grain, the ounce, and the pound are the same, but the hundredweight is equal to 112 lbs., and the ton to 2,240 lbs. (long ton).

### British or Imperial Avoirdupois Weight

7000 Grains ( <i>gr.</i> ) = 16 Ounces ( <i>oz.</i> )	=	1 Pound	. . . .	<i>lb.</i>
14 Pounds . . . . .	=	1 Stone	. . . .	<i>st.</i>
8 Stone . . . . .	=	1 Hundredweight	<i>cwt.</i>	
20 Hundredweight . . . . .	=	1 Ton	. . . .	<i>T.</i>

### Troy Weight

Troy Weight is used in weighing gold, silver, and jewels ; in philosophical experiments.

The **measuring unit** is the pound,

TABLE

24 Grains ( <i>gr.</i> ) . . . . .	= 1 Pennyweight . . . . .	<i>dwt.</i>
20 Pennyweights. . . . .	= 1 Ounce . . . . .	<i>oz.</i>
12 Ounces . . . . .	= 1 Pound . . . . .	<i>lb.</i>

The value of diamonds and other jewels is estimated by carats.

A carat is the weight of four grains.

### Apothecaries Weight

Apothecaries Weight is used by druggists and physicians in compounding medicines, but drugs and medicines are bought and sold by avoirdupois weight.

The **measuring unit** is the pound.

The pound, ounce, and grain are the same as in troy weight.

TABLE

20 Grains . . . . .	= 1 Scruple . . . . .	<i>sc.</i> or $\varnothing$
3 Scruples . . . . .	= 1 Dram . . . . .	<i>dr.</i> or $\mathfrak{s}$
8 Drams . . . . .	= 1 Ounce . . . . .	<i>oz.</i> or $\mathfrak{z}$
12 Ounces . . . . .	= 1 Pound . . . . .	<i>lb.</i>

### Apothecaries Fluid Measure

Apothecaries Fluid Measure is used in mixing liquid medicines.

TABLE

60 Minims, or Drops ( <i>m</i> ). . . . .	= 1 Fluid Drachm. . . . .	<i>f</i> $\mathfrak{z}$
8 Fluid Drachms . . . . .	= 1 Fluid Ounce . . . . .	<i>f</i> $\mathfrak{z}$
20 Fluid Ounces . . . . .	= 1 Pint . . . . .	<i>O.</i>
8 Pints . . . . .	= 1 Gallon . . . . .	<i>Cong.</i>

### Comparative Table of Weights

	TROY.	AVOIRDUPOIS.	APOTHECARIES.
1 Pound	= 5760 Grains	= 7000 Grains	= 5760 Grains.
1 Ounce	= 480 Grains	= $437\frac{1}{2}$ Grains	= 480 Grains.
	175 Pounds	= 144 Pounds	= 175 Pounds.

### Measures of Capacity

TABLE

2 Pints ( <i>pt.</i> ) . . . . .	= 1 Quart . . . . .	<i>qt.</i>
4 Quarts . . . . .	= 1 Gallon . . . . .	<i>gal.</i>
2 Gallons . . . . .	= 1 Peck . . . . .	<i>pk.</i>
4 Pecks . . . . .	= 1 Bushel. . . . .	<i>bush.</i>

A gill equals  $\frac{1}{4}$  of a pint.

The prime unit or standard measure of capacity is the gallon, containing 10 Dominion standard pounds of distilled water weighed under certain conditions. Under these conditions,

1 cu. ft. of water = 62.356 lbs.

1 gallon (10 lbs.) = 277.118 cu. inches.

Again, since 1 cu. ft. of water = 62.356 lbs., which is 997.696 ozs., the weight for ordinary purposes is taken as 1000 ozs.

Cisterns, reservoirs, and the like are often measured in barrels.

1 barrel (*bbl.*) =  $31\frac{1}{2}$  gallons.

2 barrels = 1 hogshead (*hhd.*).

1 barrel of flour = 196 lbs.

1 barrel of pork or beef = 200 lbs.

The legal bushel of grain and some other substances is determined not by measure, but by weight, as indicated in the following table :

14 lbs. Blue Grass Seed .	= 1 Bushel.
34 lbs. Oats . . . .	= 1 Bushel.
36 lbs. Malt . . . .	= 1 Bushel.
40 lbs. Castor Beans . .	= 1 Bushel.
44 lbs. Hemp Seed . .	= 1 Bushel.
48 lbs. Barley . . . .	= 1 Bushel.
48 lbs. Buckwheat . .	= 1 Bushel.
48 lbs. Timothy Seed . .	= 1 Bushel.
50 lbs. Flax Seed . . .	= 1 Bushel.
56 lbs. Indian Corn . .	= 1 Bushel.
56 lbs. Rye . . . . .	= 1 Bushel.
60 lbs. Wheat . . . . .	= 1 Bushel.
60 lbs. Beans. . . . .	= 1 Bushel.
60 lbs. Red Clover Seed .	= 1 Bushel.
60 lbs. Potatoes . . . .	= 1 Bushel.
60 lbs. Turnips . . . .	= 1 Bushel.
60 lbs. Carrots . . . .	= 1 Bushel.
60 lbs. Parsnips . . . .	= 1 Bushel.
60 lbs. Beets . . . . .	= 1 Bushel.
60 lbs. Onions . . . . .	= 1 Bushel.
70 lbs. Bituminous Coal .	= 1 Bushel.

A quarter of Wheat (British Measure) is equal to 8 bushels or 480 lbs.

Measures of Extension

Measures of Extension are those used to ascertain how long a line is, or in calculating the size (extent) of a surface or solid.

A line has only one dimension—length.

Linear or Line Measure

In measuring length, linear or line measure is used.

TABLE

12 Inches ( <i>in.</i> )	. . . . .	= 1 Foot	. . . . .	<i>ft.</i>
3 Feet	. . . . .	= 1 Yard	. . . . .	<i>yd.</i>
5½ Yards, or 16½ Ft.	. . . . .	= 1 Rod	. . . . .	<i>rd.</i>
320 Rods	. . . . .	= 1 Mile	. . . . .	<i>mi.</i>

EQUIVALENTS

1 Mile = 320 Rods = 1,760 Yards = 5,280 Feet = 63,360 Inches.

Surveyors' Measure

Gunter's Chain, used by land surveyors, is 4 rods, or 66 feet long, and consists of 100 links, each 7.92 inches long.

TABLE

7.92 Inches	. . . . .	= 1 Link	. . . . .	<i>lk.</i>
25 Links	. . . . .	= 1 Rod	. . . . .	<i>rd.</i>
4 Rods, or 66 Feet	. . . . .	= 1 Chain	. . . . .	<i>ch.</i>
80 Chains	. . . . .	= 1 Mile	. . . . .	<i>mi.</i>

The following measures are used only occasionally, or for special purposes :

The **line** =  $\frac{1}{12}$  inch.

The **size** =  $\frac{1}{8}$  inch, used by shoemakers.

The **nail** =  $2\frac{1}{4}$  inches =  $\frac{1}{16}$  yard, formerly used in cloth measure.

*The word is now obsolete as a term of measurement.*

The **hand** = 4 inches, used in measuring the height of horses.

The **fathom** = 6 feet and  
The **cable-length** = 120 fathoms, } used by sailors.

The **rod, pole, or perch** =  $5\frac{1}{2}$  yards, used in measuring land, but not by surveyors.

The **furlong** = 220 yards =  $\frac{1}{4}$  mile.

The **league**, not a fixed length, but in England commonly = 3 miles.



The **geographical** or **nautical mile**, called also a minute of mean latitude, is  $\frac{1}{10800}$  of the earth's semi-circumference from pole to pole. Its length is 6,077 feet, but for rough approximations it is taken as = 6,000 feet = 1,000 fathoms.

### Square Measure

Square Measure is used in measuring surfaces; as of land, boards, painting, plastering, etc.

Area or Surface has **length** and **breadth** only, and is the space or surface included within any given lines.

A square inch, square foot, or square yard, is a square, each side of which is respectively 1 inch, 1 foot, or 1 yard in length.

TABLE

144 Square Inches ( <i>sq. in.</i> ) . . . . .	=	1 Square Foot . . . . .	<i>sq. ft.</i>
9 Square Feet . . . . .	=	1 Square Yard . . . . .	<i>sq. yd.</i>
30 $\frac{1}{4}$ Square Yards . . . . .	=	1 Square Rod . . . . .	<i>sq. rd.</i>
160 Square Rods . . . . .	=	1 Acre . . . . .	<i>A.</i>
640 Acres . . . . .	=	1 Square Mile . . . . .	<i>sq. mi.</i>

### Surveyors' Square Measure

This measure is used by surveyors in computing the area of land.

TABLE

625 Square Links . . . . .	=	1 Pole ( <i>sq. rod</i> ) . . . . .	<i>P.</i>
16 Poles ( <i>sq. rods</i> ) . . . . .	=	1 Square Chain . . . . .	<i>sq. ch.</i>
10 Square Chains . . . . .	=	1 Acre . . . . .	<i>A.</i>
640 Acres . . . . .	=	1 Square Mile . . . . .	<i>sq. mi.</i>

### Cubic Measure

Cubic Measure is used in measuring **solids** or **volume**.

A **solid** is that which has **length**, **breadth**, and **thickness**.

A **cube** is a regular solid bounded by **six equal squares** called **faces**. Hence **length**, **breadth**, and **thickness** are equal to each other.

TABLE

1728 Cubic Inches ( <i>cu. in.</i> ) . . . . .	= 1 Cubic Foot . . . . .	<i>cu. ft.</i>
27 Cubic Feet . . . . .	= 1 Cubic Yard . . . . .	<i>cu. yd.</i>
40 Cubic Feet of Round Timber, or . . . . .	} = 1 Ton . . . . .	<i>T.</i>
50 Cubic Feet of Hewn Timber . . . . .		
16 Cubic Feet . . . . .	= 1 Cord Foot . . . . .	<i>cd ft.</i>
8 Cord Feet, or 128 Cubic Feet . . . . .	= 1 Cord of Wood. <i>Cd.</i>	
24 $\frac{3}{4}$ Cubic Feet . . . . .	= 1 Perch of Stone } <i>Pch.</i>	or Masonry . }

NOTE 1.—A cubic yard of earth is called a load.

NOTE 2.—A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord; and a cord foot is 1 foot in length of such a pile.

NOTE 3.—A foot of lumber, or a board foot, is the unit of measurement in lumber. It is 1 foot long, 1 foot wide, and 1 inch thick.

### Measure of Time

Time is the measure of duration.

Time is naturally divided into **days** and **years**. The former are measured by the revolution of the earth on its axis; the latter by its revolution around the sun.

TABLE

60 Seconds ( <i>sec.</i> ) . . . . .	= 1 Minute . . . . .	<i>min.</i>
60 Minutes . . . . .	= 1 Hour . . . . .	<i>hr.</i>
24 Hours . . . . .	= 1 Day . . . . .	<i>da.</i>
7 Days . . . . .	= 1 Week . . . . .	<i>wk.</i>
365 Days . . . . .	= 1 Common Year . . . . .	<i>c. yr.</i>
366 Days . . . . .	= 1 Leap Year . . . . .	<i>l. yr.</i>
12 Calendar Months. . . . .	= 1 Civil Year . . . . .	<i>yr.</i>
100 Years . . . . .	= 1 Century. . . . .	<i>C.</i>

The **unit** of time is the *solar day*; it includes one revolution of the earth on its axis, and is divided into 24 hours, counting from midnight to midnight again.

A **solar year** is the exact time required by the earth to make one complete rotation around the sun,—365 days, 5 hours, 48 minutes, 49.7 seconds, or about  $365\frac{1}{4}$  days.

The solar year is divided in the calendar into 365 days, called a *common year*, except every fourth year, when one day is added to the month of February and the year is called a *leap year*. Since the fraction that is disregarded when 365 days is counted as a year is less than one fourth of a day, the addition of a day every fourth year is not exactly accurate. The slight error still existing is corrected by excluding from the leap years the centennial years which are not divisible by 400. Hence, to find whether any year is a leap year or not :

*Divide the number of centennial years by 400 and all other years by 4 ; if there is no remainder, the year is a leap year.*

The civil year includes both **common** and **leap** years, and is divided into 12 calendar months, viz. :

January ( <i>Jan.</i> ) . . . 31 Days	July ( <i>July</i> ) . . . 31 Days
February ( <i>Feby.</i> ) . . 28 Days	August ( <i>Aug.</i> ) . . . 31 Days
In Leap Year 29 Days	September ( <i>Sept.</i> ) . 30 Days
March ( <i>Mar.</i> ) . . . 31 Days	October ( <i>Oct.</i> ) . . . 31 Days
April ( <i>Apr.</i> ) . . . 30 Days	November ( <i>Nov.</i> ) . 30 Days
May ( <i>May</i> ) . . . 31 Days	December ( <i>Dec.</i> ) . 31 Days
June ( <i>June</i> ) . . . 30 Days	

### Standard Time

During the year 1883 the principal railroads of Canada and the United States adopted what is known as the “**Standard Time System.**” This system divides Canada and the United States into four sections or timebelts, each covering 15 degrees of longitude,  $7\frac{1}{2}^{\circ}$  of which are east and  $7\frac{1}{2}^{\circ}$  are west of the governing or standard meridian, and the time throughout each belt is the same as the astronomical or local time of the governing meridian of that belt.

The governing meridians are the 75th, the 90th, the 105th, and the 120th, west of the Greenwich Observatory, London, England, and as these meridians are just  $15^{\circ}$  apart, there is a

difference in time of **exactly** one hour between any one of them and the one next on the east, or the one next on the west ; the standard meridian next on the east being one hour faster, and the one next on the west one hour slower. Hence, the  $60^\circ$  of longitude is four hours, the  $75^\circ$  five hours, the  $90^\circ$  six hours, the  $105^\circ$  seven hours, and the  $120^\circ$  eight hours slower than Greenwich time, making five different standards of time between the Atlantic and the Pacific Oceans, viz. : Intercolonial, Eastern, Central, Mountain, and Pacific.

### Circular or Angular Measure

Circular Measure is used principally in surveying, navigation, astronomy, and geography, for reckoning latitude and longitude, determining locations of places and of vessels, and in computing difference of time.

Every circle, great or small, is divided into the same number of equal parts ; as quarters, called quadrants ; twelfths, called signs ; three hundred and sixtieths, called degrees, etc. Consequently the parts of different circles, although having the same names, are of different lengths.

The unit is the degree, which is  $\frac{1}{360}$  part of the circumference of any circle.

TABLE

60 Seconds (")	. . . . .	= 1 Minute	. . '
60 Minutes	. . . . .	= 1 Degree	. . °
30 Degrees	. . . . .	= 1 Sign	. . . S.
12 Signs, or $360^\circ$	. . . . .	= 1 Circle	. . C.

### Miscellaneous Tables

#### COUNTING

12 Things	= 1 Dozen.
12 Dozen	= 1 Gross.
12 Gross	= 1 Great Gross.
20 Things	= 1 Score.

#### PAPER

24 Sheets	= 1 Quire.
20 Quires	= 1 Ream.
2 Reams	= 1 Bundle.
5 Bundles	= 1 Bale.



## BOOKS

2 Leaves = 1 Folio.

4 Leaves = 1 Quarto, or 4to.

8 Leaves = 1 Octavo, or 8vo.

12 Leaves = 1 Duodecimo, or 12mo.

The terms **folio**, **quarto**, **octavo**, denote the number of leaves into which a sheet of paper is folded in making books.

## REDUCTION OF DENOMINATE NUMBERS

In reduction the *unit* or *denomination* of a number changes, but not the *value*. When the change is from a higher to a lower denomination the process is called *reduction descending*, and when from a lower to a higher, *reduction ascending*.

To reduce denominate numbers from higher to lower denominations.

ILLUSTRATION.—Change 1 bushel, 1 peck, 1 pint to pints.

1 bu. 1 pk. 0 qts. 1 pt. .

4	
4	pecks
1	peck
5	pecks
8	
40	quarts
2	
80	pints
1	pint
81	pints.

SOLUTION.—Since 1 bushel = 4 pecks, 1 bushel and 1 peck = 5 pecks. Since 1 peck = 8 quarts, 5 pecks =  $5 \times 8$  quarts = 40 quarts. Since 1 quart = 2 pints, 40 quarts =  $40 \times 2$  pints = 80 pints, and 80 pints + 1 pint = 81 pints; therefore, 1 bushel, 1 peck, 1 pint = 81 pints.

## RULE

*Multiply the units of the highest denomination given by that number which will reduce it to the denomination next lower, and add the units, if any, of that denomination. Continue in this manner until the required denomination is reached.*

## SERIES 37

Reduce to the lowest denomination named :

- |                                   |  |
|-----------------------------------|--|
| 1. 4 mi. 17 rd. 3 yd. 1 in.       | 11. 4 T. 5 cwt. 3 lb. 1 oz.              |
| 2. £52 10s. 4d.                   | 12. 6 lb. 8 oz. 11 dwt.                  |
| 3. 11 bu. 5 pt.                   | 13. 20 rd, 5 in.                         |
| 4. 6 bu. 1 pk. 3 qt.              | 14. 15 sq. rd. 5 sq. yd. 3 sq. ft.       |
| 5. 43° 7' 23".                    | 15. 10 A. 31 sq. rd. 5 sq. yd. 4 sq. ft. |
| 6. 15 gal. 3 qt. 1 gi.            | 16. 7 wk. 210 hr. 5 min. 31 sec.         |
| 7. 6 l.t. 50 lb. 2 oz.            | 17. 8 cu. yd. 2 cu. ft.                  |
| 8. 7 cu. yd. 4 cu. ft. 11 cu. in. | 18. 6 lb. 3 dwt. 4 gr.                   |
| 9. 190 sq. rd. 15 sq. in.         | 19. 1 bbl, 2 gal. 1 pt.                  |
| 10. 2 mi. 10 ch. 43 li.           | 20. 2 mi. 15 rd. 11 ft. 10 in.           |

Reduction from a lower denomination to a higher.

ILLUSTRATION.—Reduce 473 pt. to bushels.

$$\begin{array}{r}
 2 \overline{) 473} \text{ pt.} \\
 \underline{8236} \text{ qt.} + 1 \text{ pt.} \\
 4 \overline{) 29} \text{ pk} + 4 \text{ qt.} \\
 \underline{7} \text{ bu.} + 1 \text{ pk.} \\
 473 \text{ pt.} = 7 \text{ bu. 1 pk.} \\
 \quad \quad 4 \text{ qt. 1 pt.}
 \end{array}$$

SOLUTION.—Since 2 pt. equal 1 qt., 8 qt. 1 pk., and 4 pk. 1 bu., the successive divisors for reducing given pints to bushels are 2, 8, and 4 respectively.

Divide 473 pt. by 2 and the result is 236 qt. with a remainder of 1 pt. ; divide 236 qt. by 8 and the result is 29 pk, with a remainder 4 qt. ; divide 29 by 4 and the result is 7 bu. with a remainder 1 pk.

Write the last quotient and the several remainders in order and the required result is 7 bu. 1 pk. 4 qt. 1 pt.

## RULE

*Divide the given number by the number of the same denomination required to make one of the next higher denomination, and consider the quotient as units of the higher denomination, and the remainder as units of the lower denomination.*

*Proceed in like manner with each successive quotient until the required denomination is reached.*

*The last result and the several remainders written in order will be the answer required.*

## SERIES 38

Reduce :

1.	813551	ounces	to tons, etc.
2.	47920	grains	to pounds, etc.
3.	23769	inches	to miles, etc.
4.	17150	pounds	to long tons, etc.
5.	4276	pints	to gallons, etc.
6.	185760	seconds	to days, etc.
7.	278644	cubic inches	to cubic yards, etc.
8.	32359	farthings	to £, etc.
9.	477960	cubic feet	to cords, etc.
10.	213546	sheets of paper	to reams, etc.
11.	2368	pence	to half-crowns, etc.
12.	23754	grains, troy,	to pounds, etc.
13.	45630	mills	to dollars, etc.
14.	4700356	links	to miles, etc.
15.	2562	pints	to bushels, etc.

To reduce a fractional denominate number to a lower denomination.

ILLUSTRATION.—Reduce  $\text{£} \frac{7}{16}$  (.4375) to integers of lower denominations.

## SOLUTION

$$12\text{d.} \times 20 \times \frac{7}{16} = 105\text{d.}$$

$$105\text{d.} = 8\text{s. } 9\text{d.}$$

or

$$20\text{s.} \times \frac{7}{16} = \frac{35}{4}\text{s.} = 8\frac{3}{4}\text{s.}$$

$$12\text{d.} \times \frac{3}{4} = 9\text{d.}$$

$$\therefore \text{£} \frac{7}{16} = 8\text{s. } 9\text{d.}$$

or

$$\begin{array}{r} 7 \\ 20 \end{array}$$


---

$$16)140(8\text{s.}$$

$$128$$


---

$$12$$

$$12$$


---

$$16)144(9\text{d.}$$

$$144$$

$$12\text{d.} \times 20 \times .4375 = 105\text{d.}$$

$$105\text{d.} = 8\text{s. } 9\text{d.}$$

or

$$\text{£}.4375$$

$$20$$


---

$$\text{s.}8.7500$$

$$12$$


---

$$\text{d.}9.0000$$

$$\therefore \text{£}.4375 = 8\text{s. } 9\text{d.}$$

## SERIES 39

Reduce to lower denominations :

- |                                |                                      |
|--------------------------------|--------------------------------------|
| 1. $\frac{3}{4}$ of a day.     | 8. $\frac{1}{160}$ of a rod.         |
| 2. $\frac{7}{12}$ of a mi.     | 9. .625 of a mile.                   |
| 3. $\frac{4}{7}$ of a mo.      | 10. .727 of a ton.                   |
| 4. $\frac{7}{11}$ of an acre.  | 11. .625 of an acre.                 |
| 5. $\frac{5}{12}$ of a cu. yd. | 12. .4225 of a cord.                 |
| 6. $\frac{8}{9}$ of a gal.     | 13. .5375 of a gal.                  |
| 7. $\frac{1}{320}$ of a bu.    | 14. $\frac{3}{7}$ of a pound, Apoth. |

To reduce a fractional denominate number to a higher denomination.

ILLUSTRATION 1.—Reduce 9s. 6d. 3 far. to the decimal of a pound sterling.

$$\begin{array}{r}
 (a) \\
 4 \overline{) 3 \text{ far.}} \\
 \underline{12} \phantom{.} 6.75 \text{d.} \\
 20 \overline{) 9.5625 \text{s.}} \\
 \underline{\pounds.478125}
 \end{array}$$

SOLUTION (a).—The successive divisors to reduce farthings to pounds are 4, 12, and 20 respectively. Dividing 3 far. by 4, the result is .75d. Putting with this the 6d., the result is 6.75. Dividing 6.75d. by 12, the result is .5625s. Putting with this the 9s., the result is 9.5625. Dividing by 20, the result is .478125 pounds sterling. Or,

$$\begin{array}{l}
 (b) \\
 9\text{s. } 6\text{d. } 3 \text{ far.} = 459 \text{ far.}
 \end{array}$$

$$\pounds 1 = 960 \text{ far.}$$

$$459 \div 960 = \pounds.478125$$

SOLUTION (b).—In 9s. 6d. 3 far. there are 459 far., and in  $\pounds 1$  there are 960 far. Hence, 9s. 6d. 3 far. is  $\frac{459}{960}$  of a pound sterling.  $\pounds \frac{459}{960} = \pounds.478125$ .

ILLUSTRATION 2.—Reduce  $\pounds 19$  9s. 6d. 3 far. to dollars and cents.

$$\begin{array}{l}
 \pounds 19 \text{ 9s. 6d. 3 far.} \\
 = \pounds 19.478125 \\
 \pounds 1 = \$4.86\frac{2}{3} \\
 \pounds 19.478125 = 19.478125 \text{ times } \$4.86\frac{2}{3} \\
 = \$94.79354 + \\
 = \$94.79
 \end{array}$$

SOLUTION. — By previous illustration 9s. 6d. 3 far. are equal to  $\pounds.478125$ . Then  $\pounds 19.478125$ , at the par value of  $\pounds 1$  ( $\$4.86\frac{2}{3}$ ), are equivalent to  $\$94.79$ .



ILLUSTRATION 3.—Reduce 4 yd. 2 ft. 6 in. to the fraction of a rod.

(a)

$$6 \div 12 = \frac{1}{2} \text{ ft.}$$

$$\frac{5}{6}(2\frac{1}{2}) \div 3 = \frac{5}{6} \text{ yd.}$$

$$\frac{2^2}{6}(4\frac{5}{6}) \div 5\frac{1}{2} = \frac{2^2}{3^2} \text{ rd.}$$

(b)

$$4 \text{ yd. } 2 \text{ ft. } 6 \text{ in.} = 174 \text{ in.}$$

$$1 \text{ rd.} = 198 \text{ in.}$$

$$174 \div 198 = \frac{174}{198} = \frac{2^2}{3^2}$$

SOLUTION (a).—The successive divisors to reduce inches to rods are 12, 3, and  $5\frac{1}{2}$  respectively. 6 in. divided by 12 equal  $\frac{1}{2}$  ft. Putting with this the 2 ft., the result is  $2\frac{1}{2}$  ft.  $2\frac{1}{2}$  ft. divided by 3 equal  $\frac{5}{6}$  yd. Putting with this the 4 yd., the result is  $4\frac{5}{6}$  yd.  $4\frac{5}{6}$  yd. divided by  $5\frac{1}{2}$  equal  $\frac{2^2}{3^2}$  rd. Or,

SOLUTION (b).—4 yd. 2 ft. 6 in. equal 174 in. 1 rd. equals 198 in. 4 yd. 2 ft. 6 in. is, therefore,  $\frac{174}{198}$  of 1 rd., or  $\frac{2^2}{3^2}$  rd.

### SERIES 40

1. What fractional part of a bushel is 3 pk. 4 qt. 2 pt. ?
2. What decimal part of a pound troy is 8 oz. 10 dwt. ?
3. What fractional part of 2 hhd. 20 gal. is 30 gal. ?
4. What decimal part of  $2\frac{1}{2}$  mi. is 10 rd. 4 ft. ?
5. Reduce 3d. 3 far. to the fraction of a shilling.
6. Reduce 8 oz. 2 dr. 2 sc. 10 gr. to the decimal of a lb.
7. Reduce 20 rd. 12 ft. 10 in. to the fraction of a mile.
8. Reduce 2 sq. yd. 6 sq. ft. 100 sq. in. to the fraction of a square rod.

Reduce to Canadian Currency.

9. £47 12s. 6d.

13. £176 9s.  $8\frac{1}{2}$ d.

10. £250 15s. 9d.

14. £29 17s.  $7\frac{3}{4}$ d.

11. £75 10s. 11d.

15. £212 9s.  $5\frac{1}{4}$ d.

12. £124 12s. 9d.

### Addition of denominate numbers.

ILLUSTRATION.—Add 1 bu. 2 pk. 1 pt., 1 bu. 1 qt., 1 bu. 3 pk. 7 qt. 1 pt.

bu. bk. qt. pt.

1 2 0 1

1 0 1 0

1 3 7 1

---

3 5 8 2 = first sum.

4 2 1 0 = equivalents.

6 pecks = 1 bushel and 2 pecks.

SOLUTION.—Add each denomination separately, and the simple denominations are 3 bushels, 5 pecks, 8 quarts, and 2 pints. Changing to equivalents, the 2 pints = 1 quart, which added to the 8 quarts = 9 quarts; 9 quarts = 1 peck and 1 quart remaining. Write the 1 quart remaining under the column of quarts and add the 1 peck to the 5 pecks. Write the 2 pecks under the column of

pecks, and add the 1 bushel to the 3 bushels above. Therefore, the equivalent of 3 bushels, 5 pecks, 8 quarts, and 2 pints = 4 bushels, 2 pecks, 1 quart.

In practice omit the first sum and write only the equivalents.

### RULE

1. *Write the numbers so that units of the same name shall stand in the same column.*

2. *Begin at the right, add each column, and reduce the sum to the next higher denomination; write the remainder, if any, and add the quotient with the next column.*

### SERIES 41

1.				2.				3.			
£	s.	d.	far.	lb.	oz.	dwt.	gr.	mi.	rd.	yd.	ft. in.
75	5	8	3	5	9	12	9	5	175	3	1 8
13	16	5	1	13	4	16	8	17	248	4	2 6
96	8	11	2	41	6	8	15	25	315	1	1 11
52	13	9	1	71	11	18	22	41	214	3	1 7
7	2	4	2	56	7	13	19	50	16	2	2 9
33	18	8	3	72	8	14	16	48	296	4	1 5

4. Add 5 gal. 3 qt. 1 pt., 7 gal. 2 qt., 2 qt. 1 pt., and 8 gal. 1 pt.

5. An importer paid £375 8s. 6d. for an invoice of cutlery, £251 5s. 8d. for an invoice of files, and £73 17s. 6d. for an invoice of saws. The charges for transportation amounted to £19 2s. 4d. What was the total cost of the goods in sterling money?

6. Add 5 ton 13 cwt. 19 lb. 4 oz., 7 ton 18 cwt. 39 lb. 5 oz., 2 ton 12 cwt. 78 lb. 11 oz., and 5 ton 12 cwt. 94 lb. 7 oz.

7. What is the sum of  $\frac{3}{4}$  bu.,  $\frac{7}{8}$  pk., and  $\frac{1}{2}$  qt.?

8. Add £.832, .0125s., 5.275s., and .17d.

9. What is the sum of .9375 gal., 2.125 gal., .5 qt., and .25 pt.?

## Subtraction of denominate numbers.

ILLUSTRATION.—From 71 rd. 3 yd. 2 ft. 6 in. take 15 rd. 5 yd. 2 ft. 8 in.

rd.	yd.	ft.	in.	
71	3	2	6	
15	5	2	8	
<hr/>				
55	2½	2	10	
		½=1	6	
<hr/>				
55	3	1	4	

SOLUTION.—Write the numbers so that the units of the same denominations stand in the same vertical columns, and beginning at the right subtract as in simple numbers.

Since 8 in. cannot be subtracted from 6 in., take 1 ft. (12 in.) from 2 ft. and add it to 6 in., making 18 in. 18 in. minus 8 in. leaves 10 in., which write as inches in the remainder.

Inasmuch as 1 ft. was added to 6 in., there is but 1 ft. remaining in the minuend. Since 2 ft. cannot be subtracted from 1 ft., take 1 yd. (3 ft.) from the 3 yd. and add it to the 1 ft., making 4 ft. 4 ft. minus 2 ft. leaves 2 ft., which write as feet in the remainder.

Inasmuch as 1 yd. was added to 1 ft., there are but 2 yd. remaining in the minuend. Since 5 yd. cannot be subtracted from 2 yd., take 1 rd. (5½ yd.) from 71 rd. and add to the 2 yd., making 7½ yd. 7½ yd. minus 5 yd. leaves 2½ yd., which write as yards in the remainder. 70 rd. minus 15 rd. leaves 55 rd., which write as rods in the remainder.

Reducing ½ yd. to lower denominations and adding, the required result is found to be 55 rd. 3 yd. 1 ft. 4 in.

## SERIES 42

1.				2.				3.			
lb.	oz.	dwt.	gr.	mi.	rd.	yd.	ft.	bu.	pk.	qt.	pt.
75	6	13	12	12	75	3	2	18	1	3	1
42	9	18	23	8	318	4	1	16	3	2	1

4. A merchant sold 25 gal. 2 qt. 1 pt. of vinegar from a barrel containing 46 gal. 1 qt. 1 pt. What quantity remained?

5. A farmer engaged to deliver 15 ton 16 cwt. 75 lb. of hay, and subsequently delivered 9 ton 18 cwt. 34 lb. How much hay had he yet to deliver?

6. An English merchant's sales for a year amounted to £15,396 5s. 8d. 2 far., and his purchases £18,352 2s. 9d. 1 far. How much greater were his purchases than his sales?

7. Find the difference between .522 yd. and .02345 mi.

8. Subtract  $\frac{3}{4}$  sq. yd. from  $\frac{5}{8}$  sq. rd.

### Multiplication of denominate numbers.

ILLUSTRATION.—Each of seven bars of silver weighs 15 lb. 5 oz. 13 dwt. 16 gr. Find the total weight.

lb.	oz.	dwt.	gr.
15	5	13	16
			7
<hr/>			
108 lb.	3 oz.	15 dwt.	16 gr.

SOLUTION.—Write the multiplier under the lowest denomination of the multiplicand, and multiply as in simple numbers, thus :

16 gr.  $\times 7 = 112$  gr. = 4 dwt. 16 gr. Put down 16 under gr. Carry 4 to dwt.

13 dwt.  $\times 7 + (4 \text{ dwt. carried}) = 95$  dwt. = 4 oz. 15 dwt. Put down 15 under dwt. Carry 4 to oz.

5 oz.  $\times 7 + (4 \text{ oz. carried}) = 39$  oz. = 3 lb. 3 oz. Put down 3 under oz. Carry 3 to lb.

15 lb.  $\times 7 + (3 \text{ lb. carried}) = 108$  lb. Put down 108 under lb.

### RULE

*Multiply each denomination separately and change the partial products to units of higher denominations.*

### SERIES 43

1. Multiply 7 bu. 3 pk. 5 qt. 1 pt. by 18.
2. 24 times 8 gal. 3 qt. 1 pt. equals what ?
3. How much wood is there in 36 piles, each measuring 3 Cd. 40 cu. ft. 1020 cu. in. ?
4. Multiply 1 wk. 5 da. 6 hr. 30 min. 45 sec. by 80.
5. If 60 acres 40 sq. rd. 9 sq. yd. 6 sq. ft. 100 sq. in. be multiplied by 72, what will be the product ?
6. Find the product of 2 ton 8 cwt. 40 lb. 12 oz. multiplied by 50.
7. 32 acres of oats averaged 42 bu. 3 pk. 5 qt. 1 pt. Find the entire crop.
8. What quantity of wine will be required to fill 48 barrels if each barrel holds 31 gal. 2 qt. 1 pt.

9. How far will an engine travel in making 20 round trips between two places 25 mi. 80 rd. 4 yd. 2 ft. 6 in. apart ?

10. How much silver will a miner gather in 60 days, if his average daily gathering is 2 oz. 15 dwt. 20 gr. ?

**Division of denominate numbers.** (1) Where divisor is abstract number.

ILLUSTRATION.—Divide 5 bu. 1 pk. 3 qt. by 4.

	bu.	pk.	qt.	pt.	
4)	5	1	3	0	quotient of 1 bushel and an undivided remainder of 1 bushel; reduce this remainder to pecks (4) and add to the 1 peck of the dividend, obtaining 5 pecks, which divided by 4 gives 1 peck and an undivided remainder of 1 peck. Reduce the 1 peck remaining to quarts (8), and adding to the 3 quarts of the dividend gives 11 quarts, which divided by 4 gives 2 quarts and an undivided remainder of 3 quarts. 3 quarts = 6 pints, which divided by 4 gives 1½ pints.
	1	1	2	1½	

### RULE

*Divide as in simple numbers, and when necessary reduce remainders to lower denominations.*

Should the highest dividend order not contain the divisor, reduce its units to the order next lower, and so proceed to the end.

### SERIES 44

Find the quotient of

1. 17 bu. 3 pk. ÷ 9.

3. £28 15s. 5d. 2 far. ÷ 6.

2. 17 ton 9 cwt. 20 lb. ÷ 16.

4. 15 yr. 5 da. 15 hr. ÷ 8.

5. If 7 lb. 5 oz. 12 dwt. 18 gr. of silver be made into 6 plates of equal weight, what will be the weight of each ?

6. From the sum of £25 4s. 10d. and £10 5s. 2d. take their difference, divide the result by 5, and reduce the quotient to Canadian money.

7. I sold 98 cd. 96 cu. ft. of wood for \$395, and in so doing lost \$98.37½. What did the wood cost per cord ?



8. Reduce \$5,164.28 to equivalent British money.

$$\begin{array}{r}
 \$4.86\frac{2}{3} \quad \$5164.28 \\
 \underline{3} \qquad \qquad \underline{3} \\
 1460 \quad 1549284 \quad £1061 \\
 \underline{\phantom{1460}} \quad \underline{1460} \qquad \qquad \qquad \\
 \phantom{1460} \quad 8928 \\
 \underline{\phantom{1460}} \quad \underline{8760} \\
 \phantom{1460} \quad 1684 \\
 \underline{\phantom{1460}} \quad \underline{1460} \\
 \phantom{1460} \quad 224 \\
 \underline{\phantom{1460}} \quad \underline{20} \\
 1460 \quad 4480 \quad (3s. \\
 \underline{\phantom{1460}} \quad \underline{4380} \qquad \qquad \qquad \\
 \phantom{1460} \quad 100 \\
 \underline{\phantom{1460}} \quad \underline{12} \\
 1460 \quad 1200 \quad (1d.
 \end{array}$$

Hence \$5164.28 = £1061 3s. 1d.

9. Reduce \$185 to equivalents in British money ; \$2,500.

Where divisor is denominate number.

ILLUSTRATION.—Divide 39 bu. 1 pk. 1 qt. 1 pt. by 5 bu. 3 pk. 3 qt. 1 pt.

### SOLUTION

39 bu. 1 pk. 1 qt. 1 pt. = 1875 pts.  
5 bu. 3 pk. 3 qt. 1 pt. = 375 pts.  
 $1875 \div 375 = 5$

NOTE.—In this solution both quantities have been reduced to the lowest denomination mentioned in either, viz., the pint, but the same result would have been obtained had both expressions been reduced to any of the other denominations of the table, as the bushel, peck, gallon, or quart.

## RULE

Reduce both quantities to the same denomination and divide in the ordinary way. The result will show how many times the one quantity is contained in the other.

## SERIES 45

1. How often will a wheel 5 ft. 6 in. in circumference turn in going 11 miles ?
2. How many bags, each holding 2 bu. 2 pk. 1 gal. 3 qt., will be required to hold 1,003 bu. 1 gal. 3 qt. ?
3. How many parcels, each containing 1 lb. 4 oz., can be made from 1 ton 35 lb. of tea ?
4. 798 ton 3 cwt. 64 lb. of hay is divided among a certain number of people so that each receives 5 ton 3 cwt. 66 lb. How many people are there ?
5. How many fields, each containing 45 acres 100 sq. rd. 25 sq. yd., can be made out of a tract of land containing 684 acres 72 sq. rd. 12 sq. yd. ?
6. How many portions of time, each equal to 1 day 14 hr. 57 min. 33 sec., are contained in 365 days 5 hr. 48 min. 45 sec. ?
7. Among how many persons must £641 14s. 11d. 1 far. be divided, so that the share of each may be £2 15s. 6d. 3 far. ?
8. If a steamer, on a voyage of 3,087 mi. 6 rd. 3 yd., average 14 mi. 93 rd. 2 yd. per hour, how many days will it take her to make the trip ?

# INVOLUTION AND EVOLUTION

## INVOLUTION

**Involution** is the operation of finding any power of a given number.

A **Power** of a number is the product obtained by using it a number of times as a factor.

A **Square** of a number is the second power of that number or the product obtained by multiplying the number by itself. Thus the square of 4 =  $4 \times 4 = 16$ .

The **Cube** of a number is the third power of the number or the product obtained by multiplying together three factors, each of which is the number. Thus the cube of 4 =  $4 \times 4 \times 4 = 64$ .

The **Exponent** is a figure written to the right of the number and a little above it, indicating the number of times the factor is to be used. Thus in  $4^2$  the exponent 2 indicates that the factor 4 is to be used twice.

### SERIES 46

1. What is the square, or second power, of 25 ?
2. What is the cube, or third power, of 5 ? 24 ? 168 ?
3. What is the fourth power of 4 ? 8 ? 16 ? 25 ?
4. What is the third power of  $\frac{3}{4}$  ?  $\frac{5}{6}$  ?  $\frac{7}{8}$  ?
5. Expand the following :  $6^3$ ,  $9^2$ ,  $8^4$ ,  $10^5$ ,  $23^3$ .
6. Find the required power of the following :  
 $(\frac{2}{9})^3$ ,  $(\frac{4}{5})^4$ ,  $(3\frac{1}{2})^3$ ,  $(14\frac{2}{7})^2$ ,  $(5.06\frac{1}{4})^3$ .
7. Find the difference between  $35^3$  and  $90^2$ .

## EVOLUTION

**Evolution** is the operation of finding any root of a given number.

The **Square Root** of a given number is that number whose square is the given number.

EXAMPLES.—4 is the square of 2,  $\therefore$  2 is the square root of 4 ; 9 is the square of 3,  $\therefore$  3 is the square root of 9 ; 100 is the square of 10,  $\therefore$  10 is the square root of 100.

The **Cube Root** of a given number is that number whose cube is the given number.

EXAMPLES.—8 is the cube of 2,  $\therefore$  2 is the cube root of 8 ; 125 is the cube of 5,  $\therefore$  5 is the cube root of 125 ; 1000 is the cube of 10,  $\therefore$  10 is the cube root of 1000.

There are two ways of denoting Evolution. In the first or older notation, the square root of a given number is denoted by prefixing the symbol  $\sqrt{\phantom{x}}$  to the given number ; the cube root is denoted by prefixing  $\sqrt[3]{\phantom{x}}$ .

EXAMPLES.— $\sqrt{64}$  denotes the square root of 64 ;  $\sqrt[3]{64}$  denotes the cube root of 64.

The second method employs fractional exponents.

Thus,  $64^{\frac{1}{2}}$  denotes the square root of 64 ;  $64^{\frac{1}{3}}$  denotes the cube root of 64.

**To extract the square root of a number.**

### RULE

1. *Point off the given number into periods of two figures each, beginning at the decimal point.*
2. *Find the highest square contained in the left-hand period and place its root to the right of the number, in the place occupied by the quotient in division.*
3. *Subtract the square of the digit put in the root, from the left-hand period, and to the remainder bring down the next period to the right, for a new dividend.*
4. *Double the part of the root already found for a TRIAL DIVISOR.*
5. *Find how many times the trial divisor is contained in the dividend, exclusive of the right-hand digit, and place the figure thus obtained both in the root and also to the right of the trial divisor.*
6. *Multiply the divisor thus completed by the digit last put in the root ; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*
7. *Again, double the part of the root already found for a new TRIAL DIVISOR ; proceed as in 5 and 6, and continue the process until all the periods are brought down.*

ILLUSTRATION.—What is the square root of 22420225 ?

	22   42   02   25)4735
	16
87	642
	609
943	3302
	2829
9465	47325
	47325

SOLUTION.—Here 22 is the left-hand period, and the highest square in 22 is 16, of which the square root is 4. We place 4 in the root and subtract 16 from 22. This leaves a remainder 6, to which we bring down the next period, 42, and thus obtain 642 for the new dividend. Our next step is to find the trial divisor, which we obtain by doubling the part of the root already found. This gives us 8 (= 4 doubled), and we ask how many times 8 will go into 64 (the dividend exclusive of the right-hand digit). Bearing in mind that we are to put the digit thus obtained both in the root and in the divisor, and that the complete divisor will be over 80, we find

that the required digit is 7, which we accordingly place both in the root and in the divisor. The complete divisor is 87, which multiplied by 7, gives 609, and this subtracted from 642, gives a remainder 33, to which we bring down the next period, 02, and thus get 3302 for the next dividend.

Again, doubling the part of the root already found, we obtain 94 (= 47 doubled) for a trial divisor, and as this will go into 330 (the dividend exclusive of the right-hand digit) 3 times, we place 3 both in the root and in the divisor.

Multiplying the 943 thus obtained by 3, subtracting and bringing down the next period, we get 47325 for the next dividend. The next trial divisor is 946 (= 473 doubled), which will go into 4732 (the dividend exclusive of the right-hand figure) 5 times; and we therefore place 5 both in the root and in the divisor. Multiplying and subtracting, we find no remainder. 4735 is therefore the square root of 22420225.

PROOF.— $4735 \times 4735 = 22420225$ .

NOTE 1.—If there is a *remainder* after the root of the last period is found, annex *periods of ciphers*, and proceed as before. The figures of the root thus obtained will be *decimals*.

NOTE 2.—If the trial divisor is not contained in the dividend, annex a *cipher* both to the root and to the divisor, and bring down the next period.

NOTE. 3—It sometimes happens that the remainder is *larger* than the divisor; but it does not necessarily follow that the *figure* in the root is too *small*.



### To extract the square root of a decimal.

#### RULE

*Begin at the units' place, and proceed towards the left and right, to separate into periods of two figures each, then extract the root as in whole numbers.*

NOTE 1.—The *left-hand* period in *whole numbers* may have but *one* figure ; but in *decimals*, each period must have *two* figures. Hence, if the number of decimals is odd, a *cipher* must be annexed to complete the period.

NOTE 2.—It must be kept in mind that no period should contain an integer and decimal, and that, if there is an odd number of decimal places in the given number, the last period must be completed by annexing a cipher.

### To extract the square root of a fraction.

#### RULE

*Reduce the fraction to its simplest form and find the square root of each term separately.*

NOTE 1.—If the denominator of the given fraction, when reduced, is an imperfect square, reduce the fraction to a *decimal*, and proceed as above.

NOTE 2.—Mixed numbers should be reduced to improper fractions, or the fractional part to a decimal.

#### SERIES 47

Find the square root of

1. 576	9. 39601	17. 11.9025
2. 1225	10. 88209	18. 41249.61
3. 2025	11. 751689	19. .697225
4. 4356	12. 106929	20. 54.6121
5. 11025	13. $\frac{225}{289}$	21. 3
6. 18225	14. $\frac{169}{625}$	22. 376.4
7. 15129	15. $\frac{256}{361}$	23. 12.823
8. 103041	16. $\frac{196}{841}$	24. 4367.2

25. Find the length in rods of a square 10-acre field.

26. A park, in the form of a rectangle 220 yards wide and 528 yards long, has a path around it and from corner to corner.

How much will a person, who wishes to cross the park diagonally, save by "cutting across" ?

27. An electric light pole broke in such a way that the top struck the ground 33 feet from the base of the pole. What was the height of the pole, the broken part being 65 feet long ?

28. A ladder, 41 feet long, stands upright against a wall. Find how far the bottom of the ladder must be pulled out to lower the top 1 foot.

29. A ladder, 40 feet long, is placed so as to reach a window 24 feet high on one side of a street, and from the same spot it will reach a window 32 feet high on the other side of the street. Find the width of the street.

30. It requires 432 yards of carpet,  $\frac{3}{4}$  of a yard wide, to cover a square room. What is the length of the room ?

31. A room is 24 feet long, 18 feet broad, and 7 feet high. What length of string will reach from any corner of the floor to the farthest corner of the ceiling ?

32. Find the difference in the length of fence necessary for a rectangular field 850 feet long and 306 feet wide and a square field of the same area.

33. A boy was flying a kite with a string 975 feet long. If the distance from where the boy was standing to a point directly under the kite was 759 feet, how high was the kite ?

34. A square space, containing 150 sq. yds. is to be lengthened by 4 feet 3 inches in one of its dimensions and shortened by 3 feet 4 inches in the other. What will then be its area ?

35. A ladder, 42 feet long, placed with its foot 24 feet from a wall, reaches within 3 feet of the top. How near the wall must the foot of the ladder be brought that it may reach the top ?

36. The surface of a small park is 540 sq. rd. and its length is to its breadth as 5 to 3. What are its dimensions ?

37. Find the difference between the perimeter of a square field, containing  $22\frac{1}{2}$  acres, and the perimeter of a rectangular field of equal area, the length of the latter field being to its width as 4 to 3.

38. If it costs \$448 to fence a square field at \$4.40 a rod, what would it cost to fence the same amount of land in the form of a rectangle whose sides are in the ratio of 9 to 16 ?

To extract the cube root of a number.

QUESTION.—What is the cube root of 184,608,795,384 ?

(a)	(b)	(c)	
	5		184,608,795,384
			125
			<hr/>
15	6	7500	59608
		936	
		<hr/>	50616
		8436	<hr/>
			8992795
168	9	940800	
		15201	8604009
		<hr/>	<hr/>
		956001	388786384
1707	4	97128300	388786384
		68296	<hr/>
		<hr/>	
		97196596	

Divide the number into periods of three figures each, beginning at the decimal point.

(1) The first period is 184, and the nearest perfect cube to 184 is 125, which is the cube of 5. Then 5 is the first digit in our answer.

Put 5 in column (b), the answer column.

Subtract 125 from 184, and bring down second period.

(2) Multiply 5 by 3, and put the result, 15, in column (a).

Now multiply 5 squared by 300 = 7500. Put this in column (c), opposite 15.

Now divide 7500 into 59608. We find it goes 7 times. Now if we put 7 in column (b), and multiply 157 by 7, and add result to 7500, we get 8599. But if we multiply 8599 by 7, our result is greater than 59608. Therefore 7 is too great for our second digit of answer.

So dividing 59608 by 7500, take 6 for quotient instead of 7.

Put 6 down in column (*b*) as second figure of answer.

Multiply 156, that is, the 15 in column (*a*) and 6 in column (*b*), by 6, and add result to 7500. Thus we get 8436.

Now multiply 8436 by 6, and we get 50616.

Subtract this from 59608, and our remainder is 8992, to which we attach the third period.

(3) Now to find third figure of answer.

Multiply 15 by 10, and to it add  $6 \times 3$ , or 18. 6 is the second figure of column (*b*), and 15 the first in column (*a*).

Thus we get in column (*a*) 168.

Square 56, and multiply result by 300 = 940800.

This divided into 8992795 gives 9.

Put down 9 in column (*b*) as third figure of answer.

Multiply 1689 (168 in first column, 9 in second) by 9, = 15201, and add result to 940800 = 956001.

Now multiply 956001 by 9 = 8604009, and subtract this from 8992795 = 388786.

(4) To 388786 attach fourth period.

Multiply 168 by 10, and add  $9 \times 3$ , or 27.

Thus  $1680 + 27 = 1707$ . Put this in column (*a*). Square 569, and multiply by 300 = 97128300.

Divide this into 388786384 = 4.

Put 4 in column (*b*) as fourth figure in answer, and proceed as in (2) and (3).

Thus our answer is 5694 exactly.

NOTE 1.—If there is a *remainder* after the root of the last period is found, annex periods of ciphers, and proceed as before. The root figures thus obtained will be *decimals*.

NOTE 2.—If a trial divisor is *not contained* in the dividend, put a *cipher* in the root, *two ciphers* on the right of the divisor, and bring down the next period.

NOTE 3.—If the product of the divisor completed into the figure last placed in the root *exceeds* the dividend, the root figure is too large. Sometimes the remainder is *larger* than the divisor completed; but it does not necessarily follow that the root figure is *too small*.

### To extract the cube root of a decimal.

#### RULE

*Begin at the units' place, and proceed both toward the left and right to separate into periods of three figures each, then extract the root as in whole numbers.*

NOTE.—The *left-hand* period in *whole numbers* may have but *one* or *two* figures, but in *decimals* each period must have *three* figures. Hence, ciphers must be annexed to the right of the decimal to complete the periods, when necessary.

### To extract the cube root of a fraction.

#### RULE

*Reduce the fraction to its lowest terms, then extract the root of its numerator and denominator.*

NOTE 1.—When the denominator is not a *perfect cube*, the fraction should be reduced to a decimal, and the root of the decimal be found as above.

NOTE 2.—A *mixed* number should be reduced to an improper fraction.

#### SERIES 48

Find the cube root of

1. 4096	11. 122615327232
2. 32768	12. 389.017
3. 74088	13. .194104539
4. 493039	14. 48228.544
5. 250047	15. 2
6. 614125	16. 14.6
7. 14706125	17. $\frac{1331}{1728}$
8. 84027672	18. $\frac{250}{686}$
9. 354894912	19. $\frac{3}{4}$
10. 673373097125	20. $405\frac{28}{125}$

Find the fourth roots of

21. 531441	22. 4100625	23. 1575.2961
------------	-------------	---------------

24. Find the sixth root of 4826809.

25. What are the dimensions of a cubical block whose volume is 35,937 cu. in. ?



26. A bin, whose capacity is 6,750 cu. in., is twice as long as it is wide or high. What are its dimensions ?

27. A pile of wood has the form of a rectangular solid, 168 feet long, 126 feet wide, and 28 feet high. What would be the length of one of its edges if it were in the form of a cube ?

28. How large a cube could be made by using the total contents of a rectangular mound of earth that is 216 feet long, 48 feet wide, and 36 feet high ?

29. The dimensions of a rectangular box are as 1 : 2 : 3. If the box has the same capacity as one 6 ft. long, 4 ft. 2 in. wide, and 3 ft. 9 in. deep, find its measurements.

30. The cubical base of a monument contains 274,625 cu. ft. How many square feet in one of its faces ?

31. There are 231 cu. in. in a gallon. Find the inside dimensions correct to tenths of an inch of a cubical tank that will hold 1,000 gals.

32. If 160,103,007 cubical blocks of stone, each one foot square, were placed in a cubical pile, what would be the length of each edge ?

# PRACTICAL MENSURATION

**Mensuration** treats of the measurement of lines, angles, surfaces, and solids.

## SURFACES

A **Line** is that which has length, but not breadth or thickness.

A **Straight Line** is one that does not change direction. It is the shortest distance between two points.

A **Curved Line** is one that changes its direction at every point.

It is one of which no three consecutive points are in the same direction.

**Parallel Lines** are lines that have the same direction and are equally distant at all points.

A **Horizontal Line** is one that is parallel to the horizon or water level.

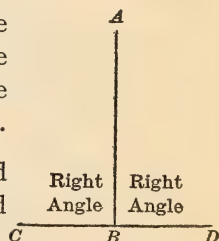
A **Vertical Line** is one that is perpendicular to a horizontal plane.

A **Surface** is that which has length and breadth only.

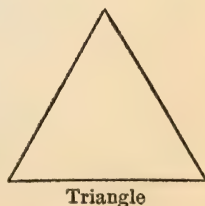
If a straight edge laid anywhere upon a surface touches at every point, the surface is a plane surface or a plane.

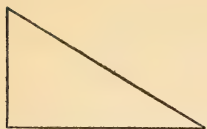
A **Right Angle** is an angle formed when one straight line meets another so as to make the adjacent angles equal. The lines forming the angles are said to be *perpendicular to each other*.

In the accompanying diagram  $ABC$  and  $ABD$  are right angles, and the lines  $AB$  and  $CD$  are perpendicular to each other.



A **Triangle** is a plane figure with three plane sides and three plane angles. The side on which the triangle stands is the *base*, the opposite corner the *vertex*, and the shortest distance from the vertex to the base, or the base extended, is the *height* or *altitude* of the triangle.



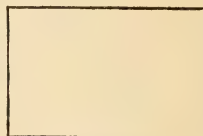


A **Right-angled Triangle** is a triangle having a right angle.

The **Hypotenuse** of a right-angled triangle is the side opposite the right angle.

The **Perpendicular** is the side that forms a right angle with the base.

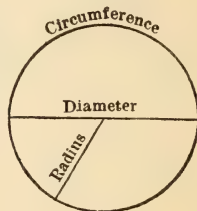
A **Rectangle** is a plane figure having four straight sides and four square corners. When the four sides are equal, the figure is usually called a *square*.



Rectangle

The **Perimeter** of any plane figure is the length of the line or lines inclosing it.

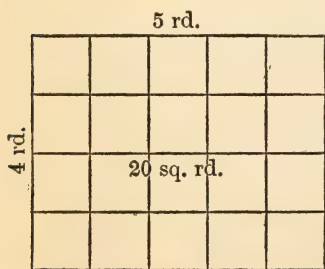
When a plane figure is bounded by a curved line, every point of which is equally distant from the centre, it is called a *circle*. The perimeter of a circle is called its *circumference*; a line passing through the centre and terminating in the circumference, the *diameter*; one-half of the diameter, the *radius*.



A **Polygon** is a plane figure having more than four straight sides.

**To find the area of a rectangle or square.**

ILLUSTRATION.—What is the area of a garden 5 rd. long by 4 rd. wide ?



SOLUTION

$$1 \text{ square rod} \times 4 \times 5 = 20 \text{ sq. rds.}$$

RULE

*Multiply the length by the breadth and the result will be the area.*

To find the area of a triangle when the base and altitude are given.

ILLUSTRATION.—Find the area of a triangle whose base and altitude are 6 ft. and 8 ft. respectively.

SOLUTION.—In the accompanying diagram assume that the base ( $CB$ ) is 6 ft. and the altitude ( $AD$ ) is 8 ft. It will be seen that the altitude divides the triangle into two right-angled triangles, each of which is one-half of a rectangle whose sides are 8 ft. and 3 ft. Two triangles, each one-half of a rectangle 8 ft. by 3 ft., are equal to one rectangle 8 ft. by 3 ft. The area of the triangle given is, then, the product of these two dimensions, or 24 sq. ft.



#### RULE

*Multiply one-half the base by the altitude.*

To find the area of a triangle when the sides are given.

ILLUSTRATION.—What is the area of a triangle whose sides are 12 ft., 16 ft., and 18 ft. ?

SOLUTION.— $(12 + 16 + 18) \div 2 = 23$   
 $23 - 18 = 5$                        $23 \times 5 \times 7 \times 11 = 8855$   
 $23 - 16 = 7$   
 $23 - 12 = 11$                        $\sqrt{8855} = 94.1$  sq. ft. Ans.

#### RULE

*From half the sum of the three sides subtract each side separately ; then multiply the half sum and the three remainders together, and extract the square root of the product.*

To find the hypotenuse of a right-angled triangle.

ILLUSTRATION.—The base of a right-angled triangle is 12 ft. and the perpendicular is 9 ft. What is the hypotenuse ?

SOLUTION.— $12^2 = 144$   
 $9^2 = 81$   
 $144 + 81 = 225$   
 $\sqrt{225} = 15$

#### RULE

*To the square of the base add the square of the perpendicular ; the square root of the sum will equal the hypotenuse.*

**To find the base or perpendicular.**

ILLUSTRATION.—The hypotenuse of a right-angled triangle is 25 in. and the perpendicular is 15 in. What is the base ?

**SOLUTION**

$$25^2 = 625$$

$$15^2 = 225$$

$$625 - 225 = 400$$

$$\sqrt{400} = 20$$

**RULE**

*From the square of hypotenuse subtract the square of the given side ; the square root of the remainder will be the required side.*

**THE CIRCLE****Principles.**

1. The circumference = the diameter  $\times$  3.1416 nearly.
2. Therefore the diameter = the circumference  $\div$  3.1416 nearly.
3. The area of a circle = the square of the radius  $\times$  3.1416 nearly.
4. The area of a circle = the circumference  $\times$  half the radius = 3.1416 times the square of the radius.
5. Therefore the radius of a circle = square root of (the area  $\div$  3.1416) nearly.

NOTE.—The fraction of  $3\frac{1}{7}$  is commonly used in place of the decimal 3.1416, and is near enough for common practical operations.

**To find the circumference of a circle.**

ILLUSTRATION 1.—What is the circumference of a circle whose radius is 14 ft. ?

**SOLUTION**

$$14 \text{ ft.} \times 2 = 28 \text{ ft. diameter,}$$

$$28 \text{ ft.} \times 3\frac{1}{7} = 88 \text{ ft. Ans. (Prin. 1.)}$$

**RULE**

*Multiply the diameter by 3.1416, or  $3\frac{1}{7}$ .*



To find the diameter of a circle.

ILLUSTRATION 2.—The circumference of a circle is 352 ft. What is the diameter ?

SOLUTION

$$352 \div 3\frac{1}{7} = 112 \text{ ft. Ans. (Prin. 2.)}$$

RULE

*Divide the circumference by 3.1416, or  $3\frac{1}{7}$ .*

To find the area of a circle when the diameter is given.

ILLUSTRATION 3.—What is the area of a circle whose diameter is 28 ft. ?

SOLUTION 1

$$\begin{aligned} 28 \text{ ft.} \div 2 &= 14 \text{ ft. radius,} \\ 14 \times 14 \times 3\frac{1}{7} &= 616 \text{ sq. ft. Ans. (Prin. 3.)} \end{aligned}$$

SOLUTION 2

$$\begin{aligned} 28 \text{ ft.} \times 3\frac{1}{7} &= 88 \text{ ft. the circumference. (Prin. 1.)} \\ 28 \text{ ft.} \div 2 &= 14 \text{ ft. the radius.} \\ 88 \times \frac{1}{2} &= 616 \text{ sq. ft. Ans. (Prin. 4.)} \end{aligned}$$

RULE

*Multiply the square of the radius by 3.1416 or  $3\frac{1}{7}$ . Or*

*Multiply the circumference by  $\frac{1}{4}$  of the diameter, or  $\frac{1}{2}$  the radius.*

To find the radius, diameter, and circumference when the area is given.

ILLUSTRATION 4.—The area of a circle is 616 square feet. Find the radius, diameter, and circumference.

SOLUTION

$$\begin{aligned} \text{Radius} &= \sqrt{616 \div 3\frac{1}{7}} = 14 \text{ ft. (Prin. 5.)} \\ 14 \text{ ft.} \times 2 &= 28 \text{ ft. the diameter.} \\ 28 \text{ ft.} \times 3\frac{1}{7} &= 88 \text{ ft. the circumference. (Prin. 1.)} \end{aligned}$$

RULE

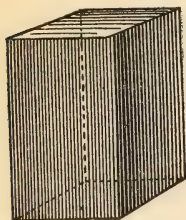
*Divide the area by 3.1416 or  $3\frac{1}{7}$  and extract the square root of the quotient. Multiply the radius by 2. Multiply the diameter by 3.1416 or  $3\frac{1}{7}$ .*

## SOLIDS

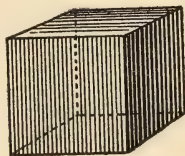
A **Solid** is that which has length, breadth, and thickness.

A **Rectangular Solid** is a solid bounded by six rectangular sides or faces. When these sides are squares, the figure is called a *cube*.

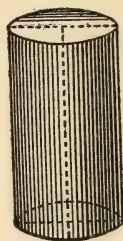
A **Cylinder** is a circular body of uniform diameter whose ends are equal and parallel circles.



Rectangular Solid.



Cube.

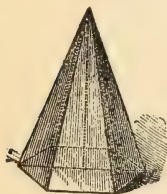


Cylinder.

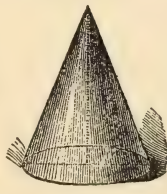
A **Pyramid** is a solid whose base is a polygon and whose sides terminate in a point called the *vertex*.

A **Cone** is a solid which has a circle for its base, and terminates in a point called the *vertex*.

The **Altitude** of a pyramid or cone is the perpendicular distance from the base to the vertex.



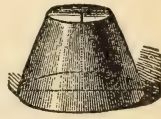
Pyramid.



Cone.



Frustum of a pyramid.



Frustum of a cone.

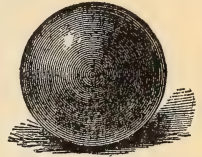
The **Slant Height** of a pyramid is the distance from the vertex to the middle point of any side of the base.

A **Frustum** of a pyramid or cone is the part which is left after the top is cut off by a plane parallel to the base.

The **Altitude** of a frustrum is the perpendicular distance between its ends.

The **Slant Height** of a frustrum of a pyramid is the distance between the middle points of two parallel sides of one of its faces.

A **Sphere** or **Globe** is a solid terminated by a curve surface, every part of which is equally distant from a point within, called the *centre*.



Sphere or Globe.

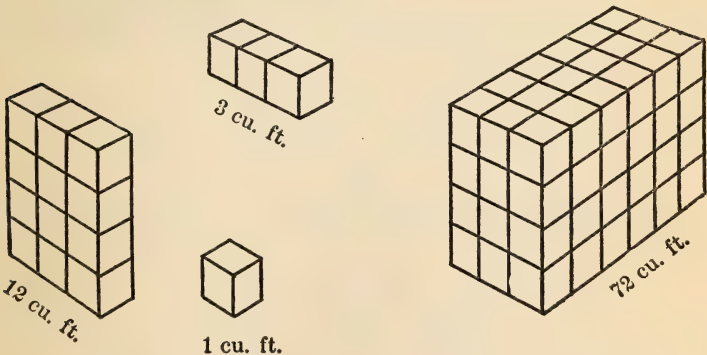
The **Diameter** of a sphere is a straight line drawn through its centre and terminated at both ends by the surface.

A **Hemisphere** is one-half a sphere.

The **Radius** of a sphere is a straight line drawn from its centre to any point in its surface.

To find the solid contents of a rectangular solid.

ILLUSTRATION.—What is the volume of a solid 6 ft. long, 4 ft. high, and 3ft. wide ?



SOLUTION.—1 cu. ft.  $\times 3 \times 4 \times 6 = 72$  cu. ft.

### RULE

*Multiply the length by the breadth by the thickness and the result will be the solid contents.*

To find the solid contents of a cylinder.

ILLUSTRATION.—Find the cost of digging a round well 25 ft. deep and 8 ft. in diameter, at 35c per cubic yard.

SOLUTION

Area of base in square feet =  $(4 \times 4 \times 3\frac{1}{2})$  cu. ft.

Solid contents of well in cu. ft. =  $(4 \times 4 \times 3\frac{1}{2}) \times 25$

Solid contents of well in cu. yd. =  $\frac{4 \times 4 \times 3\frac{1}{2} \times 25}{27}$

Cost of digging =  $\frac{35c. \times 4 \times 4 \times 3\frac{1}{2} \times 25}{27} = \$16.30$

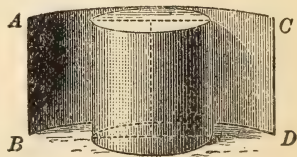
RULE

*Multiply the area of the base by the height of the cylinder.*

The **Lateral Surface** of a cylinder is the surface of its curved sides.

The lateral surface of a cylinder is equal to the surface of a rectangular body, the length and height of which are equal to the circumference and height of the cylinder.

Thus, the lateral surface of the cylinder in the accompanying diagram is the area of the rectangle described by  $A B C$  and  $D B$  back of the cylinder. Hence,



To find the area of the lateral surface of a cylinder.

RULE

*Multiply the circumference of the base by the height of the cylinder.*

### MISCELLANEOUS

To find the convex surface of a pyramid or cone.

RULE

1. *Multiply the perimeter by one-half the slant height.*
2. *To find the entire surface add the area of the base to the area of the convex surface.*

To find convex surface of a frustum of a cone or pyramid.

RULE

1. Multiply one-half the sum of the perimeters of the ends by the slant height.

2. To find the entire surface, add the areas of the ends to the area of the convex surface.

To find the volume of a cone or pyramid.

RULE

Multiply area of the base by one-third the altitude.

To find the volume of the frustum of a cone or pyramid.

RULE

$(A + a + \sqrt{A \times a}) \times h \times \frac{1}{3}$ , where "A" stands for the area of the larger end, "a" for the area of the smaller end, and "h" for the perpendicular height.

To find the surface of a sphere.

Multiply the square of the diameter by  $3\frac{1}{4}$ .

To find the volume of a sphere.

Multiply the cube of the diameter by  $3\frac{1}{4}$ , and divide the result by 6.

To find the number of gallons in a cistern.

Find the volume in cubic inches and divide the result by 277.

NOTE.—There are 277 cubic inches in one gallon.

To find the number of bushels of wheat in a bin or pile.

Find the volume in cubic inches and divide the result by 2150.42.

NOTE.—There are 2150.42 cubic inches in one bushel.

To find the mean diameter of a cask (nearly).

Add to the head diameter  $\frac{2}{3}$ , or, if the staves are but little curved,  $\frac{3}{8}$  of the difference between the head and bung diameters.

To find the volume of the cask in gallons.

Multiply the square of the mean diameter by the length (both in inches), and this product by .0034.



## SERIES 49

1. How many square feet are there in a floor 22 yards long and 5 yards wide ?

2. How many acres are there in a field 220 yards long and 40 rods wide ?

3. How many square yards are there in the walls of a room 15 ft. 6 in. long, 12 ft. wide, and 8 ft. 6 in. high ?

4. How many square yards are there in the floor of the room in question 3 ?

5. How many square feet of glass are there in a box containing 72 panes, each 12 in. x 16 in. ?

6. How many bricks 8 in. long and 4 in. wide will pave a yard that is 116 feet long and 46 ft. wide ?

7. What will it cost to pave a roadway 80 ft. long and 15 ft. wide at \$1.50 per square yard ?

8. A certain rectangular piece of land measures 1,500 links by 200 links. How many acres does it contain ?

9. A man bought a farm 198 rods long and 150 rods wide at \$32 an acre. What did the farm cost ?

10. Which would be the more economical and by how much : to pave a walk 4 ft. wide and 200 ft. long with stone at 22 cents per square yard or with brick at \$1.02 per square yard ?

11. How many granite blocks 12 in. by 18 in. will be required to pave a mile of roadway 42 ft. in width ?

12. Find the cost at 16 cents per square yard of making a walk 4 ft. wide around the outside of a lot 10 rods long and 130 ft. wide ?

13. If the walk in the preceding question had been around the inside of the lot, how much would it have cost ?

14. The top of a table 4 ft. 8 in. long contains  $1\frac{7}{8}$  square yards. How wide is it in inches ?

15. A farm cost \$8,250 at \$75 an acre. If it is 968 yards long, how many rods wide is it ?

16. How much would it cost to fence a field 40 rods long and 30 rods wide at 9 cents a yard ?

17. Two fields contain 40 acres each. One is in the form of a square and the other is 4 times as long as it is wide. Find the difference in the cost of fencing them at 45 cents per rod.
18. How many square feet in the upper surface of a board 16 ft. long and 6 in. wide at one end and 14 in. wide at the other ?
19. A field containing 8 acres is 5 times as long as it is wide. Find its perimeter in yards.
20. Find the cost of paving a road of the uniform breadth of 4 yards around the inside of a rectangular 2-acre lot, 20 rods wide, at 24 cents a square yard.
21. A rectangular piece of ground is 60 yards long and contains  $\frac{1}{3}$  of an acre. It contains a grass plot bordered by a walk 6 ft. wide. Find the area of the plot.
22. A square space containing 256 square yards is to be lengthened by 4 ft. 3 in. in one of its dimensions and shortened by 3 ft. 4 in. in the other. What will then be its area ?
23. Find the dimensions of a rectangle containing 1,014 sq. ft., if its length is to its width as 2 is to 3.
24. An electric light is 18 ft. above the ground. What will be the length of the shadow of a man 6 ft. in height if he stands 16 ft. from the post on which the light is placed ?
25. A rectangular garden  $2\frac{1}{2}$  chains wide contains  $\frac{3}{4}$  of an acre. How many feet long is it ?
26. A rectangular field is 40 rods in length and 30 yards in width. Find in feet the side of a square of equal area.
27. Find the difference between the perimeter of a square field containing  $22\frac{1}{2}$  acres and the perimeter of a rectangular field of equal area, the length of the latter being  $1\frac{1}{3}$  times its width.
28. A board is 8 in. broad ; what length of board must be cut off to make 1 square yard of surface (on one side) ?
29. The perimeters of a rectangle and a square are each 40 in. Find the difference in their areas, if the sides of the rectangle are in the ratio of 1 to 3.
30. The rent of a square field at \$12 an acre is \$132.24. Find the cost of putting a fence around it at 35 cents a yard.

31. At 10 cents a square foot what will it cost to lay sod on a triangular yard whose sides are 8, 15, and 17 ft. respectively ?

32. The sides of an iron plate are 20, 21, and 29 ft. respectively. What is it worth at \$2.70 a sq. yd. ?

33. The hypotenuse of a right-angled triangle is 50 ft. and the base is 40 ft. What is (a) the perpendicular (b) the area of the triangle ?

34. A ladder 17 feet long was placed so as to just reach the top of a building 15 ft. high. How far from the base of the building was the ladder placed ?

35. A triangular sail whose edges are, respectively, 33, 56, and 65 ft., was made of canvas that cost 80 cents a square yard. What did the sail cost ?

36. What is the distance diagonally across a floor that is 40 ft. long and 30 ft. wide ?

37. Two upright poles are respectively 57 ft. and 82 ft. high. If the poles are 60 ft. apart, what must be the length of a line that reaches from the top of one to the top of the other ?

38. Find the area of a triangle whose base is 12 ft. 6 in. and altitude 6 ft. 9 in.

39. What is the cost of a triangular piece of land whose base is 15.48 ch. and altitude 9.67 ch. at \$60 an acre ?

40. At 40 cents a square yard find the cost of paving a triangular court, its base being 105 ft. and its altitude 21 yds.

41. The base of a triangular field, containing 1 acre, is  $90\frac{1}{2}$  yards in length. What is the altitude ?

42. The sides of a triangle are 13, 14, and 15 ft. respectively. Find the perpendicular from the opposite angle on the 14 ft. side.

43. The distance from the centre of the hub of a wheel to the outer edge of the felly is 1 ft. 9 in. How long must the tire be ?

44. If it is 12 ft. 10 in. around the base of a tree, what is its diameter ?

45. Find, in inches, the side of the greatest square stick of timber which can be cut from a tree whose circumference is 9 ft. 2 in. ?

46. The radius of a circle is 2 ft. Find the whole perimeter of its semicircle.

47. The whole perimeter of a semicircle is 90 in. Find its radius.

48. What is the diameter of a wheel which turns around 1,000 times in going a mile ?

49. The difference between the diameter and the circumference of a circle is 12 ft. Find its area.

50. A cow is tethered to a post driven in the centre of a lot 100 ft. square ; the tether is just long enough for her to reach the fence. How much of the surface of the lot is she unable to crop ?

51. The diameter of a carriage wheel is  $24\frac{1}{2}$  in. Find how many turns the wheel makes in going one mile.

52. Find the cost of paving a  $10\frac{1}{2}$  ft. drive around the outside of a circular plot 28 yards in diameter, at 65 cents a square yard.

53. Find the difference in feet between the perimeters of a circular and a square field, if each contains 2 acres.

54. Find the area of a uniform walk 2 yards wide around a circular pond which contains  $15\frac{2}{3}$  acres.

55. Find the width of a circular path containing 120 square yards which surrounds a circular pond whose circumference is 220 yards.

56. The radius of the outer boundary of a ring is 14 in. and its area is 462 sq. in. Find the circumference of the inner boundary.

57. A circular pond, which is 4 miles in diameter, has a driveway around it. A man wishing to reach a point directly across the pond from where he stands can drive at the rate of ten miles an hour and row at the rate of 6 miles an hour. How many minutes can he save by going the quicker way ?

58. The radius of a circle is 8 feet. Find the circumference of another circle of  $\frac{1}{5}$  the area.

59. Find the side of a square which is equal in area to a circle whose circumference is 55 inches.

60. How many cubic yards will a box hold if it is 10 ft. 6 in. long, 4 ft. wide, and 2 ft. 4 in. deep ?

61. The ice on a pond, whose area is  $\frac{1}{2}$  an acre, is 10 inches thick. How many tons of ice may be taken from the pond supposing a cubic foot of ice to weigh 56 pounds ?

62. How many bricks 8 in. long, 4 in. wide, and 2 in. thick can be packed in a box 4 ft. 6 in. long, 3 ft. wide, and 2 feet deep ?

63. A room contains 1,536 cu. ft. of space. If it is 16 ft. long and 12 ft. wide, how high is it ?

64. Rain falling uniformly for 5 hours on a roof, whose horizontal dimensions are 10 yds. by 15 ft., fills a tank 6 ft. 3 in. by 3 ft. by 2 ft. 6 in. Find the depth of the rainfall per hour.

65. The side of a square field is 48 rods. Find the length of the side of a square field containing two and a quarter times as much land.

66. A lot 96 ft. long and 60 ft. wide is filled with cordwood piled to a height of 6 ft. How many cords are there in the yard ?

67. A farmer owes a merchant \$105. They agree to pay it in cordwood at \$5.00 a cord. The yard in which the wood is to be piled is 28 ft. long and 16 ft. wide. How high must the wood be piled so that there may be enough to pay the debt ?

68. If a cubic foot of water weighs 1,000 ozs., and a gallon of water weighs 10 lbs., how many gallons will be required to fill a rectangular tank 12 ft long, 4 ft. wide, and 4 ft. deep ?

69. Find the number of acres in a triangular field whose sides are 1,056 yards, 1,980 yards, and 2,244 yards.

70. One blackboard in a school is 4 ft. wide and 16 ft. long, the other is  $3\frac{1}{2}$  ft. wide and 24 ft. long. How many feet must I cut off the length of the larger blackboard so that the remainder will have the same area as the smaller one ?

71. Find the cost of gilding the entire outside surface of a covered box, 3 ft. long, 2 ft. 6 in. wide, and 1 ft. 9 in. deep, at \$1.20 per square foot.

72. Find the length of the diagonal of a square field containing 30 acres.



73. To drain a swamp in Dereham, the Township Council had a ditch dug 1 mile long, 3 ft. deep, 6 ft. wide at the surface, and 4 ft. wide at the bottom. Find the total cost at 9 cents per cubic yard.

74. How many miles must be travelled by a team in ploughing lengthwise a piece of land 60 rods long and 40 rods wide, if each furrow is 10 in. wide ?

75. The Manufacturers and Liberal Arts Building of the Columbian Fair was in the form of a rectangle and covered an area of 30 acres, 76 sq. rds., 19 sq. yds., 7 sq. ft. The building was 787 feet long. How many feet in length was it ?

76. A load of wood 10 ft. long, 3 ft. 8 in. wide, and 3 ft. high, was sold for \$3. (a) What was the price per cord ? (b) At \$4 per cord what would the load be worth ?

77. How much will it cost to paint the outside and both floors of a two-storey cottage, 36 ft. long, 33 ft. wide, and 18 ft. high, at 10 cents per square yard, the walls to be 18 in. thick and no allowance to be made for cornices, openings, or partitions ?

78. A gravel-bed whose surface has an area of 4 acres contains gravel to an average depth of 6 ft. How many miles of road 11 ft. wide can be covered from the gravel in the bed if it be spread on to a uniform depth of 8 inches ?

79. A cord of wood and one hundred bushels of grain fill equal spaces. A cubic bin whose edge is 12 ft. contains 45,900 lbs. of grain. Find the weight of one bushel of this grain.

80. Find the expense of sodding a plot of ground, which is 40 yards long and 100 ft. wide, with sods each a yard in length and a foot in width, the sods, when laid, costing 75 cents per hundred.

81. The whole surface of a rectangular solid is 1,000 sq. in. ; if its length and breadth are respectively 1 ft. 3 in. and 1 ft. 2 in., find its height.

82. A box is made of plank 2 inches thick and without a lid, Its outside measurements are : depth, 16 in. ; width, 18 in. ; and length, 24 inches. How much will it cost to have it painted, inside and outside, at 9 cents per square foot ?

83. How many gallons will a can hold if it is 22 in. in diameter and 3 ft. high ?

84. What is the volume of a triangular prism whose length is 12 ft. and one of the equal sides of its equilateral ends is 3 ft. ?

85. Find the number of cords of wood in a cylindrical stick of timber, the length being 40 ft. and the circumference 22 ft.

86. A garden roller is 3 ft.  $7\frac{1}{2}$  in. wide and 5 ft.  $10\frac{3}{4}$  in. in circumference. How much ground does it pass over in making three complete revolutions ?

87. A sphere 8 in. in diameter is placed in a cubical box whose interior dimensions are 8 in. How much water will the box then hold ?

88. I have a cylindrical tank which contains 160 gallons. It is 6 ft. 5 in. in diameter. How deep is it ?

89. How many square inches of surface are there in a stove pipe 22 in. in circumference and 12 ft. long ?

90. How many square yards of canvas will be required to make a conical tent 9 ft. high and having a base of 4 ft. radius, no allowance being made for seams ?

91. A conical tin vessel has a lid ; the diameter of the lid is 24 in. and the depth of the vessel is 18 in. How many square feet of tin does the whole surface present ?

92. How many gallons of water would the vessel in the preceding question hold ?

93. Find the weight of gunpowder required to fill a hollow sphere 9 in. in diameter, supposing that 30 cu. in. of gunpowder weigh one pound ?

94. A locomotive, running at the rate of 35 miles per hour, has a driving wheel which makes 4 revolutions in one second. Find the diameter of the wheel.

95. The driving wheel of a locomotive, of diameter 7 ft., makes  $1\frac{1}{2}$  revolutions in 1 second. Find the rate of the locomotive in miles per hour.

96. How many pieces of money,  $\frac{3}{4}$  of an inch in diameter and  $\frac{1}{8}$  of an inch thick, can be coined from material in the form of a cube whose edge is 3 inches ?

97. The height of a cylinder is to the diameter of its base as  $3 : 2$ ; if its volume is 320 cubic inches, find its height.

98. A sphere of radius 7 in. is equal in volume to a cone of height 14 in. Find the radius of the base of the cone.

99. A spherical shell, internal diameter 14 in., is filled with water. Its contents are poured into a cylindrical vessel whose internal radius is 14 in. Find the depth of the water in the cylinder.

100. Find the volume and the area of a cone whose slant height is 10 in. and the diameter of whose base is 6 in.

101. What will it cost to gild a ball 12 inches in diameter at 10 cents a square inch?

102. Find the number of cubic feet in a log 30 ft. long and 2 ft. in diameter at the larger and 1 ft. 10 in. at the smaller end.

103. A spherical shell is 9 in. in diameter and its thickness is 1 in. Find the volume of the shell.

104. Find the whole surface of a hemispherical bowl whose inner diameter is 4 in. and outer 6 in.

# PRACTICAL MEASUREMENTS

## CARPETING

Carpet is sold by the linear yard. Oilcloth and linoleum are generally sold by the square yard.

In finding the number of yards of carpet required for a room it is necessary to know whether the strips are to run lengthwise or crosswise. Economy in matching and appearance usually decide this.

Fractional lengths may be bought, but not fractional widths. In finding the cost of carpet we must find the whole number of strips, and add one to this number for all fractions.

In finding the length of each strip it is customary to allow a certain amount for waste in matching the figures of the pattern.

ILLUSTRATION.—How many yards of Brussels carpet,  $\frac{3}{4}$  of a yard wide, laid lengthwise of the room, will be required to cover a floor 22 ft. by 17 ft. 4 in., if the waste in matching be 6 in. on each strip ?

$$17 \text{ ft. } 4 \text{ in.} = \frac{52}{9} \text{ ft.} = \frac{52}{9} \text{ yd.}$$

$$\frac{52}{9} \text{ yd.} \div \frac{3}{4} = \frac{52}{9} \times \frac{4}{3} = \frac{208}{27} = 7\frac{8}{27}$$

$7\frac{8}{27}$  strips is practically 8 strips.

$$22 \text{ ft.} + 6 \text{ in.} = 22\frac{1}{2} \text{ ft.}$$

$$22\frac{1}{2} \text{ ft.} \times 8 = 180 \text{ ft.} = 60 \text{ yd.}$$

SOLUTION.—Since the strips run lengthwise of the room, to find the number of strips, divide the width of the room by the width of the carpet.  $\frac{52}{9}$  yd. divided by  $\frac{3}{4}$  yd. equals  $7\frac{8}{27}$ , the number of times one strip required. Since

fractional widths of carpet cannot be bought, drop the fraction and add 1 to the whole number ; 8 strips are required, and  $\frac{8}{27}$  of a strip may be cut off or turned under. The length of the room is 22 ft., and there is a waste of 6 in. on each strip for matching ; hence, the length to be bought for each strip is  $22\frac{1}{2}$  ft. If there are  $22\frac{1}{2}$  ft. in each strip, in the 8 strips there are 8 times  $22\frac{1}{2}$  ft., or 180 ft., or 60 yds.

### SERIES 50

1. How many yards of Axminster carpeting  $\frac{3}{4}$  of a yard in width, and laid lengthwise of the room, will be required to cover a floor  $21\frac{3}{4}$  ft. long and  $18\frac{3}{4}$  ft. wide, making no allowance for waste in matching the design ?

2. How many yards of carpeting 27 in. wide will be required for a room 17 ft. 6 in. by 15 ft. 5 in., if the strips run crosswise, and 7 in. be wasted in matching each strip ?

3. What will be the cost of the carpet border for a room 16½ ft. by 21 ft., if the price be 62½c per yard ?

4. Find the cost of carpeting a room 28 ft. 10 in. long by 17 ft. 8 in. wide, with carpet  $\frac{3}{4}$  of a yard wide, at \$1.80 per yard, if the strips run lengthwise of the room, and 9 in. per strip be wasted in matching.

5. A parlor, 20 ft. by 17 ft., is carpeted with a carpet 1 yard wide, running lengthwise, at \$1.20 per yard, surrounded with a carpet border 1 foot wide, at 75 cents a yard. Find the total cost.

6. Find the cost of covering the floor of a hall, 24 ft. long by 8 ft. wide, with oilcloth at 75c a square yard.

7. Find the cost of the stair carpet at \$1.20 a yard, for a flight of stairs of 22 steps, 11 in. wide, with 7 in. rise.

8. How many yards of carpeting  $\frac{3}{4}$  of a yard wide will be required to carpet a room 32 ft. long by 25 ft. wide, if the lengths of carpet are laid across the room and 8 in. are lost on each strip in matching the pattern ? How many yards if the strips are laid lengthwise and 6 in. are lost in matching ? If the carpet is laid in the more economical way, what will be the cost at \$2.55 per yard ?

9. A room is 36 ft. long and 24 ft. wide. Find the difference in the expense of carpeting it with carpet a yard wide at \$1.40 per yard and with carpet 27 in. wide at \$1.15 per yard, the carpet to run crosswise of the room in both cases.

10. If it cost \$57.60 to carpet a room 20 ft. long, with carpet 30 in. wide, at \$1.20 per yard, find the width of the room, the strips running crosswise.

## PAPERING

Wall paper is 18 in. wide, and is put up in single rolls of 8 yards and double rolls of 16 yards.

Any part of a roll is usually counted as a whole roll, although in



the more expensive papers some dealers will sell part of a roll. But this depends upon the quality of the paper and the dealer.

### METHOD OF ESTIMATING

#### (a) For the Walls :

1. Find the distance around the room, less the width of doors and windows.

2. Find the number of strips required, by dividing the width of one strip into the net distance around the room.

3. Find the number of strips that can be cut from a full roll by dividing the height of the room into the length of a roll.

4. Find the number of rolls required by dividing the number of strips required for the room by the number of strips in a roll.

#### (b) For the Ceiling :

If the strips are to run lengthwise, get the number of strips by dividing the width of the room by the width of a strip. If the strips are to run crosswise, get the number of strips by dividing the length of the room by the width of a strip. Having got the number of strips required, proceed as for the walls.

#### (c) For the Border :

Calculate the distance around the room, making no allowances for the doors and windows.

NOTE 1.—Where the width of the doors and windows is not given, allow 3 feet for each.

NOTE 2.—When the height of the room is not given, allow 2 rolls for every 5 strips.

ILLUSTRATION.—How many double rolls of paper will be required for the sides and ends of a room 24 ft. long, 18 ft. wide, and 8 ft. high, with one door and three windows each  $3\frac{1}{2}$  ft. wide, making no allowance for waste in cutting ?

## SOLUTION

$24 \text{ ft.} + 18 \text{ ft.} \times 2 = 84 \text{ ft.}$ , the perimeter of the room.

$3\frac{1}{2} \text{ ft.} \times 4 = 14 \text{ ft.}$ , the width of the doors and windows.

$84 \text{ ft.} - 14 \text{ ft.} = 70 \text{ ft.}$ , or  $23\frac{1}{3} \text{ yd.}$ , the length of the regular surface of the walls.

A double roll of paper is  $\frac{1}{2} \text{ yd.}$  wide and 48 ft. long.

$23\frac{1}{3} \div \frac{1}{2} = 46\frac{2}{3}$ , or practically 47, the number of strips necessary for the regular surface.

$48 \text{ ft.} \div 8 \text{ ft.} = 6$ , the number of strips in each double roll.

$47 \div 6 = 7\frac{5}{6}$ , or practically 8 double rolls.

Hence, 8 is the required number of double rolls of paper.

## SERIES 51

1. How many strips of paper will go around a room 20 ft. by 24 ft. ?

2. How many strips of paper are required for a room 30 ft. by 24 ft., if there are four windows and two doors, allowing 3 ft. for each ?

3. How many single rolls of paper will it take to cover the ceiling of a room 60 ft. long, 45 ft. wide, if there be no waste in matching ?

4. How many double rolls are required for a hall 21 ft. long, 18 ft. wide, and 12 ft. high, with a cornice 1 ft. deep ?

5. How many rolls of wall paper will be required for the walls of a rectangular room 20 ft. by 16 ft., with a 12 ft. ceiling, there being one door 3 ft. 8 in. wide, and two windows each 4 ft. 2 in. wide ?

6. What is the cost of paper, at \$1.25 per double roll, for a room 18 ft. long, 12 ft. wide, and 9 ft. high above the baseboard, allowing for one door and two windows each  $3\frac{1}{2} \text{ ft.}$  wide ?

7. How many rolls of paper 8 yards long and 18 in. wide will be required for the walls of a room 20 ft. long, 15 ft. wide, and having a height of 8 ft. 9 in., allowing for one door 3 ft. by 7 ft., and for two windows 3 ft. by 6 ft., and a baseboard 9 in. high ?

8. At \$1.90 per double roll, what will be the cost of papering a parlor 20 ft. square and 8 ft. high from the baseboard, allowing for one door 3 ft. by 7 ft., and three windows each 3 ft. by 6 ft. ?

9. Allowing for three windows, each 42 in. by 7 ft., and two doors each 4 ft. by 11 ft., what will be the cost, at \$1.80 per single roll, of papering a room 24 ft. long, 18 ft. wide, and 12 ft. high from the baseboard ?

10. Find the cost of the wall paper at 80 cents a roll, and bordering at 7 cents a yard, for a room 18 ft. 9 in. long by 16 ft. 5 in. wide, with the ceiling 10 ft. 9 in. above the baseboards, allowing for two doors each 3 ft. 8 in. wide, and three windows each 3 ft. 6 in. wide.

## LUMBER

The unit of measurement for lumber is the board foot. It is 1 ft. long, 1 ft. wide, and 1 in. thick.

Lumber less than 1 in. in thickness is considered inch lumber in measuring.

In measuring the width of a board, a fraction greater than a half runs the width up to the next even number ; a fraction less than a half is rejected. Thus, a board  $4\frac{5}{8}$  in. wide is considered 5 in. wide ; while a board  $4\frac{3}{8}$  in. wide is considered 4 in. wide.

Lumber is usually sold at so much a thousand feet, board measure.

To find the number of board feet or feet of lumber in a board, plank, joist, etc., multiply together the three numbers representing the number of times the unit (1 board feet) is repeated in each direction.

ILLUSTRATION.—Find the number of board feet in a plank 10 ft. long, 15 in. wide, and 3 in. thick.

### SOLUTION 1

The plank is 10 ft. long, and a board foot is 1 ft. long, so the unit is repeated 10 times in length.

The plank is 15 inches wide, and a board foot is 1 ft. or 12 inches wide, so the unit is repeated  $1\frac{1}{4}$  times in width.

The plank is 3 inches thick, and a board foot is 1 inch thick, so the unit is repeated 3 times in thickness.

So the number of board feet in the plank  $= 10 \times 1\frac{1}{4} \times 3 = 37\frac{1}{2}$  board feet.

### SOLUTION 2

$$\frac{\text{Contents of plank}}{\text{Contents of board foot}} = \frac{10 \text{ (ft.)} \times 15 \text{ (in.)} \times 3 \text{ (in.)}}{1 \text{ (ft.)} \times 12 \text{ (in.)} \times 1 \text{ (in.)}} = 37\frac{1}{2} \text{ bd. ft.}$$

## SHORT METHOD

*Multiply the length in feet by the width and thickness in inches, and divide the product by 12, and the result will be the number of board feet of lumber.*

In charging, or billing lumber, the number of pieces are entered first, then the thickness and width in inches, then the feet in length. For example, in recording 6 pieces, 4 in. thick by 6 in. wide and 20 ft. long the form would be thus: 6 pcs. 4 in.  $\times$  6 in. — 20 ft., and would be called off by the salesman, "6 four-by-sixes — 20 ft.," *four-by-sixes* being the name by which he selects and sells stock.

Instead of writing "inches" and "feet," lumber billing clerks use (") for inches, and (') for feet; thus, 3 in. by 4 in.—17 ft. long, is written, 3"  $\times$  4" — 17'.

When the width of a board tapers uniformly, the average width is found by finding one-half the sum of the two ends.

## SERIES 52

1. How many feet of lumber in a floor 15 ft. long, 12 ft. wide, and 1 in. thick?

2. A bridge 84 ft. long and 20 ft. wide, is covered with oak plank 2½ in. thick. What is it worth at \$16 per thousand?

3. How much lumber 1 in. thick will be required to cover a walk 5 ft. wide around the outside of a lawn 300 yards long and 200 yards wide?

4. What is the number of board feet in a stick of timber 30 ft. long and 18 in. square?

5. A 2 in. plank, 9 in. wide at one end and 15 in. wide at the other, tapers gradually. How much lumber is there in it, the length being 18 ft.?

6. At \$20 per thousand, find the whole cost of 5 scantling 20 ft. long, 4 in. wide, 3 in. thick; 9 scantling 18 ft. long, 5 in. wide, 4 in. thick; 6 scantling 14 ft. long, 6 in. wide, 5 in. thick.

7. At \$32.50 per thousand, what will be the cost of  
8 scantlings 3"  $\times$  4"—18'; 12 scantlings 4"  $\times$  5"—16';  
8 scantlings 5"  $\times$  6"—14'.

8. At \$19.50 per thousand, what will be the total cost of  
9 boards 1"  $\times$  2"—14'; 6 boards 1½"  $\times$  18"—16';  
15 boards 2"  $\times$  14"—20'; 8 boards 1¼"  $\times$  12"—18'.

9. At \$24 per thousand, what will be the cost of the lumber required to inclose a field 40 rods square with a board fence if the boards are 15 ft. long, 5 in. wide, and 1 in. thick, and the fence five boards high ?

10. At \$21.50 per thousand, what will be the cost of the lumber in a line fence 160 rods long, if the boards are 11 ft. long, 7 in. wide, and 1 in. thick, and the fence four boards high ?

11. What will it cost to fence 10 miles of railway, both sides, with six rounds of 6 in. boards at \$15 per thousand feet ?

12. What will it cost, at \$16 per thousand, to fence a field 40 rods by 60 rods with one round of 12 in. boards and five of 6 in. boards ?

13. What will be the cost per mile to fence a railway with six strands of barbed wire, which weighs 1 lb. per rod, at 8 cents a pound ?

14. Find the cost of a quarter-mile of fence, with the posts 8 ft. apart, a 12 in. base, a  $2 \times 4$  rail at top, and five rows of 6 in. boards. The posts cost 10 cents each, and the lumber \$12 per thousand.

15. If lumber is \$20 per thousand, find the cost of the boards and scantling required for a sidewalk 54 ft. long and 4 ft. wide. The boards are 1 in. thick, and are laid on two rows of scantling 2 in.  $\times$  4 in.

16. A contractor undertakes to lay a sidewalk 8 ft. wide, on both sides of a street one-eighth of a mile in length. The plank used is to be 2 in. thick, and the walk is to be supported by three continuous lines of scantling 4 in. square. Determine the cost of the lumber at \$15 per thousand feet board measure.

17. A lot, 60 feet wide and 120 feet long, is to be enclosed on the two sides and the back by a tight board fence 6 ft. high. The posts are to be placed 6 ft. apart, and to cost 15 cents each. There are to be two string pieces of scantling, 2 in. thick and 4 in. wide, from post to post, on which to nail the boards. Lumber is \$15 per 1,000 ft.

(a) Find the cost of the posts.

(b) Find the cost of the boards.

(c) Find the cost of the scantlings.



18. A certain sidewalk is 250 yards long, 10 ft. wide, and made of plank 2 in. thick. The planks rest on three continuous lines of scantling 3 in.  $\times$  6 in. Find the cost of the material at \$18 per thousand.

19. At \$16 per thousand, find the cost of the 2 in. plank for a sidewalk half a mile long and 6 ft. wide.

20. Find the cost of 720 boards, 14 ft. long, 8 in. wide, and  $1\frac{1}{2}$  in. thick, at \$12 per thousand feet.

21. At \$15 per thousand, board measure, what will be the cost of 2 in. plank for a 4 ft. sidewalk, half a mile long ?

22. At \$12 per thousand, what will be the cost of 2 in. plank for a 3 ft. sidewalk on the street sides of a rectangular corner lot 55 ft. wide and 108 ft. 8 in. long ?

23. What length of 2 in. plank, 18 in. wide, will contain 48 board feet ?

24. What length of 3 in. plank, 10 in. wide, will contain 40 board feet ?

25. What is the width of a 2 in. plank, 16 ft. long, which contains 40 board feet ?

26. What is the thickness of a piece of timber, 40 ft. long and 15 in. wide, that contains 400 ft. of lumber ?

27. What will be one-half of the cost of a line fence 40 rods long and five boards high, of boards 16 ft. long, 1 ft. wide, and 1 in. in thickness, the posts to be placed 8 ft. apart at a cost of \$16 per thousand for lumber and \$25 per hundred for posts ?

28. At \$18 per thousand for lumber and \$22 per hundred for posts, what will be the cost of lumber and posts for enclosing a lot  $40 \times 160$  ft. with a picket fence, the pickets being 4 ft. long, 3 in. wide, and 1 in. thick, allowing 3 in. space between pickets, the posts being placed 8 ft. apart, two  $2 \times 4$ 's being used as stringers, and a baseboard 10 in. wide extending below the pickets ?

## ROOFING

The unit in measuring for flooring or roofing is the square. A square contains 100 square feet.

Shingles are 16 in. long, and average 4 in. in width. They are generally laid 4 in. to the weather. Hence each shingle will

cover 16 square inches, and 9 will be required for each square foot, or 900 for each square of roofing.

Allowing for waste, 1,000 shingles, laid 4 in. to the weather, are estimated to cover a square.

Shingles are sold by the bunch, each bunch containing 250 shingles ; hence four bunches are required for a square of roofing.

Dealers will not sell part of a bunch.

### SERIES 53

1. Making no allowance for waste, how many shingles, laid  $4\frac{1}{2}$  in. to the weather, will be required for a square of roofing ?
2. Making no allowance for waste, how many shingles, laid 5 in. to the weather, will be required for a square of roofing ?
3. How many squares of roofing are there in a double roof, 45 ft. long, with 20 ft. rafters ?
4. At \$10 per square, what will be the cost of the slate for a double roof 45 ft. long and 28 ft. wide ?
5. How many bunches of shingles are required for a shed roof 40 ft. long and 3 ft. wide ?
6. How many bunches of shingles are required for a double roof, 70 ft. long, with 30 ft. rafters ?
7. Find the cost, at \$4.50 per thousand, of the shingles necessary for a double roof, 56 ft. long, with rafters 25 ft. long.
8. Find the cost, at \$4.20 per thousand, of the shingles for the roof of a building, 52 ft. 8 in. long and 32 ft. wide, having a gable 12 ft. high and the rafters having an 18 in. heel.
9. A building, 64 ft. long and 36 ft. wide, has a gable 14 ft. high. In roofing the building the rafters were given an 18 in. heel. Find
  - (a) The number of bunches of shingles required for the roof.
  - (b) The value of the sheeting, at \$16 per thousand.
  - (c) The value, at \$23 per thousand, of the rafters, if they are 2 in.  $\times$  4 in., and are 2 ft. from centre to centre.

10. How many shingles, 18 in. long and 4 in. wide, lying one-third to the weather, are required to shingle a double roof, 54 ft. long, with rafters 22 ft. long, the first row of shingles being double and no allowance being made for waste ?

## LATHING AND PLASTERING

Standard laths are 4 ft. long,  $1\frac{1}{2}$  in. wide, and are laid  $\frac{1}{2}$  in. apart. 1,000 laths of standard size will, therefore, cover about 74 square yards. Allowing for waste, however, 1,000 laths are estimated to cover 70 square yards.

Laths are put up in bundles of 100 and 50. So a bundle of 100 is estimated to cover 7 square yards, and a bundle of 50 to cover  $3\frac{1}{2}$  square yards.

But all laths are not of standard size. Some are only 32 in. long and some are only  $1\frac{1}{4}$  in. wide.

In reckoning the amount of laths required, calculate the superficial area to be lathed, and deduct the area of all openings.

In reckoning the cost of labor for lathing and plastering, it is customary to deduct only one-half the area of the openings ; but there is no set rule to this effect, and it is well to have a distinct understanding with the contractor on this point before the job is commenced.

### SERIES 54

1. How many bundles of 100 standard laths are required for the walls and ceiling of a room 27 ft. long, 18 ft. wide, and 10 ft. high, containing two doors 7 ft. by 4 ft., and four windows 6 ft. by 4 ft. ?

2. In the preceding question, had the lath been 32 in. long, and put up in bundles of 50 each, how many bundles would have been required ?

3. Making no allowance for waste, how many square yards would 1,000 laths, 32 in. long and  $1\frac{1}{4}$  in. wide, cover if laid three-eighths of an inch apart ?

4. Find the cost of plastering a room 30 ft. long, 27 ft. wide, and 12 ft. high, at 20 cents a square yard, if there are two doors each 3 ft. 6 in. by 7 ft., and three windows each 3 ft. 4 in. by 6 ft., and

the plasterer be allowed pay for one-half the area of the openings for his trouble in plastering around them.

5. A room, 16 ft. long, 14 ft. 6 in. wide, and 10 ft. high, has a skirting board 1 ft. high, two doors 7 ft. by 3 ft. 10 in., and two windows 6 ft. by 4 ft. Find the number of bundles of 50 standard laths required to lath it.

6. Find the cost, at 22 cents a square yard, of plastering the walls of a room 23 ft. 6 in. long, 15 ft. wide. and 8 ft. 4 in. high, having one door 7 ft. 6 in. by 3 ft. 8 in., three windows 6 ft. 9 in. by 4 ft. 2 in., and a skirting board 11 in. high, allowing one-half the area of the openings.

7. Find the cost, at 22 cents a square yard, of plastering a room 18 ft. long, 15 ft. 6 in. wide, and 10 ft. 4 in. high, containing two doors 7 ft. 4 in. by 4 ft., two windows 6 ft. by 3 ft. 10 in., one mantel-piece 5 ft. by 3 ft. 6 in., and a 10 in. baseboard (deduct one half the area of the openings).

## BRICK AND STONE WORK

It may be stated at the outset that no set rule can be given for the calculation of either the amount of material required or the amount to be expended in labor in either of these cases. Bricks are of varying dimensions. They are laid according to specifications in varying thicknesses of mortar, and the judgment of the contractor, in facing the particular conditions, so often modifies even his own general plan of figuring, that it would be absurd to say that if certain methods of calculation are followed, we will get results as contractors in Canada or any other place would estimate them.

What is true of brick work is just as true of stone work. There are many kinds of stone work and many places where the judgment of the contractor is brought into play. The best that we can hope to do in a work of this character is to lay down certain general rules, which at least have the sanction of good authority, in the hope that the student who learns to apply his ability under these rules will have no difficulty in making calculations, no matter what changes he may be told to make, according to the part of the



country in which he is working or the peculiar methods of the contractor for whom he is working.

### Stone Work

The perch is a common unit of measurement. In stone work it is  $16\frac{1}{2}$  ft. long,  $1\frac{1}{2}$  ft. wide, and 1 ft. high, and contains exactly  $24\frac{3}{4}$  cu. ft. In estimating 25 cu. ft. are figured as a perch. Stone masonry is measured by two systems : quarryman's measurement and mason's measurement. The quarryman's measurement is concerned with the figuring of the quantity of material to be used. The mason's measurement is more concerned with figuring the amount of work to be done, and the consequent price to be paid for it.

In measuring for material, the actual contents of the wall are measured. That is, all openings are taken out and all corners are measured single. In buying the stone, it may be bought by the cord, which is of the same dimensions as a cord of wood ; namely, 8 ft. by 4 ft. by 4 ft., or 128 cu. ft. It is estimated that 4 perches or 100 cu. ft. of wall will take one cord of stone, one barrel of lime, or, say,  $2\frac{1}{2}$  bushels, and 5 barrels of sand.

In estimating the amount of work to be done, corners are always doubled. That is, the girt measurement of the walls is taken. No allowance is made for openings less than 3 by 5 ft., and only half the amount of openings larger than 3 by 5 ft.

### Brick Work

Brick walls are of various degrees of thickness. For instance, there is the wall which is  $4\frac{1}{2}$  inches thick, or, as it is sometimes called, the half-brick wall. There is the 9-inch wall, which is sometimes called the single brick wall. There is the 13-inch wall or the brick and a half wall, the 18-inch wall or the double brick, the 22-inch wall or the  $2\frac{1}{2}$  brick, and so on according to the thickness of the wall. In consequence of variations in size of bricks, no exact rule for the volume of laid brick can be given. The following scale is, therefore, a fair average :

- 7 bricks to the superficial foot in 4-inch wall.
- 14 bricks to the superficial foot in 9-inch wall.
- 21 bricks to the superficial foot in 13-inch wall.
- 28 bricks to the superficial foot in 18-inch wall.
- 35 bricks to the superficial foot in 22-inch wall.

If the wall is thicker than 22 inches, for each additional  $4\frac{1}{2}$  inches in thickness add 7 bricks per square foot.

The labor and material of brick work are estimated by the 1000 brick. In measuring up brick walls, it is not customary to deduct for openings less than 2 feet square. To allow for corners, in getting the superficial area of the walls, from the girt measurement subtract four times the thickness of the wall and then multiply by the height.



## SERIES 55

1. How many cords of stone will be required for the foundations of a house 39 ft. by 27 ft., the stonework to be 6 ft. high and 18 in. thick ?

2. In the preceding question, for how many cubic yards would the masons be paid ?

3. How many cords of stone are required for the walls of a house 40 ft. long, 27 ft. wide, and 20 ft. high, the wall to be 1 ft. thick, if it is to have four doors 7 ft. 6 in. by 4 ft., and eight windows 5 ft. by 4 ft. ?

4. In the preceding question, for how many cubic yards will the men who do the work be paid ?

5. A house is to be built 35 ft. long, 28 ft. wide, and 22 ft. high, and is to have ten windows  $4\frac{1}{2}$  ft. by 6 ft., and five doors 8 ft. by 4 ft. 6 in., the wall to be 9 in. thick. How many bricks will be required ?

6. If the wall in the preceding question were made 13 in. thick, how many bricks would be required ?

# PERCENTAGE

## TERMS USED IN PERCENTAGE CALCULATIONS.

The **Prime Cost** of an article is the first cost.

The **Gross Cost** is the prime cost plus charges of freight, drayage, etc.

The **Selling Price** is the price for which an article is sold.

The **Profit** or **Gain** is the amount by which the selling price exceeds the gross cost.

The **Loss** is the amount by which the gross cost exceeds the selling price.

A **Trade Discount** is an allowance or deduction made by a dealer or manufacturer from catalogue or list prices.

An **Agent** is one who acts under authority for another.

The **Principal** is the one for whom an agent acts.

A **Commission Agent** is one who buys and sells goods or property, or collects money for his principal.

A **Broker** is an agent who effects purchases or sales in the interests of a buyer or seller. He brings the buyer and seller together, as it were, and is chiefly engaged in the case of stock transactions.

**Commission** is the name applied to the commission agent's charge.

**Brokerage** is the name applied to the broker's charge.

A **Consignment** is the property received by a commission agent to be sold for a principal.

The **Consignor** is the person who ships the goods.

The **Consignee** is the commission agent who receives the goods to be sold.

A **Del Credere Agent** is one who guarantees to his principal the payment of sales made on time.

**Guaranty** is the charge made for guaranteeing sales made on time.

The **Gross Proceeds** of a sale or collection is the total amount received by the agent before deducting charges.

The **Net Proceeds** is what remains after all charges have been deducted.

An **Account Sales** is the statement rendered by the agent to his principal, showing sales and all charges and the net proceeds.

An **Account Purchase** is the statement rendered by an agent to his principal, showing cost of goods bought and the charges, together with the total amount of the purchase.

**Duties or Customs** are taxes levied by the Dominion Government on goods imported into Canada. Duties are levied for the purpose of raising revenue and protecting home industry.

An **Ad Valorem Duty** is a certain percentage of the actual cost of the goods imported.

A **Specific Duty** is a tax assessed at a certain sum per foot, yard, gallon, or other weight or measure. Upon certain goods both kinds of duties are levied.

A **Tariff** is a list of goods, with the rates of duties imposed on the same.

The **Free List** includes goods that are exempt from duty.

A **Preferential Duty** is a duty lower than the common, imposed on goods from certain countries. Thus, goods imported from Great Britain or British possessions are given a preferential duty in Canada.

A **Surtax** is a duty higher than the common, imposed on goods from certain countries. Thus, goods imported from Germany are subject to surtax.

A **Drawback** is a refund made where goods once imported, and upon which import duty has been paid, are exported.

A **Bonded Warehouse** is a place for the storage of goods upon which duties have not been paid.

**Insurance** is a contract in which one party to the contract agrees for a certain consideration to make up loss which another party may sustain. It is distinguished as Property Insurance, Life Insurance, Accident Insurance, and Health Insurance.

The **Policy** is the name applied to the written form of the contract made between the insurance company and the party insured.

A **Valued or Closed Policy** is one in which the amount insured is definitely determined when the insurance is effected. Houses and furniture are insured in this way.

An **Open Policy** is one upon which additional insurance may be entered at any time.

The **Premium** is the amount paid for the insurance.

**Adjustment of Fire Insurance Losses.**—Under the ordinary form of policy, the insurance company undertakes to pay the full amount of the loss or damage, provided such loss does not exceed the sum mentioned in the policy. The policy, however, may contain an average clause, in which case the payment made is such proportion of the loss as the amount of the insurance bears to the total value of the property. Again, the policy may contain a “co-insurance clause.” As an example, we quote the following 80% co-insurance clause :

“ It is a part of the consideration of this policy, and the basis upon which the rate of premium is fixed, that the insured shall maintain insurance concurrent in form with this policy, on each and every item of the property hereby insured, to the extent of at least eighty per cent. of the actual cash value thereof, and that, failing so to do, the insured shall be a co-insurer to the extent of an amount sufficient to make the aggregate insurance equal to eighty per cent. of the actual cash value of each and every item of the property hereby insured, and, in that capacity, shall bear his, her, or their proportion of any loss that may occur.”

**Short Rates** are rates of premium charged when the term of insurance is less than a year.

In case a policy is terminated at the request of the insured, he is charged the short rate premium. If the policy is terminated by the company, the lower long rate will be charged, and the company will refund the premium for the unexpired time of the policy.

**Marine Insurance** refers to insurance of vessels and their cargoes against the dangers of navigation.

**Adjustment of Marine Insurance Losses.**—Such losses are usually settled by the insurance company paying only such proportion of the losses as the sum insured is to the entire value of the vessel.

**Salvage** is an allowance made to those rendering aid in saving vessels or cargoes from marine disasters.

**Taxes** are sums collected by a municipality for the payment of municipal indebtedness. The tax is a sum assessed either on the person, property, or income of an individual.

A **Poll Tax** or **Head Tax** is the sum collected from each male citizen liable to taxation without regard to property or income.

A **Property Tax** is a tax on real or personal estate, and is assessed at a given rate, generally at so many mills on the dollar.

An **Income Tax** is a tax assessed on the individual according to his income. Generally speaking, a certain part of the income is exempt, and above that it is liable to taxation.

**Interest** is the use of money.

The **Principal** is the money which is loaned, and for the interest, or use of which, a certain sum in cash has to be paid.

The **Amount** is the sum of the principal plus the interest.

A **Joint Stock Company** is an association of individuals who have obtained incorporation enabling them to transact business as a single individual.

**Methods of Incorporation.**—Joint stock companies are formed in Canada in two ways: (a) By special Act of Parliament, either of the Parliament of Canada or the Legislature of the Province in which business is to be conducted; (b) by the letters patent issued under the Companies Act. In some of the Provinces registration takes the place of letters patent.



The **Charter** or **Letters Patent** is the document, issued under the seal of the Department of State or the Provincial Secretary's Department, constituting certain persons therein mentioned, and others who may become associated with them, a joint stock company.

The **Authorized Capital of the Company** is the amount designated in the charter as the limit to which the company may invite subscriptions for stock.

The **Subscribed Capital** is that part of the authorized capital which at any date has been subscribed for, or allotted to, shareholders.

The **Paid-up Capital** is that part of the subscribed capital which at any date has been paid for by subscribers thereto.

A **Share** is one of the equal divisions into which the authorized capital is divided. Unless specially mentioned to the contrary, a share is worth \$100.

The **Par Value of a Share** is the value fixed upon it by the charter, which mentions the amount of the authorized capital, the number of shares into which it is divided, and the value of each.

The **Market Value of a Share** is the figure for which the share may be sold in the stock market. Thus, a \$100 share may sell for \$90 or for \$150. This is all a question of the standing of the company and the way in which its stock is sought for.

A **Stock Certificate** is a paper issued by a company to one who has bought stock, and certifies as to the number of shares to which the holder is entitled.

**Preferred Stock** is stock which is entitled to a preference over any ordinary stock. This preference usually takes the shape of a prior claim on gains for the purpose of dividend, but it may extend in any other direction, such as a right to elect a stated number of the board of directors.

**Common Stock** is the ordinary stock of the company not entitled to any preference.

**Cumulative Preferred Stock** is stock which is preferred in the matter of dividends, and which, further, is entitled to have dividends which are not paid in one year cumulate as a charge against profits in succeeding years. Thus, if the stock were a 5% preference stock it means that each year 5% dividend is to be paid. If the 5% is not forthcoming the first year, the dividend to be paid the second year would really be 10%. If not paid in the first or second year, the dividend for the third year would really be 15%.

The **Dividend** is simply a part of the profits of the company set aside by the directors as the amount to be divided among the shareholders as a return for the money that they invested. The dividend must not be paid unless there is a net gain from which to pay it.

An **Assessment** is the opposite of a dividend, and is the amount contributed by the shareholders to meet losses or expenses of the company. It will be noted that once a shareholder has paid for his stock, he cannot be forced to pay any additional amounts. The assessment is, therefore, a voluntary contribution.

A **Call** is an order issued by the board of directors requesting those shareholders who have not fully paid for their stock to pay a certain amount on such stock.

An **Instalment** is the amount paid in answer to a call.

A **Premium** is the amount by which the market value of stock exceeds its par value.

A **Discount** is the amount by which the market value is less than the par value.

A **Bond** or **Debenture** is a written promise to pay a sum of money, with a fixed rate of interest, at or before a specified time. Bonds are simply the promissory notes of corporations, such as joint stock companies, cities, towns, villages, etc.

**First Mortgage Bonds** are bonds which are secured by a first mortgage on the property of the corporation issuing the bonds. In the same way we may have **Second Mortgage Bonds**.

**Registered Bonds** are those issued to a certain person whose name is registered in the books of the company as the owner of such bonds. The interest will be paid only to the registered owner or his attorney.

**Coupon Bonds** are bonds having small coupons or detachable certificates attached to the bond. These coupons may be clipped off at the time the interest falls due, and, on being presented, entitle the holder to the interest amount.

### BASIS OF CALCULATION

The term, "Per cent.," usually written %, is an abbreviation of the Latin words, *Per centum*, which signify "by the hundred."

Some values in business are estimated by the pound (sugar, for example), some by the bushel (wheat, for example), others by the hundreds (use of money, for example).

**Commission.**—An agent is hired to do a certain work. He is paid 5% commission. It means that for every \$100 worth of business done the agent is paid \$5.

**Discount.**—A man pays a debt before it is due, and he is allowed 5% discount. It means that for every \$100 that he owed he will only have to pay \$95.

**Profit and Loss or Loss and Gain.**—Goods are sold at 5% profit. It means that as often as goods costing \$100 are sold, \$5 more than cost is received as a selling price. If 5% were lost, \$5 less than cost would be received.

**Insurance.**—Property is insured, and a premium of 5% is paid. It means that for every \$100 worth of property insured \$5 is paid to the insurance company. These premiums form the fund from which the insurance company pay for all losses of insured property.

**Exchange.**—A draft is bought at a bank, and 5% exchange is paid. It means that for issuing a \$100 draft the bank will charge the one who buys the draft \$5 for the convenience.

**Taxes.**—A man owns a city lot, and pays a tax rate of 5%. It means that for every \$100 assessed value he must pay into the city treasury \$5 as his contribution to the city expenses.

**Dividends.**—Stock in a certain company yields 5% dividends. It means that for every \$100 worth of stock held, a man is entitled to \$5 as his share of the profits.

**Duties.**—An importer has to pay a duty of 5%. It means that in bringing goods into Canada from some other country the importer must pay to the Canadian Government \$5 for every \$100 worth of goods imported.

**Interest.**—Money is borrowed and interest at the rate of 5% per annum is charged. It means that for every \$100 borrowed, \$5 must be paid for its use every year.

**ILLUSTRATION.**—What is 5% of \$225 ?

\$225	SOLUTION.—5% means 5 per hundred, or 5 out of every
.05	hundred, which is $\frac{5}{100}$ or .05. The question becomes a simple
—	one in multiplication of decimals.
\$11.25	

**NOTE.**—The circumstance which gives rise to the calculation of the percentage cannot alter the method of finding it. It may be we want to know the interest on \$225 at 5%, or we may want to figure the commission on a sale of \$225 at 5%, or we may want to find the duty on goods worth \$225 at 5%. The process of getting the result is the same for all cases.

## QUESTIONS OF THE FIRST ASPECT

### SERIES 56

1. ILLUSTRATION.—What is 36% of \$480 ?

SOLUTION

100%	of it equals	\$480
1%	of it equals	\$4.80
36%	of it equals	$\$4.80 \times 36 = \$172.80$
	or	
		$\$480 \times .36 = \$172.80.$

2. What is 24% of 375 yards ?

3. What is 76% of 793 ?

4. A man, who is worth \$75,000, has 35% of his wealth invested in real estate. What is the value of his real estate ?

5. In a certain school there are enrolled 460 pupils, of whom 45% are boys. How many girls are enrolled ?

6. A rancher, who owned 640 sheep, lost 15% of them in a storm. How many sheep had he remaining ?

7. A merchant bought goods for \$458.75, and sold them for 36% more than he paid for them. What did he receive for the goods ?

8. A farmer sold 420 bushels of wheat at \$0.97 per bushel, and 20% more oats than wheat at \$0.43 per bushel. What did he receive for both.

9. A man invests \$17,280 as follows : 25% in real estate  $37\frac{1}{2}\%$  in bank stock, and the remainder in city lots. How much did he invest in each ?

10. What will  $37\frac{1}{2}\%$  of 480 bushels of wheat cost at \$1.25 per bushel ?

11. A man, buying a house and lot, paid \$1,500 for the lot and  $37\frac{1}{2}\%$  more than that for the house. What did both cost ?

12. A farm contained 320 acres ; 25% of it was sold at \$45 per acre, 40% at \$37.50 per acre, and the balance at \$30 per acre. How much was received for the farm ?

13. A man has a yearly salary of \$2,400, and spends  $33\frac{1}{3}\%$  of it the first year, 45% the second, and 64% the third. How much does he save in the three years ?

14. A man deposited in a bank \$1,875 ; he withdrew 40% of the deposit, and with  $9\frac{1}{3}\%$  of the amount withdrawn purchased a gun. What was the cost of the gun ?

15. A man, having a salary of \$3,600, spends 20% of it for board,  $12\frac{1}{2}\%$  for clothing, 10% for books and lecture fees, 5% for incidentals, and deposits the remainder in the bank. How much does he deposit ?

16. I owed a man \$1,450, and paid him 40% of it at one time, 20% of the remainder at another time, and 10% of what then remained at another time. How much did I then owe him ?



17. A rancher owned 2,070 cattle, and sold  $33\frac{1}{3}\%$  at one time, 40% of the remainder at another time, and  $8\frac{1}{3}\%$  of what then remained at another time. What is the value of the remainder at \$36 each?

18. A farmer had an orchard containing 225 trees, and he paid for picking and packing the apples  $3\frac{1}{3}\%$  of their value. If they averaged  $8\frac{1}{2}$  barrels to the tree, and he sold them at \$1.20 per barrel, how much did he pay for picking and packing them?

19. A man, dying, left an estate of \$150,000, with instructions that after his widow received her share of  $33\frac{1}{3}\%$ , 10% was to be given to his brother, and the remainder was to be divided equally among his four children. How much did each child receive?

ILLUSTRATION.—Goods are invoiced at \$720, with discounts of 25%, 20%, and 10% off. Find the net selling price.

## SOLUTION

$$\begin{array}{r} \$720 \\ 180 \quad = 25\% \text{ of } \$720 \\ \hline \end{array}$$

$$\begin{array}{r} \$540 \\ 108 \quad = 20\% \text{ of } \$540 \\ \hline \end{array}$$

$$\begin{array}{r} \$432 \\ 43.20 = 10\% \text{ of } \$432 \\ \hline \end{array}$$

$$\$388.80 = \text{Net price.}$$

$$\begin{array}{l} \text{Or} \\ 100\% - 25\% = 75\% \\ 100\% - 20\% = 80\% \\ 100\% - 10\% = 90\% \end{array}$$

$$.75 \times .80 \times .90 = .54$$

$$\$720 \times .54 = \$388.80$$

NOTE.—The order in which the discounts of any series are considered is not material, a series of 25%, 20%, and 10% being the same as one of 20%, 10%, and 25%, or 10%, 25%, and 20%, etc.

Find the net amount of the following bills:

20. \$625, less 20% and 10%.

21. \$432, less  $33\frac{1}{3}\%$  and 25%.

22. \$327.85, less 50% and  $12\frac{1}{2}\%$ .

23. \$45.50, less 60%, 20%, and  $2\frac{1}{2}\%$ .

24. \$316.80, less 40%, 25%, and 10%.

25. \$421, less  $37\frac{1}{2}\%$ , 5%, and  $2\frac{1}{2}\%$ .

26. \$360, less 50%, 20%, and 5%.

27. \$324, less  $33\frac{1}{3}\%$ , 20%, and 10%.
28. \$243.50, less 5%,  $2\frac{1}{2}\%$ , and  $2\frac{1}{2}\%$ .
29. \$325.50, less 25%,  $12\frac{1}{2}\%$ , and 10%.
30. \$348.20, less 20%, 25%, and 10%.
31. \$127, less  $66\frac{2}{3}\%$ , 10%, and 5%.
32. \$850, less 30%, 20%, and 10%.
33. \$426.25, less  $33\frac{1}{3}\%$ , 10%, and  $8\frac{1}{3}\%$ .

34. One merchant offers to sell neckties for \$12 a dozen, with discounts of 20%,  $12\frac{1}{2}\%$ , and 10%; another offers the same grade for \$12 a dozen, with discounts of 25% and  $16\frac{2}{3}\%$ . Which is the better offer, and how much would be saved on 35 dozen neckties?

35. Bought 75 carpet sweepers at \$2.40 each, less 25%, 20%, and 10%, and sold them at the same price, less 20%, 15%, and 10%. What was my profit?

36. The catalogue price of hats is \$3.60, subject to discounts of 25%, 10%, and 10%. How many hats can be bought for \$349.92?

ILLUSTRATION.—What single discount is equivalent to 25%,  $33\frac{1}{3}\%$ , and 10% off?

SOLUTION

\$1.00

.25 = 25% of \$1.00

---

\$ .75 = Net after first discount.

.25 =  $33\frac{1}{3}\%$  of .75

---

\$ .50 = Net after second discount.

.05 = 10% of .50

---

\$ .45 = Net after third discount.

\$1.00 - .45 = .55, or 55% = single discount.

Or

\$1.00	\$1.00	\$1.00
--------	--------	--------

.25	.33 $\frac{1}{3}$	.10
-----	-------------------	-----

---

\$ .75 ×	.66 $\frac{2}{3}$ ×	.90	= .45
----------	---------------------	-----	-------

\$1.00 - .45 = .55

What single discounts are equivalent to the following discount series?

37. 20% and 10%.

38. 25% and 20%.

39.  $33\frac{1}{3}\%$  and 10%.

40. 30%, 20%, and 10%.

41. 50%, 20%, and 5%.

42. 20%, 25%, and 10%.

ILLUSTRATION.—Find mentally a single discount equivalent to 20% and 10%.

SOLUTION.—From the sum of the discounts subtract  $\frac{1}{100}$  of their product, and the result will be the required discount.

$$\text{Thus } 20 + 10 - \frac{1}{100} \text{ of } 20 \times 10 = 30 - 2 = 28$$

$\therefore$  the single discount is 28%.

When a third discount is given, combine it with the result obtained from the other two.

By inspection find a single rate of discount equivalent to the following discount series :

43. 30% and 10%.

45. 10% and  $12\frac{1}{2}\%$ .

44.  $33\frac{1}{3}\%$  and 6%.

46. 40%, 20%, and 10%.

47. 25%, 8%, and 5%.

48. Rice Lewis & Co., Toronto, sold Ingram & Davey, St. Thomas, Apr. 2, 1908, on account 30 da., 2% 10 da. : 15 cultivators listed at \$7.50 each, less 20% and 10%; 25 doz. table knives listed at \$9.75, less 10%; 5 doz. pocket knives. 3 doz. at \$6.50 and 2 doz. at \$7.50, less  $33\frac{1}{3}\%$  on each;  $\frac{3}{4}$  doz. cheese knives at \$9.75, less  $16\frac{2}{3}\%$ . Find the net amount due on the bill 9 days after date.

49. John J. Doane, St. Thomas, bought of Heintzman & Co., Toronto, Apr. 1, 1908, on account 60 da., 5% 10 da. : 5 pianos at \$450 each, 6 pianos at \$575 each, 4 pianos at \$250 each, less a discount of 40 % from each list price ; 10 organs at \$125 each, less 25% and 10% from the list price. Find the amount of the bill to render, also the amount to be remitted if the bill is paid April 8, 1908.

50. A grocer bought 10 bbls. of sugar, each weighing 330 lbs., at  $4\frac{1}{2}\text{c}$  per pound, and sold them so as to gain  $16\frac{2}{3}\%$ . Find the gain and the selling price.

51. A stock of goods, consisting of \$25,000 worth of groceries, was sold at a loss of  $12\frac{1}{2}\%$ , and 15% of the selling price was in uncollectible accounts. What was the total loss sustained ?

52. A produce dealer paid \$320 for apples, \$90 for onions, and \$120 for potatoes. He sold the apples at a gain of 25%,

the onions at cost, and the potatoes at 95% of their cost. Did he gain or lose, and how much?

53. A man bought three horses, paying respectively \$240, \$300, and \$500. He sold the first at 125% of its cost, the second at a loss of 10%, and the third at a gain of 15%. Did he gain or lose, and how much?

54. A dry-goods merchant bought a bill of goods amounting to \$175. He sold 14 $\frac{2}{7}$ % of the bill, and realized a gain equal to 50% of the cost of the whole bill. If the remainder of the stock was sold for \$100, what was the gain or loss?

55. ILLUSTRATION.—An agent sold goods to the amount of \$1,580. Find his commission at 2%.

\$1580	SOLUTION.—The value of the sales is \$1,580. The
.02	rate is .02. Then, $\$1,580 \times .02 = \$31.60$ , the com-
<hr/>	mission.
\$31.60	

56. ILLUSTRATION.—What are the net proceeds on a sale of goods amounting to \$200, at 3% commission?

\$200	SOLUTION.—The sale is \$200. The rate is .03. $1 - .03$
.97	$= .97$ . Then $\$200 \times .97 = \$194$ , the net proceeds.
<hr/>	The commission may be found and then deducted
1400	from the value of the sales. It will give the same result
1800	as the method just explained.
<hr/>	
\$194.00	

57. A real estate agent sold a farm of 90 acres at \$125 per acre on a commission of 2%. What was the amount of his commission? How much did he turn over to his principal?

58. An agent sold 450 barrels of flour at \$6.25 per barrel on a commission of 3 $\frac{1}{3}$ %. What was his commission?

59. A collector succeeded in collecting 80% of a doubtful account of \$1,500. If he charged 7 $\frac{1}{2}$ % commission, how much did he turn over to his principal?

60. My Montreal agent buys for me 4,500 bushels of wheat at 83 $\frac{1}{2}$ c per bushel. How much should I remit him to cover the cost of the wheat and his commission of 5%?

61. ILLUSTRATION.—An agent purchased goods for \$750 at a commission of  $3\frac{1}{3}\%$ . What was the principal's entire cost, the other expenses being \$25.75 ?

\$ 750
1.03 $\frac{1}{3}$
—
2250
750
250
—
\$775.00
25.75
—
\$800.75

SOLUTION.—The purchase price is \$750. The rate is .03 $\frac{1}{3}$ .  $1 + .03\frac{1}{3} = 1.03\frac{1}{3}$ .  $\$750 \times 1.03\frac{1}{3} = \$775$ . Then,  $\$775 + \$25.75 = \$800.75$ , the entire cost.

The commission may be found and then added to the value of the goods purchased.

62.

## ACCOUNT PURCHASE

TORONTO, May 24, 1906.

Purchased by C. A. NORMAN.

For account and risk of C. D. JONES.

12	Bags Bran .....	\$0.50	*		
30	Bu. Clover Seed .....	\$7.50	***		
12	" Timothy Seed .....	\$2.25	**		
40	" Flax " .....	\$ .90	**	***	
CHARGES.					
	Drayage .....		1 25		
	Commission, $1\frac{1}{2}\%$ .....		***	*	**
	Charge your acct .....			***	**

63. In accordance with the above form, prepare an Account Purchase of 4 mats Java Coffee, 266 lbs. @  $21\frac{1}{2}c$ ; 4 half-chests Y. H. Tea, 220 lbs. @  $42c$ ; 3 half-chests Oolong Tea, 130 lbs. @  $20c$ ; 1 hhd. N. O. Molasses, 63 gal. @  $46c$ . Charges as follows : Drayage, \$2.25; Commission,  $1\frac{1}{3}\%$ . Commission Merchants, Student & Co.; bought for Teacher & Co.; present date and place.



64. Rule a piece of paper, copy the following Account Sales, and make the necessary extensions, footings, etc. :

TORONTO, Aug. 3, 1908.

Sold for Account of S. L. BOWLING. By EDWARD HESS & Co.

1896									
July	6	150	lbs.	" Snowflake "	5 <sup>60</sup>	***			
"	11	80	"	" New Carlisle "	5 <sup>25</sup>	***			
"	25	100	"	" Snowflake "	6 <sup>00</sup>	***			
Aug.	3	70	"	" New Carlisle "	5 <sup>50</sup>	***	****		
CHARGES									
July	"	5		Transportation, 400 bbls. @ 31c	.....	***			
"	"	5		Cartage, 300 " @ 6c	.....	**			
Aug.	"	3		Storage, 300 " @ 3c	.....	*			
"	"	3		Commission and Guaranty, 4%	.....	***	***	***	***
Net Proceeds					.....			****	***

65. Account Sales of 5,000 bushels wheat by Rutherford and Marshall for account of the Produce and Provision Co., of Stratford; Jan. 1, 2,000 bu. at \$1.05; Jan. 20, 2,000 bu. at \$1.10; Jan. 30, 1,000 bu. at \$1.02. Charges: cartage, \$25; storage, \$12.50; guaranty,  $\frac{1}{4}\%$ ; commission, 2%. Find the net proceeds.

66. ILLUSTRATION.—How much will it cost to insure a store and contents for \$42,000 at  $1\frac{1}{2}\%$ ?

SOLUTION.—

\$42,000 = the amount insured.

$1\frac{1}{2}\%$  of \$42,000 = \$630, the premium charged.

67. A merchant insured his warehouse, worth \$10,800, at  $\frac{3}{5}\%$  of its value. What was paid for premium if the rate of insurance was  $\frac{5}{8}\%$ ?

Sum insured ( $\frac{3}{5}$  of \$10,800)  $\times$  rate of premium ( $\frac{5}{8}\%$ ) = premium (?).

68. The stock of a manufacturing company was insured for \$75,000 at the following rates:  $\frac{1}{2}$  of the stock at  $\frac{5}{8}\%$ ,  $\frac{2}{3}$  of the

remainder at  $\frac{3}{4}\%$ , and the remainder at  $\frac{7}{8}\%$ . What was the total premium paid ?

69. A store is valued at \$12,000 and the contents at \$18,000. Find the cost of insuring  $\frac{3}{4}$  of the value of the store at  $\frac{4}{5}\%$ , and  $\frac{2}{3}$  of the value of the contents at  $\frac{3}{8}\%$ .

70. An insurance company, having insured a block of buildings for \$200,000 at 75c per \$100, re-insured \$60,000 with another company at  $\frac{3}{5}\%$ , and \$80,000 with another at  $\frac{3}{8}\%$ . What amount of premium did it receive more than it paid ?

71. ILLUSTRATION.—What is the cost of a sight draft of \$960 at  $\frac{5}{8}\%$  premium ?

#### SOLUTION

Face of draft (\$960)  $\times$  rate of premium ( $\frac{5}{8}\%$ ) = premium (\$6).

Face of draft (\$960) + premium (\$6) = cost of draft (\$966).

What is the cost of a sight draft

72. Of \$7,200 at  $\frac{1}{2}\%$  premium ?

73. Of \$8,500 at  $\frac{3}{4}\%$  premium ?

74. ILLUSTRATION.—The taxable value of the property of a town is \$675,000, and the rate of taxation is \$.007 on the dollar. What is the tax ?

SOLUTION.—If the tax on \$1 is \$.007,

$\$675,000 \times .007 = \$4,725$  on \$765,000, it will be 765,000 times \$.007, which is \$4,725.

75. The rate of taxation in a certain city was  $11\frac{1}{4}$  mills on the dollar. What tax was paid by a person whose property was assessed for \$12,000 ?

76. My property is assessed at \$6,400. At the rate of  $3\frac{1}{2}$  mills on the dollar, how much tax will I be required to pay ?

77. What amount of tax must a man pay who is assessed \$12,000 for real estate and \$4,500 for personal property, if he pays a rate of  $2\frac{1}{2}\%$  ?

78. I have property worth \$19,600, which was assessed at  $\frac{3}{4}$  of its value. What was my entire tax at  $7\frac{1}{2}$  mills on the dollar, including 3 polls at 75 cents each ?

79. My real estate was assessed at \$7,500, and my personal property \$12,500. What amount of tax did I pay thereon at \$1.18 per \$100, having been allowed a discount of 3% for prompt payment ?

80. What is my entire tax for the present year upon real estate worth \$12,500, and personal property worth \$5,000, the real estate having been assessed at  $\frac{4}{5}$  of its value, the rate of taxation being \$1.40 on the \$100?

81. ILLUSTRATION.—What income will be derived from 60 shares G.T.R. Stock, paying 6% dividends?

SOLUTION

Income from 1 share is \$6.

Income from 60 shares is  $\$6 \times 60 = \$360$ .

82. ILLUSTRATION.—What would a stockholder, who owns \$4,000 Bank of Commerce Stock, receive from a 5% dividend?

SOLUTION

\$4,000 stock = 40 shares.

40 shares at \$5 income per share = \$200.

What income will be derived from

	SHARES.	DIV.		SHARES.	DIV.
83.	70	6%	87.	120	3%
84.	120	$5\frac{1}{2}\%$	88.	110	$3\frac{1}{2}\%$
85.	150	$4\frac{1}{4}\%$	89.	75	9%
86.	65	8%	90.	126	$8\frac{1}{2}\%$

What income will be derived from

	STOCK.	DIV.		STOCK.	DIV.
91.	\$5,000	7%	95.	\$3,600	6%
92.	\$8,750	3%	96.	\$4,500	$9\frac{1}{4}\%$
93.	\$4,400	4%	97.	\$9,150	$5\frac{1}{2}\%$
94.	\$3,620	5%	98.	\$4,375	8%

What is the

99. Dividend on \$3,200 par of stock, if the rate of dividend is  $3\frac{1}{2}\%$ ?

100. Dividend on 87 shares of stock, if the rate of dividend is  $12\frac{1}{2}\%$ ?

101. Assessment on \$10,500 par of stock; rate of assessment,  $6\frac{1}{4}\%$ ?

102. Assessment on 123 shares of stock; rate of assessment,  $7\frac{1}{8}\%$ ?

103. Premium on \$1,800 par of bonds, if the rate of premium is 18%?

104. Discount on 73 shares of stock, if the rate of discount is  $8\frac{5}{8}\%$ ?

105. ILLUSTRATION. — Find the duty on an importation of 50 gallons of varnish invoiced at 97c. a gallon, the rate of duty being 20c a gallon specific and  $22\frac{1}{2}\%$  ad valorem.

(a) Specific Duty :

$$\begin{array}{r} 20c \\ 50 \\ \hline \$10.00 \end{array}$$

(b) Ad valorem Duty :

50 gals. at 97c = \$48.50,  
or for duty purposes  
this is called \$49.  
 $22\frac{1}{2}\%$  of \$49 = \$11.03.

(c) \$10 + \$11.03 = \$21.03 total  
duty.

SOLUTION.—The specific duty is simply 20c on each gallon, or \$10 on 50 gallons. The ad valorem duty is a percentage of the cost. The cost of 50 gallons at 97c is \$48.50. As the value for duty is always given to the nearest dollar, we add another dollar if the cents are 50 or more, and drop the cents if they are less than 50. \$48.50 thus becomes \$49.  $22\frac{1}{2}\%$  of \$49 is \$11.03.

The whole duty is \$10 + \$11.03, or \$21.03.

What is the specific duty upon an importation of

106. 7,835 sq. yd. of dress goods, rate of duty 25c per sq. yd. ?

107. 300 tons of medicinal roots, rate of duty 1c per pound ?

108. 1,200 tons guano, rate of duty 75c per ton ?

109. 30 doz. bottles wine, duty \$2.50 per doz. ?

110. 650 gallons brandy, rate of duty \$1.50 per gallon ?

111. 160 gross bottles, rate of duty 10c per doz. ?

112. What is the duty upon an importation of 1,500 yards flannel weighing 350 lbs. net, and valued at 60 cents per yard, the rate of duty thereon being 24 cents per pound and 35% ad valorem ?

113. I imported from the United States 7,240 bushels of corn and  $17\frac{1}{2}$  tons of hay, invoiced at \$9.50 per ton. What amount of duties had I to pay at 15c per bushel on the corn and 20% on the hay ?

114. What is the duty on an invoice of linens amounting to £3,256 sterling at 27% ?

115. What is the duty on 1,000 yards of Brussels carpet, 27 in. wide, invoiced at 6s. 9d. per yard ; duty 44c per square yard specific, and 35% ad valorem ?

116. An invoice of woollen cloth, imported from England, was valued at £956 6s. If its weight was 684 lbs., how much was the duty at 50c per pound specific, and 35% ad valorem?

117. ILLUSTRATION.—Find the interest on \$650 for 2 years at 4%.

\$650 Principal  
 .04  


---

 \$26.00 Int. for 1 yr.  
 2  


---

 \$52.00 Int. for 2 yrs.

SOLUTION.—Interest for 1 year is 4% of the principal,  $\$650 = \$650 \times .04 = \$26.00$ , and the interest for two years is twice the interest for 1 year, or  $\$26.00 \times 2 = \$52.00$ .

Find the interest on the following :

- |  |  |
|--|--|
| 118. \$75 for 2 yr. at 10%.                | 128. \$320 for 3 yr. at 6%.                |
| 119. \$80 for 3 yr. at $12\frac{1}{2}\%$ . | 129. \$620 for 2 yr. at 10%.               |
| 120. \$120 for 4 yr. at 5%.                | 130. \$800 for 2 yr. at $7\frac{1}{2}\%$ . |
| 121. \$240 for 2 yr. at $8\frac{1}{2}\%$ . | 131. \$120 for 4 yr. at 5%.                |
| 122. \$72 for 5 yr. at 6%.                 | 132. \$314 for 2 yr. at 10%.               |
| 123. \$84 for 3 yr. at 5%.                 | 133. \$215 for 3 yr. at 8%.                |
| 124. \$150 for 2 yr. at 6%.                | 134. \$32 for 1 yr. at 5%.                 |
| 125. \$240 for 3 yr. at 10%.               | 135. \$60 for 2 yr. at $6\frac{1}{2}\%$ .  |
| 126. \$320 for 5 yr. at 4%.                | 136. \$75 for 1 yr. at 6%.                 |
| 127. \$325 for 5 yr. at 4%.                | 137. \$1,200 for 2 yr. at 9%.              |

138. ILLUSTRATION.—Find the interest on \$850 for 62 days at 5%.

\$850  
 .05  


---

 \$42.50 Int. for 1 yr.  
 62  


---

 365)2635.00(\$7.21 +  
 or  
 \$7.22.

SOLUTION.—Sixty-two days is  $\frac{62}{365}$  of 1 year. The interest for 62 days is, therefore,  $\frac{62}{365}$  of the interest for 1 year, and this may be found by multiplying the interest for 1 year (\$42.50) by 62 and dividing the result by 365.

Compute the interest on the following :

139. \$840 for 63 days at 5%.  
 140. \$960 for 75 days at 6%.  
 141. \$320 for 96 days at 7%.  
 142. \$1,260 for 123 days at 8%.  
 143. \$2,480 for 85 days at 9%.



144. \$1,244 for 54 days at 10%.

145. \$2,360 for 59 days at 11%.

146. \$9,200 for 36 days at 12%.

147. ILLUSTRATION.—Find the interest on \$3,250 from April 16th, 1889, to June 18th, 1891, at 6% per annum.

(From April 16th, '89, to June 18th, '91, is 2 years and 63 days.)

SOLUTION 1

$$\begin{array}{r} \$3250 \\ .06 \\ \hline \$195.00 \\ 2\frac{63}{365} \\ \hline \$423.66 \end{array}$$

SOLUTION 2

$$\$3250 \times \frac{6}{100} \times \frac{63}{365} = \$ 33.66 \text{ Int. for 63 days.}$$

$$\$3250 \times \frac{6}{100} \times 2 = \$390.00 \text{ Int. for 2 years.}$$

$$\hline \$423.66 \text{ Int. for 2 years 63 days.}$$

Find the interest on :

148. \$800 from Jan. 15, 1890, to June 18, 1890, at 8%.

149. \$2,400 from Feb. 11, 1891, to June 3, 1892, at  $4\frac{1}{2}\%$ . At  $7\frac{1}{2}\%$ .

150. \$650 from June 17, 1892, to Jan. 21, 1894, at 7%. At 9%.

151. \$565.25 from Jan. 29, 1890, to Jan. 1, 1893, at 6%. At  $6\frac{1}{2}\%$ .

152. \$940 from June 15, 1891, to Jan. 15, 1894, at  $6\frac{1}{2}\%$ . At  $7\frac{1}{2}\%$ .

## QUESTIONS OF THE SECOND ASPECT

### SERIES 57

1. ILLUSTRATION.—A farmer, having 480 bushels of wheat, sold 120 bushels. What per cent. of his wheat did he sell ?

$$\begin{aligned} (1) \quad \frac{120}{480} &= \frac{1}{4} \\ \frac{1}{4} \text{ of } 100\% &= 25\% \end{aligned}$$

SOLUTION 1.—120 is  $\frac{1}{4}$  of 480, and since 480 is 100% of itself, 120, which is  $\frac{1}{4}$  of 480, must be  $\frac{1}{4}$  of 100%, or 25%. Or,

$$\begin{aligned} (2) \quad 120 \div 4.80 &= 25 \\ 25 \text{ times } 1\% &= 25\% \end{aligned}$$

SOLUTION 2.—Since 480 is 100% of itself, 1% of 480 would be  $\frac{1}{100}$  part of it, or 4.80. Since 4.80 is 1% of 480, 120 would be as many times 1% as 4.80 is contained times in 120, which is 25 times ; 25 times 1% is 25%.

2. I bought 152 yards of muslin and sold 19 yards of it. What per cent. of it did I sell ?

3. A dealer put up 8,000 tons of ice. During the summer 15% of it melted, and he sold 65% of the remainder. What per cent. of the whole amount did he still have ?

4. The base is 25.8 ; the percentage is 2.58. What is the per cent. ?

5. What per cent. does a man make on his money, who rents a house that cost \$7,500, for \$600 a year ?

6. A certain school enrolled 60 pupils. If the attendance is 54 pupils, what is the per cent. of attendance ?

7. In a battle in which 9,000 men were engaged, 2,000 men were killed. What per cent. of the number engaged was not killed ?

8. A man's income is \$1,500 a year, and his expenses for the same are \$1,095. What per cent. of his income does he save ?

9. The assets of a firm are \$20,280, and the liabilities are \$24,000. What per cent. of its debts can the firm pay ?

10. ILLUSTRATION.—A reaper that cost \$350 was sold at a profit of \$105. Find the rate per cent. of gain.

Cost = \$350

Gain = \$105

\$105 gained on \$350 =  $\frac{105}{350}$   
 $\frac{105}{350}$  of 100% = 30%

SOLUTION.—105 is  $\frac{105}{350}$  of 350, and as 350 is 100% of itself, 105, which is  $\frac{105}{350}$  of 350, must be  $\frac{105}{350}$  of 100 %, or 30%.

Find the rate per cent, of gain or loss when the cost is

11. \$300 and the loss is \$50. 16. \$450 and the gain is \$2.25.

12. \$500 and the gain is \$75. 17. \$900 and the loss is \$4.50.

13. \$400 and the loss is \$8. 18. \$800 and the gain is \$2.00.

14. \$750 and the gain is \$150. 19. \$1750 and the gain is \$750.

15. \$800 and the loss is \$600. 20. \$1000 and the loss is \$250.

21. What is the per cent. of profit on a piano which cost \$350 and is sold for \$420 ?

22. What is the rate per cent. of profit on merchandise which cost \$573 and is sold for \$647.49 ?

23. A stationer bought 9 reams of paper at \$2.40 per ream, and retailed it at 1 cent per sheet. What was his per cent. of gain ?

24. A grain dealer bought 735 bushels of wheat at \$1.14 per bushel, and sold the entire quantity for \$977.55. What was his per cent. of gain ?

25. A drover bought 15 horses at \$125 per head, sold 2 of them at \$127.75 per head, 8 at \$140 per head, and the remainder at \$150 per head. If his expenses in taking them to market amounted to \$5 per head, what was his per cent. of gain?

26. Oats purchased at  $45\frac{5}{8}$  cents per bushel were sold for  $54\frac{3}{4}$  cents per bushel. What was the rate per cent. of gain?

27. A grocer bought 536 gallons of vinegar for \$150.08, sold 175 gallons at 30 cents per gallon, 124 gallons at 35 cents per gallon, and the remainder for \$91.70. What was his rate per cent. of gain?

ILLUSTRATION.—An agent received \$50 commission for selling \$2,500 worth of goods. What is his rate of commission?

## SOLUTION

On \$2500, commission is \$50

On \$1, commission is \$50

---

2500

On \$100, commission is  $100 \times \frac{50}{2500}$

or \$2

The commission is 2%.

NOTE.—An agent is paid for what he does. If he buys \$500 worth of goods and ships them to his principal, his commission is on \$500. Other costs, such as freight, may bring the cost to the principal up to, say, \$600, but the agent gets his commission on \$500. Again, an agent may sell goods for \$500. His commission is on \$500. The principal may receive only \$400, after all expenses are deducted from the sale money, but the agent is paid on what he sold, \$500 worth of goods.

What is the rate per cent. of commission if

28. The sales are \$960 and the commission \$15.60?

29. The cost is \$3,264 and the commission for buying \$3.06?

30. The sales are \$3,200 and the commission for selling \$6?

31. The cost is \$460 and the commission for buying \$23?

32. The first cost of the purchase is \$275 and the gross cost \$286?

33. The total sales are \$105 and the net sales \$102.90?

34. The total sales are \$380 and the net sales \$379.05?

35. The prime cost is \$124 and the gross cost \$128.34?

36. My agent sold queensware for me, and, retaining his commission, \$90, remitted me \$1,910. What per cent. commission did I allow him ?

37. A commission merchant sold corn for \$6,000 ; he charged \$80 commission and \$135 for guaranteeing payment. What was his per cent. of commission and of guaranty ?

38. The total cost of a church was \$12,646.40, which includes the architect's commission, \$296.40. Find the per cent. of commission.

39. I sold real estate through an agent for \$12,750 ; I received \$765 commission and paid him \$573.75. What was the per cent. of commission of each ?

40. My commission for selling furniture was \$591.50 ; the net proceeds, \$16,308.50. What per cent. did I charge for my services ?

41. I bought sugar for a grocer : the whole cost, including expenses, \$69.45, and commission, \$170.50, was \$6,439.95. What per cent. commission did I get ?

42. ILLUSTRATION.—I paid \$30 for insuring a house, worth \$6,400, at  $\frac{3}{4}$  valuation. What was the rate ?

SOLUTION

$\frac{3}{4}$  of \$6400 = \$4800, the face of the policy.

\$30  $\div$  4800 = \$.00625, or  $\frac{5}{8}\%$  = the rate of insurance.

43. The premium for insuring a house, worth \$12,000, at its full value, was \$80. What was the per cent. ?

44. If \$125 are paid annually for insuring \$24,000, what is the rate per cent. ?

45. A fire insurance company charged \$129.60 for insuring a house for \$13,500. What was the rate of insurance per \$100 ?

46. I paid \$175 premium on a schooner worth \$25,000, which was insured at  $\frac{4}{5}$  of its value. What was the rate per cent. of insurance ? What was the rate of insurance in cents on the \$100 ?

47. A marine insurance company received \$484.50 for insuring a vessel worth \$80,000 at  $\frac{2}{5}$  of its value. What was the rate per cent. of insurance if \$4.50 was charged for the policy and survey ?

48. ILLUSTRATION.—A tax of \$3,750 is to be levied upon a valuation of \$1,250,000. Find the rate of taxation.

SOLUTION

$$\$3,750 \div \$1,250,000 = .003.$$

Therefore, the rate of taxation is \$.003 on \$1, or 3 mills on the dollar.

49. A tax of \$7,380 was levied upon the taxable property of a county, valued at \$2,460,000. What was the rate, and what was the tax on a farm assessed at \$4,000 ?

Find the rate of taxation in mills on \$1 if the assessed valuation is

50. \$514,000, and the gross tax \$6,233, including 130 polls at 50c.

51. \$15,000, and the gross tax \$137.25, including 3 polls at 75c.

52. \$15,387,200, and the total property tax \$169,259.20.

Find the rate of taxation in cents on \$100 if the assessed valuation is

53. \$34,000, and the total property tax is \$469.20.

54. \$1,275,000, and the total property tax is \$12,495.

55. The assessed valuation of the property of a village is \$832,000. The estimate of corporate expenses includes \$1,500 for schools, \$1,600 for streets, \$1,200 for salaries and commissions, and \$692 for sundry other expenses. What will be the rate of taxation expressed as mills on the dollar ?

56. ILLUSTRATION.—What is the rate per cent. dividend when 40 shares of stock yield an income of \$240 ?

SOLUTION 1

40 shares = \$4.000 par

(1 share = \$100 par)

On \$4,000 the dividend is \$240

On \$1 the dividend is  $\frac{\$240}{4000}$

On \$100 the dividend is  $\frac{100 \times 240}{4000}$

= \$6

The rate is 6%

SOLUTION 2.

40 shares yield \$240

1 share yields  $\frac{240}{40}$

= \$6

The rate is 6%



What is the rate per cent. of dividend when

SHARES.		INCOME.	STOCK.		INCOME.
57.	50	yield \$275	62.	\$4,500	yields \$135
58.	60	" \$300	63.	\$7,550	" \$453
59.	90	" \$390	64.	\$8,600	" \$301
60.	75	" \$450	65.	\$3,275	" \$131
61.	34	" \$170	66.	\$4,125	" \$330

If the par value of stock is

67. \$6,800, the assessment \$204, what is the per cent. of assessment ?

68. \$5,000, the dividend \$318.75, what is the per cent. of dividend ?

69. \$8,300, the premium \$1,182.75, what is the per cent. of premium ?

70. \$5,400, discount \$681.75, what is the per cent. of discount ?

71. ILLUSTRATION.—At what rate will \$4,380, in 76 days, produce \$45.60 interest ?

SOLUTION.—Let 1% equal the rate. Then interest on \$4,380 for 76 days at 1% = \$9.12.  $\$45.60 \div \$9.12 = 5$ . The interest at 1% is contained in the given interest 5 times. Therefore, the required rate is 5 times 1%, or 5%.

Find the rate of interest :

	Principal.	Interest.	Time.		Principal.	Interest.	Time.
72.	\$600	\$72	2 yr.	77.	\$100	\$24	8 yr.
73.	\$500	\$60	3 yr.	78.	\$584	\$9.28	174 da.
74.	\$300	\$60	5 yr.	79.	\$511	\$5.95	85 da.
75.	\$200	\$24	4 yr.	80.	\$1460	\$10.80	45 da.
76.	\$400	\$16	6 mo.				

## QUESTIONS OF THE THIRD ASPECT

### SERIES 58

I. ILLUSTRATION.—375 is  $12\frac{1}{2}\%$  of what number ?

(a)  $12\frac{1}{2}\%$  of the number = 375

1% of the number = 30

100% of the number = 3000

(b)  $12\frac{1}{2}\%$ , or  $\frac{1}{8}$ , of a number = 375

$\frac{8}{8} = 8$  times 375, or 3000

SOLUTION (a).—If  $12\frac{1}{2}\%$  of a number is 375, 1% of the number is  $\frac{375}{25}$  of 375, or 30, and 100%, or the whole of the number is 100 times 30, or 3000. Or,

SOLUTION (b).— $12\frac{1}{2}\%$  of a number is  $\frac{1}{8}$  of it. If  $\frac{1}{8}$  of a number is 375,  $\frac{8}{8}$ , or the whole number, is 8 times 375, or 3000. Therefore, the required result is 3000.

2. A house rents for \$360 per year, which amount represents 8% of its value. How much is the house worth?

3. A grocer sold 55 bushels of potatoes in one month, which amount represented  $5\frac{1}{2}\%$  of his stock of potatoes. How many bushels of potatoes had he at first?

4. From a cask of oil there leaked out 5.04 gallons, which was 8% of what it originally contained. How many gallons were there in the first place, and how many gallons remained in the cask?

5. A farmer sold 17 bu. 2 pk. and 1 qt. of tomatoes, which were 3% of his whole stock of tomatoes before selling. How many bushels of tomatoes had he at first?

6. From his bank account a man checked out \$75.50, which amount was 30% of his account. How much had he in the bank at first?

7. ILLUSTRATION.—By selling a farm at a gain of 20%, I realized a profit of \$850. Find the cost of farm.

SOLUTION

$$20\% \text{ of cost} = \$850$$

$$1\% \text{ of cost} = \$850$$

---


$$20$$

$$100\% \text{ of cost} = \frac{100}{20} \text{ of } \$850 = \$4,250$$

8. A bill of goods was sold for \$13 less than cost, by which a loss of 5% was sustained. What was the cost of the goods?

9. A gentleman gained 24% by selling land for \$195.60 more than he paid for it. What did he obtain for the land?

10. What must have been the cost of a stock of goods if owner, by selling at a gain of  $12\frac{1}{2}\%$ , received \$450 more than the cost?

11. By selling a lot of delaine for  $37\frac{1}{2}\%$  more than I paid for it, I gained \$135. What did it cost me?

12. A merchant lost 5% by selling gingham at 9½c a yard. Find the cost per yard.

Find cost:

	GAIN.	GAIN %.		LOSS.	LOSS %.
13.	\$3.00	10%	17.	\$2.50	30%
14.	60c.	$12\frac{1}{2}\%$	18.	\$4.80	25%
15.	$37\frac{1}{2}$ c.	$16\frac{2}{3}\%$	19.	\$1.20	$8\frac{1}{3}\%$
16.	\$5.60	40%	20.	\$3.00	$6\frac{1}{4}\%$

21. ILLUSTRATION.—An agent received \$170 for buying a house. If his commission was 5%, what was the value of the house ?

SOLUTION 1

$$5\% \text{ of value} = \$170$$

$$1\% \text{ of value} = \$170$$

---


$$5$$

$$100\% \text{ of value} = \frac{100 \times 170}{5} = \$3,400$$

Or

$$\begin{array}{r} .05) \$170.00 \\ \$ 3,400 \end{array}$$

SOLUTION 2.—The commission is \$170. The rate .05. Then,  $\$170 \div .05 = \$3,400$ , value of house.

22. I paid a grain dealer  $1\frac{1}{2}\%$  for buying corn for me at 62c per bushel. If his commission amounted to \$83.70, how many bushels did he buy ?

23. An agent charged \$433.60 for selling a consignment of canned fruit. If his rate of commission was  $2\frac{1}{2}\%$ , what was the net proceeds ?

24. What must an agent's sales for one year aggregate in order that at 3% commission his yearly income may be \$2,700 ?

25. A Mobile factor earned \$99.75 by selling cotton at  $2\frac{3}{8}\%$  commission. How many bales, averaging 560 lbs., did he sell, the price being 15c per pound ?

26. An agent charged \$13.40 for selling a consignment of merchandise. What were the gross proceeds of the sale if his rate of commission for selling was  $2\frac{1}{2}\%$  ?

27. I paid  $\frac{3}{4}\%$  brokerage to a grain broker for effecting the sale of a quantity of wheat at \$1.15 per bushel. How many bushels of wheat were sold, if the total brokerage was \$63.48 ?

28. ILLUSTRATION.—For what sum is a house insured if the premium paid is \$75, and the rate of insurance  $\frac{5}{8}\%$  ?

SOLUTION

$$\frac{5}{8}\% \text{ of policy} = \$75$$

$$1\% \text{ of policy} = \$75 \times \frac{8}{5}$$

$$100\% \text{ of policy} = 100 \times 75 \times \frac{8}{5} = \$12,000.$$

Or

$$\text{Premium } (\$75) \div \text{rate } (\frac{5}{8}\%, \text{ or } .625\%) = \$12,000.$$

29. A house is insured at  $\frac{3}{5}\%$ , and the premium is \$93.60. For how much is it insured ?

30. The cargo of a steamer is insured at  $\frac{1}{2}\%$ . For what sum is it insured, the premium being \$1,500 ?

31. Paid \$350 on a shipment of goods to insure  $\frac{3}{4}$  the value, at  $3\frac{5}{8}\%$ . What was the whole value ?

32. A manufacturing company paid \$214.80 premium for insurance on  $\frac{3}{4}$  of the cost of its building and machinery, at 60c per \$100. What was their cost ?

33. ILLUSTRATION.—The tax on a certain property was \$96.10, and the rate of taxation  $7\frac{3}{4}$  mills on the dollar. For how much was the property assessed ?

## SOLUTION

$$\begin{array}{rcl} \$0.00775 \text{ is the tax on } & \$1 & \\ \$1 \text{ is the tax on } & \frac{\$1}{.00775} & \\ \$96.10 \text{ is the tax on } & \frac{96.10}{.00775} = & \$12,400 \end{array}$$

34. What is the assessed value of a property that pays a tax of \$182, at the rate of  $3\frac{1}{4}$  mills on the dollar ?

35. A town hall, costing \$12,250, was built by a tax assessed upon the property of the town. The tax rate was 5 mills on the dollar, and the cost of collection  $2\%$ . What was the valuation ?

36. ILLUSTRATION.—What amount of stock must be held to obtain \$200 income from a  $4\%$  dividend ?

## SOLUTION

\$4 income is derived from 1 share.

$\therefore$  \$200 income is derived from  $200 \div 4 = 50$  shares.

50 shares =  $50 \times 100 = \$5,000$  stock.

What is the par value of stock if the

37. Dividend is \$170.50, and the rate of dividend  $5\frac{1}{2}\%$  ?

38. Assessment is \$1,031.25, and the rate of assessment  $8\frac{1}{4}\%$  ?

39. Premium is \$3,293.75, and the rate of premium  $19\frac{3}{8}\%$  ?

40. Discount is \$1,550.25, and the rate of discount  $13\frac{1}{4}\%$  ?

41. ILLUSTRATION.—What sum must be invested in 5% bonds, at 105, to yield an annual income of \$1,250 ?

SOLUTION

The dividend from 1 share = \$5.

$\$1,250 \div \$5 = 250 =$  number of shares.

The cost of 1 share = \$105.

The cost of 250 shares =  $\$105 \times 250$ , or \$26,250.

42. What sum must be invested at 93 to produce an income of \$1,600, the rate of dividend being 8% ?

43. How much must be invested in 8% stock, at 162, to secure an annual income of \$1,280 ?

44. If 10% mining stock is at  $87\frac{1}{2}$ , what sum must be invested in it to realize an annual income of \$1,500 ?

45. If Ontario Express Co. stock pays 4% semi-annual dividends, and is quoted at  $150\frac{1}{4}$ , how much money must be invested in it to produce an annual income of \$3,200 ?

46. ILLUSTRATION.—What principal will yield \$400 interest in 2 years at 8% ?

SOLUTION

Interest on \$100 for 2 yrs. at 8% = \$16

\$16 is the interest on  $\frac{\$100}{16}$

\$1 is the interest on  $\frac{\$100}{16}$

\$400 is the interest on  $\frac{400 \times 100}{16}$

= \$2,500

Find the principal :

	RATE.	TIME.	INTEREST.		RATE.	TIME.	INTEREST.
47.	$3\frac{1}{2}\%$	1 yr.	\$45 $\frac{1}{2}$	53.	5%	7 yrs.	\$29.75
48.	$5\frac{1}{2}\%$	1 yr.	\$41 $\frac{1}{4}$	54.	$3\frac{1}{3}\%$	$4\frac{1}{2}$ yrs.	\$94.50
49.	$4\frac{1}{2}\%$	$\frac{1}{2}$ yr.	\$25 $\frac{1}{2}$	55.	4%	$1\frac{3}{4}$ yrs.	\$68.25
50.	$3\frac{3}{4}\%$	$\frac{1}{2}$ yr.	\$3 $\frac{3}{8}$	56.	$4\frac{1}{2}\%$	$1\frac{1}{4}$ yrs.	\$47.25
51.	8%	$\frac{3}{4}$ yr.	\$18	57.	6%	$5\frac{2}{3}$ yrs.	\$170.00
52.	$2\frac{1}{2}\%$	6 yrs.	\$52 $\frac{1}{2}$	58.	$3\frac{1}{8}\%$	$4\frac{1}{4}$ yrs.	\$136.00



## QUESTIONS OF THE FOURTH ASPECT

## SERIES 59

1. ILLUSTRATION.—What number, increased by 87% of itself, is equal to 1122 ?

## SOLUTION

Represent the number by 100%.

87% = the increase.

187% = the number, increased by 87% of itself.

1122 = the number, increased by 87% of itself.

Therefore, 187% of the number = 1122.

1% =  $\frac{1}{187}$  of 1122, or 6.

100%, or the number, = 100 times 6, or 600.

Hence, the required result is 600.

2. ILLUSTRATION.—What number, decreased by 35% of itself, equals 2600 ?

## SOLUTION

Represent the number by 100%.

35% = the decrease.

65% = the number after decrease.

2600 = the number after decrease.

Therefore, 65% of the number = 2600.

1% =  $\frac{1}{65}$  of 2600, or 40.

100% = 100 times 40, or 4000, the required result.

3. 4186 is 15% more than what number ?

4. When gold is worth 12% more than currency, what is the gold value of \$725.76 in currency ?

5. A foreman, whose salary was increased 11%, receives \$1,082.25. How much did he receive before the increase ?

6. An agent sold a house and lot for \$11,002.50, which was  $12\frac{1}{2}\%$  more than it cost him. What did it cost him ?

7. During the month of December a merchant sold goods to the value of \$12,620.02, which was 9% more than his sales in November. What did his sales amount to in November ?

8. Sold a horse for \$170, which is  $6\frac{1}{4}\%$  more than it cost me. How much did it cost me ?

9. The number of students attending a certain school in 1895 was 357, which was 40% more than in 1894. What was the attendance in 1894 ?

10. What number increased by  $16\frac{2}{3}\%$  of itself equals 6825 ?

11. If  $11\frac{1}{9}\%$  of a number be added to itself, the sum will equal 2000. What is the number ?

12. A shepherd, after losing 35% of his flock, had 325 sheep remaining. How many had he at first ?

13. A dealer in dry goods spent for calico \$1,726.80, which was 20% less than the amount spent for muslin. How much did he spend for muslin ?

14. A certain man owns two farms. The first farm contains 290 acres, which is 42% smaller than the second. How many acres does the second farm contain ?

15. In the construction of a business block, 40% of the entire cost was paid for the brickwork and stonework, 20% for the carpenter work, 15% for glazing, 10% for the elevator, and the remainder, \$510, for the painting. What was the whole cost ?

16. What was the cost of a horse that sold for \$136 at a loss of  $11\frac{1}{9}\%$  ?

17. A collector, whose fee was 4%, returned to his principal \$540.96. What sum did he collect ?

18. The pressure on the surface of a steam boiler is 81 lbs. since decreasing it 10%. What was the pressure before decreasing it ?

19. ILLUSTRATION.—What must be asked for goods that a discount of 20% and 10% may be allowed, and net \$2.88 to the seller ?

SOLUTION

Let 100% = asking price.

20% and 10% = allowance.

72% = asking price after allowance.

72% of asking price = \$ 2.88

1% of asking price =  $\frac{2.88}{72}$

100% of asking price =  $\frac{100 \times 2.88}{72}$   
= \$4.00

At what price must goods be marked

20. To net \$2.72, after allowing a discount of 15% ?

21. To net \$12, after allowing a discount of 10% and 10% ?

22. To net \$3.60, after allowing a discount of 25%, 20%, and 10% ?

23. To net \$30, after allowing a discount of 10%, 10%, and 10% ?

24. To net \$14.58, after allowing a discount of 40%, 10%, and 10% ?

25. ILLUSTRATION.—A horse and carriage which sell for \$240 net a gain of 20%. Find the cost.

SOLUTION.—  $100\% = \text{cost.}$

$20\% = \text{gain.}$

---

$120\% = \text{sale} = \$240.$

$1\% = \$240 \div 120 = \$2.$

$100\% = 100 \times \$2 = \$200.$

26. A set of jewelry was sold for \$126.30. What was the cost if the seller gained 25% ?

27. A music dealer sold a piano for \$413 and gained 18%. What was his profit ?

28. A grocer sold a quantity of sugar for \$330, and thereby lost 12%. How much did he lose ?

29. A man sold two lots for \$1,800 each, gaining 25% on one lot and losing 25% on the other. Did he gain or lose, and how much ?

30. A merchant sold a bill of goods for \$89.30, and thereby lost 6%. How much did he lose ?

31. A house was sold for \$5,616 at a gain of 8%. What was the gain ?

32. A horse was sold for \$753.75, which was  $\frac{1}{2}$  of 1% more than it cost. Had it been sold for \$700, what per cent. would have been gained or lost ?

33. Sold goods for \$4,026.75, at a loss of  $3\frac{1}{4}\%$ . What would they have had to sell for to yield a profit of  $3\frac{1}{4}\%$  ?

34. ILLUSTRATION.—I sent my agent \$4,100, with instructions to deduct his commission of  $2\frac{1}{2}\%$  and invest the balance

in wheat. How much did he invest, and what was his commission ?

SOLUTION.

Represent the actual investment by 100%.

$2\frac{1}{2}\%$  = the charges for buying.

$100\% + 2\frac{1}{2}\% = 102\frac{1}{2}\%$ , the cost of the investment to the principal.

\$4,100 = the cost of the investment to the principal.

Therefore,  $102\frac{1}{2}\%$  of the cost = \$4,100.

1% = \$40.

100% = \$4,000, the actual investment in wheat.

\$4,100 - \$4,000 = \$100, the commission for buying.

35. An agent received \$6,180 to buy cotton. After deducting his commission at 3%, how much did he invest in cotton ?

36. How many yards of muslin at 5 cents a yard can my agent buy with \$609 after deducting his commission at  $1\frac{1}{2}\%$  ?

37. I bought coffee at  $1\frac{7}{8}\%$  commission and charged  $2\frac{1}{4}\%$  for guaranteeing payment. What did I pay for the coffee if the whole cost was \$1,832.60 ?

38. Having sent a Toronto agent \$1,835.46 to be invested in sugar, after allowing 3% on the investment for his commission, I received 32,400 pounds of sugar. What price per pound did the sugar cost the agent ?

39. An agent in Hamilton received \$828 to invest in prints, after deducting his commission of  $3\frac{1}{2}\%$ . If he paid  $7\frac{1}{2}c$  per yard for the prints, how many yards did he buy ?

40. A Stratford fruit dealer sent a Grimsby agent \$1,946.70, and instructed him to buy apples at \$1.40 per barrel. The agent charged 3% for buying, and shipped the purchase to his principal in six car loads of an equal number of barrels. How many barrels did each car contain ?

41. ILLUSTRATION.—What is the amount of sales when the net proceeds are \$975 and the commission  $2\frac{1}{2}\%$  ?

SOLUTION.—Let 100 % = sales.

$2\frac{1}{2}\%$  = commission.

Then  $97\frac{1}{2}\%$  = net proceeds.

$97\frac{1}{2}\%$  of sales = \$975.

1 % of sales = \$10.

100 % of sales = \$1,000.

42. The net proceeds of a consignment are \$675.50, and the rate of commission  $3\frac{1}{2}\%$ . What is the amount of the sale?

43. An agent collected part of an account for me, and, after deducting his commission of 5%, he sent me \$427.50. What per cent. of the original debt of \$900 remains unpaid?

44. A farmer received from his city agent \$496.80 as the net proceeds of a shipment of butter. If the agent's commission is 3%, and 5% charge is made for guaranty of quality to purchasers, how many pounds at 27c per pound must have been sold, and how much commission was allowed?

45. Sold clothing at  $1\frac{3}{4}\%$  commission and  $2\frac{1}{2}\%$  for guaranteeing payment. What did the clothing sell for if the net proceeds due the principal is \$2,711.64?

46. What was received for a farm the net proceeds of which amounted to \$6,993.82, after paying  $1\frac{3}{5}\%$  commission and \$263.18 other charges?

47. ILLUSTRATION.—How many shares Bank of Commerce stock selling at 21% premium can be bought for \$7,275, brokerage  $\frac{1}{4}\%$ ?

SOLUTION.—1 share at par	= \$100
Premium	= 21
	———
Market value	= 121
Brokerage	= $\frac{1}{4}$
	———
Cost to purchaser	= $121\frac{1}{4}$
$\$7,275 \div \$121\frac{1}{4} = 60$	

There are 60 shares, or \$6,000 par of stock.

How many shares may be bought for

	COST.	MAR. VAL.	BROK.		COST.	MAR. VAL.	BROK.
48.	\$13,155	225	$\frac{1}{4}\%$	52.	\$1,923	80	$\frac{1}{8}\%$
49.	\$9,760	$121\frac{3}{4}$	$\frac{1}{4}\%$	53.	\$3,850	96	$\frac{1}{4}\%$
50.	\$5,610	140	$\frac{1}{4}\%$	54.	\$12,025	240	$\frac{1}{2}\%$
51.	\$13,620	85	$\frac{1}{8}\%$	55.	\$4,134	86	$\frac{1}{8}\%$



56. ILLUSTRATION.—How many shares Bank of Commerce stock at 121 must I sell to realize \$7,245, brokerage  $\frac{1}{4}\%$ ?

SOLUTION.—Market value = \$121  
 Brokerage =  $\frac{1}{4}$   
 Amount received from one share = \$120 $\frac{3}{4}$   
 $\$7,245 \div \$120\frac{3}{4} = 60$   
 60 shares are sold.

How many shares must be sold to realize

	S. P.	MAR. VAL.	BROK.		S. P.	MAR. VAL.	BROK.
57.	\$8,505	121 $\frac{3}{4}$	$\frac{1}{4}\%$	61.	\$19,755	220	$\frac{1}{2}\%$
58.	\$10,245	85 $\frac{1}{2}$	$\frac{1}{8}\%$	62.	\$2,400	96 $\frac{1}{4}$	$\frac{1}{4}\%$
59.	\$4,314	90	$\frac{1}{8}\%$	63.	\$8,336	130 $\frac{1}{2}$	$\frac{1}{4}\%$
60.	\$4,350	87 $\frac{1}{8}$	$\frac{1}{8}\%$	64.	\$10,548	110	$\frac{1}{8}\%$

65. ILLUSTRATION.—What principal will amount to \$508 in 4 $\frac{1}{2}$  years at 6%?

SOLUTION

Let \$1 represent the principal.

\$1.27 = the amount of a dollar for 4 $\frac{1}{2}$  yrs.

\$508 = the amount of a certain principal for 4 $\frac{1}{2}$  yrs.

$\$508 \div \$1.27 = 400$ .

Since the given amount is 400 times the assumed amount, the required principal must be 400 times the assumed principal.

400 times \$1 = \$400, the required principal.

What sum must be put out at interest for

- |     |                               |              |             |
|-----|-------------------------------|--------------|-------------|
| 66. | 2 years at 4%                 | to amount to | \$540.00.   |
| 67. | 4 years at 6%                 | to amount to | \$2,480.00. |
| 68. | 6 years at 2 $\frac{1}{2}\%$  | to amount to | \$2 760.00. |
| 69. | 3 years at 3%                 | to amount to | \$87.20.    |
| 70. | 10 years at 7%                | to amount to | \$342.00.   |
| 71. | 8 years at 5%                 | to amount to | \$616.00.   |
| 72. | 102 days at 5%                | to amount to | \$4,441.20. |
| 73. | 318 days at 3%                | to amount to | \$2,246.70. |
| 74. | 75 days at 6%                 | to amount to | \$2,586.50. |
| 75. | 150 days at 2 $\frac{1}{2}\%$ | to amount to | \$3,318.75. |
| 76. | 200 days at 6 $\frac{1}{4}\%$ | to amount to | \$755.00.   |
| 77. | 85 days at 5%                 | to amount to | \$516.95.   |
| 78. | 174 days at 3 $\frac{1}{3}\%$ | to amount to | \$593.28.   |
| 79. | 312 days at 6 $\frac{2}{3}\%$ | to amount to | \$462.96.   |

## REVIEW OF PERCENTAGE

## SERIES 60

1. A grocer mixes 30 lbs. of tea worth 45 cents a pound with 20 lbs. worth 35 cents a pound. At what price per pound must he sell the mixture so as to gain 20% on his outlay ?

2. A dealer sold an article for \$6.75 and lost 10%. At what selling price would he have gained 10% ?

3. How much better is a single discount of 60% than a discount series of 25%, 20%, and 15% ?

4. How many mills on the dollar must be levied to raise a tax of \$5,632.50, the taxable value of the property being \$1,346,250, if 165 polls are taxed at \$1.50 each ?

5. A man invests \$6,000 in 5% stock at 120 ; at the end of one year, having just received the yearly dividend, he sells out at 121½. How much better off is he than if he had loaned his money at 5% per annum ?

6. A premium of \$50 is paid to insure a house for \$2,500 for 3 years. What is the yearly rate ?

7. A retail dealer sold a suit of clothes for \$29.40, making a profit of 20 per cent. If the cloth and trimmings cost six times as much as the making, find how much the tailor who made the suit received.

8. A sold a town lot to B and gained 12½%. B sold it to C for \$306 and lost 15%. What did the lot cost A ?

9. A has \$360, B has \$540. What per cent. is A's money of B's ? What per cent. is B's money of A's ?

10. What will be the cost in Halifax of a draft on Montreal for \$900, if the rate of exchange is ½% ?

11. A machinist sold two seed-drills for equal sums of money. He gained 25% on the one and lost 25% on the other. His total loss was \$9.60. Find the cost of each drill.

12. Sold stock at a discount of 12½% and made 16⅔% on my money. At what rate of discount did I buy ?

13. Assessed valuation of real property \$3,500,000, personal property \$500,700, and there being 250 polls assessed \$1.25 each, collection charges 5%, tax levy \$95,313.50, find rate of tax.

14. An agent received a consignment of wheat which he sold on a commission of  $2\frac{1}{2}\%$ ; the amount sent to his employer was equivalent to  $81\frac{1}{4}$  cents a bushel. What was his commission?

15. A grocer gained  $20\%$  by selling 20 lbs. sugar for \$1. Afterward he increased his price, giving only 18 lbs. for \$1. How much per cent. did he make at the increased price?

16. What per cent. is made by investing in  $4\frac{1}{2}\%$  stock at 75?

17. A merchant bought goods at  $25\%$  and  $20\%$  off, and sold at  $20\%$  and  $10\%$  off. What per cent. profit did he make?

18. Find the per cent. of commission on a purchase, if the gross cost is \$2,048.51, the commission \$87.30, the cartage \$20, and other charges \$1.21.

19. A merchant failed with liabilities amounting to \$50,000; his assets in merchandise are \$20,000; in real estate, \$8,000; in accounts receivable, \$12,000. If the assignee received  $10\%$  of the assets for making the settlement, how many cents on the dollar will the creditors receive?

20. A merchant has a stock of goods valued at \$1,200, and a store building valued at \$9,000. At  $\frac{3}{4}\%$ , what amount of premium does he pay on a policy if the insurance is on a  $\frac{3}{4}$  valuation?

21. What is the price of a  $7\%$  stock paying  $5\%$  on the money invested, brokerage  $\frac{1}{8}\%$ ?

22. If my goods had cost  $20\%$  more my rate of gain would have been  $25\%$  less. Find my gain per cent.

23. A fruit dealer bought 50 boxes of oranges, receiving a discount of 20, 10, and  $5\%$ . What was the price of the oranges per box, his discounts amounting to \$49.77?

24. A man's salary is \$1,700, and his net income is \$1,675.95 after paying income tax on all over \$400. What is the rate?

25. At what price must I mark cloth which cost me \$2.40 per yard, so that after throwing off  $20\%$  I may still make a gain of  $20\%$ ?

26. If  $50\%$  of A's money equals  $40\%$  of B's, and  $90\%$  of B's equals  $75\%$  of C's, and  $120\%$  of C's equals  $50\%$  of D's, how much has each, if  $25\%$  of D's is equal to \$360?

27. A commission agent retained \$22 out of \$1,782 sent him for investment. What rate of commission did he charge?

28. Having received a stock dividend of 5%, I find I am now the owner of 189 shares. How many shares did I own at first ?

29. At 50% ad valorem, the duty on 1,580 yards of silk was \$1,975. What was the invoice price per yard, and for what price per yard must the importer sell it to gain 20% ?

30. A merchant buys \$10.50 worth of sugar. He uses a pound weight  $\frac{1}{4}$  ounce too light and marks his goods at a profit of 15%. Find his gain.

31. James Todd made an assignment. He owed A \$4,500, B \$15,000, C \$3,500, and D \$12,000. His assets amounted to \$26,360. If B is a preferred creditor and the expenses for settling are \$1,360, what per cent. of their claim will A, C, and D receive ?

32. In building a house, the cost of the material was to the cost of the labor as 3 is to 2. Had 5% more for material and 10% less for labor been paid, the house would have cost \$44,550. What was the cost ?

33. A farmer received from his city agent \$490 as the net proceeds of a shipment of butter. If the agent's commission is 3%, delivery charges \$6.80, and 5% charge is made for guaranty of quality to purchasers, how many pounds, at 27c per pound, must have been sold, and how much commission was allowed ?

34. A man bought 400 sheep at a certain price per head. He sold  $\frac{3}{8}$  of them at a gain of 20%,  $\frac{3}{10}$  of them at a gain of 15%, and the remainder at a loss of 10%, gaining on the whole \$217. How much did he pay for the sheep per head ?

35. A broker invests \$6,450.75 in stock at  $68\frac{1}{2}$  on  $\frac{1}{8}$  % commission. What are his charges ?

36. A city hall costing \$36,750 was built by a special tax upon the property of the city. The rate of taxation being 5 mills on the dollar and the cost of collection 2%, find the total assessment.

37. A cigar dealer imported 4,000 lbs., net, of cigars, invoiced at \$1.25 per lb. How much duty did he pay at \$3 per lb. and 25% ad valorem ?

38. Bought oranges at the rate of 10 cents per dozen, and sold them at the rate of 5 oranges for 11 cents. Find my gain per cent.



39. The net profits of a business for two years were \$6,400, the second year's profit being  $66\frac{2}{3}\%$  greater than the profits of the first year. What were the profits each year?

40. Find the cost of a draft on Winnipeg for \$1,397.60 bought when exchange was  $\frac{3}{8}\%$ .

41. A grain dealer sold a quantity of rye and wheat for \$1,320, gaining  $33\frac{1}{3}\%$  on the rye and  $12\frac{1}{2}\%$  on the wheat. He received 20% more for the wheat than for the rye. What was his total gain?

42. A company took a risk at  $1\frac{1}{2}\%$ , reinsured 40% of it at  $1\frac{3}{4}\%$ , and 40% of the remainder at  $1\frac{7}{8}\%$ . What rate did the company receive on the amount of risk it carried?

43. Find the alteration in income occasioned by shifting \$4,500 stock from the 5 per cents. at  $115\frac{1}{8}$  to the 3 per cents. at  $91\frac{7}{8}$ , the brokerage being  $\frac{1}{8}\%$  on each transaction.

44. A person buys a lot of land at \$37.50 an acre, and by selling it in allotments finds the value increased three-fold, so that he clears \$375 and retains 30 acres for himself. How many acres did he buy?

45. After I have spent 50% of my money and 25% of the remainder, I find that I have left \$3,375. How much money had I at first?

46. J. M. Brown became insolvent; the schedule showed that he owed A \$4,000, B \$3,500, C \$6,000, and D \$4,500; the assets in real estate amounted to \$5,000, in accounts receivable \$7,650, and in bills receivable \$2,000. If the expenses of bankruptcy were \$650, how much did D receive in settlement of his claim?

47. A business man reckons his percentages on his turnover. If the turnover is \$36,729.28, wages \$15,387.20, salaries \$8,756.80, gross profit \$26,239.50, and net profit \$13,729.20, calculate his percentage of wages, salaries, gross profit, and net profit,

48. A gentleman owns  $\frac{2}{5}$  of a steamboat, which is insured for  $\frac{2}{3}$  of its value at  $1\frac{1}{2}\%$ . If he pays a premium of \$480, what is the value of the steamer?

49. A merchant imported 400 pieces of linoleum of 40 yds. each, 30 in. wide, valued at 25c a sq. yd. What duty did he pay at 35% ad valorem?



50. My broker bought R. and O. stock for me at  $124\frac{3}{4}$ , and immediately sold same at  $143\frac{1}{4}$ , remitting as net profit \$1,341. How many shares did he buy, a brokerage of  $\frac{1}{8}\%$  being allowed on buying and the same on selling ?

51. A merchant, in buying certain goods, uses a pound weight  $\frac{1}{4}$  oz. too heavy, and, in selling them, a pound weight  $\frac{1}{4}$  oz. too light. Find his gain per cent. from his dishonesty.

52. Divide \$700 into two parts, such that the simple interest on one part for 3 years at  $5\%$  per annum may be equal to the simple interest on the other part for 6 years at  $3\frac{1}{2}\%$  per annum.

53. What is my per cent. gain if  $20\%$  of what I receive for an article is gain ?

54. I bought a bicycle for \$96. I wish to make  $25\%$  profit and yet offer  $33\frac{1}{3}\%$  and  $10\%$  off. How much must I ask for it ?

55. Which is the better investment,  $5\%$  stock at 109 or  $6\%$  stock at 130 ?

56. An insurance company charged \$37.50 for insuring a house worth \$6,000 for 2 years. If their rate was  $\frac{1}{2}\%$ , what fraction of the value of the house was insured ?

57. An agent sold a consignment of flour for \$4,800, and invested the proceeds, less both commissions, in tea on a commission of  $4\%$ . If his total commission was £300, find the rate for selling.

58. A merchant engages a lawyer to collect his accounts, agreeing to pay him  $2\frac{1}{4}\%$  of the sum collected. If  $\frac{2}{9}$  of the accounts prove worthless, and the lawyer receives \$75.60 for collecting the balance, find the total amount of the merchant's accounts.

59. I have \$1,500 of my employer's money. Find the amount of the largest draft I can remit him if exchange is at  $\frac{1}{4}\%$ .

60. A merchant's expenses are as follows : Rent \$600, clerk hire \$1,500, sundry items \$400, advertising \$250. What must be the amount of his annual sales in order that they may average  $16\frac{2}{3}\%$  above the cost, and allow him a gain of \$2,000 above expenses ?

61. A merchant sent his agent \$3,075, with instructions to deduct his commission of  $2\frac{1}{2}\%$ , and invest the remainder in flour

at \$6 per barrel. If the cost of freight and insurance amounts to \$125, at what price must the flour be sold per barrel to make a clear profit of 15% ?

62. A druggist gives a pound troy of certain goods instead of a pound avoirdupois. What is his gain and the customer's loss per cent. ?

63. Find the income derived from \$22,831.50 invested in Bank stock, which sells at 184 and pays a dividend of 8% per annum, brokerage being  $\frac{1}{8}\%$ .

64. A man drew 20% of his money from the bank, and paid 20% of the money thus withdrawn for three months' rent at the rate of \$960 per year. How much has he left in the bank ?

65. Bought a bill of hardware amounting to \$1,560 : of this \$125 less 10% and 10% ; \$268 less 40% and 10% ; \$600 less 25% and 20% ; the balance of the bill  $33\frac{1}{3}\%$  and 10%. What is the net price ?

66. A merchant sold an article at a loss of 8%, but had he sold it for \$1.05 more he would have gained 7%. What price did he sell it at ?

67. A man invests \$12,000 in 3% stock at 75 ; he sells out at 80, and invests  $\frac{1}{3}$  of the proceeds in  $3\frac{1}{2}\%$  stock at 96, and the remainder at 5% par. Find the change in his income.

68. The property of a town is assessed at \$2,600,000 ; the rate of taxation is 17 mills on the dollar ; it costs 2% of the tax for collection. Find the net amount received by the town.

69. A man sold 2 lots for \$445, gaining  $12\frac{1}{2}\%$  on one and losing  $12\frac{1}{2}\%$  on the other. Find the cost of each, if he gained \$5 on the whole transaction.

70. An attorney collected an account, and, after deducting his commission of  $7\frac{1}{2}\%$ , and exchange 75c on draft, he remitted his principal \$548.70. What was the amount of the collection and how much was the agent's commission ?

71. A merchant sold a quantity of silk and satin for \$630, gaining 25% on the silk and 20% on the satin. If he received 10% more for the silk than for the satin, what was his total gain ?

72. My income is \$1,900 annually, of which \$500 is exempted from taxation. What income tax do I pay if the rate is  $1\frac{3}{5}\%$  ? What is my net income ?

73. A money lender has \$1,500 out at 8%, \$1,200 at  $7\frac{1}{2}\%$ , and \$1,000 at 6%. Find the percentage he receives on the average.

74. If the pig iron of Canada is equal to that of foreign countries, what must be the invoice price of the latter to compete with us when our market price is \$43 a ton, the specific duty being equal to  $13\frac{1}{3}\%$  ad valorem, and the freight and charges \$9 a ton ?

75. A grocer sold goods to the amount of \$8.40, and gained  $16\frac{2}{3}\%$ . Had he gained 20%, find what the goods would have sold for.

76. A builder pays 3 times as much for material as for labor ; had he paid 10% more for material and 6% less for labor, his contract would have cost him \$3,637.92. What was his contract price ?

77. The 6 per cents. are at  $91\frac{1}{2}$  and the 7 per cents. at 102. A person has a sum of money to invest which will give him \$7,000 more of the former stock than of the latter. Find the difference in income he could obtain by investing in these stocks.

78. A merchant had 500 barrels of flour insured for  $\frac{3}{4}$  of its value at  $2\frac{1}{2}\%$ , paying \$75 premium. At what price per barrel must he sell to gain 25% of cost, as well as premium paid ?

79. What is the difference between a discount of 40% and three discounts of 20, 10, and 10% ?

80. A town requires \$23,832.90 to meet expenses for the year ; they pay  $2\frac{1}{2}\%$  for collection. What must be the rate if the taxable property of the town is \$1,800,000, and an allowance of 3% is made for uncollectible taxes ?

81. Find the gross proceeds of a sale made by an agent charging  $2\frac{1}{2}\%$  for commission, 5% for guaranty, \$17.65 for cartage, \$11.40 for storage, and \$3.25 for insurance, if the net proceeds remitted amount to \$1,714.10.

82. An insurance company took a risk at  $2\frac{1}{4}\%$ , and re-insured  $\frac{3}{5}$  of the risk at 2%. The premium received exceeded the premium paid by \$42. Find the amount of the risk.

83. A collector obtained 75% of a doubtful account amounting to \$1,750. What was his per cent. of commission if, by agreement with the principal, the commission was to be 50% of the net proceeds remitted?

84. The profits of a grocery business for three years were \$5,632. The second year's profits were 20% greater than the first, and the third year's profits were 10% greater than the second. What were the profits of each year?

85. At what price must I mark goods which cost \$16.20 so that I can allow a discount of 10% on marked price, and still make a profit of 10% on cost, allowing 10% of sales for bad debts?

86. I buy 400 shares of mining stock at  $134\frac{3}{4}$ , and sell 125 shares at  $138\frac{1}{2}$ , and the remainder at  $136\frac{7}{8}$ . What is my net gain, allowing  $\frac{1}{8}\%$  brokerage each way?

87. A man, whose income is \$2,500, finds that his net income, after paying the tax of  $18\frac{3}{4}$  mills on the dollar, is \$2,466.25. How much of his income is exempt from taxation?

88. Three-quarters of the selling price is equal to 5% less than cost. Find gain per cent. at which goods were sold.

89. I bought a drug store and a grocery store for \$7,000, and I get \$550 rent per annum for the two. I make 7% on the cost of the drug store and 9 per cent. on the cost of the grocery store. Find the cost of each.

90. An agent receives a certain percentage of the gross rental for letting a house, and assumes responsibility for collecting the rent. On a house let at \$38 a month for three years, he fails to collect one month's rent, and in the three years his net commission is \$37.24. Find the percentage allowed him.

91. A building is assessed for  $\frac{3}{4}$  of its value and the rate of taxation is 17 mills on the dollar. What will be the amount of the tax if it costs \$72 to insure the building for  $\frac{3}{5}$  of its value at  $1\frac{1}{2}$  per cent.?

92. If Canadian Pacific Railway stock pays a semi-annual dividend of  $2\frac{1}{2}\%$ , what per cent. do I realize by purchasing same at  $107\frac{1}{2}$ , brokerage  $\frac{1}{8}\%$ ?

93. If 9% of the cost price of an article is equal to 7% of the selling price, find the gain per cent.



94. A lad earned \$21.16 collecting accounts for a physician. He was allowed  $5\frac{3}{4}\%$ . What amount did he collect ?

95. A firm failed with assets amounting to \$630 and liabilities amounting to \$1,250. If the expenses of the assignment are 5% of the amount distributed to the creditors, how many cents on the dollar will the estate pay ? How much will Smith receive, who has a claim against the estate of \$160 ?

96. A man sold his 5 per cents. at  $78\frac{1}{8}$ , and invested the proceeds in 6 per cents. at  $103\frac{7}{8}$ . His change in income being \$385, find how much 5% stock he had, brokerage  $\frac{1}{8}\%$  each way.

97. A market woman buys apples at the rate of 100 for 40 cents. 10% of her apples is lost by decay. What per cent. does she gain by selling 5 for 3 cents ?

98. A dealer shipped 400 bushels of wheat which cost him \$1.40 per bushel, 800 bushels which cost \$1.62 $\frac{1}{2}$ , and 300 bushels which cost \$1.20, to his agent, who sold the first lot at 20% gain, the second at 15% gain, and the third at  $4\frac{1}{6}\%$  loss. The agent's commission was 3%, and the other charges were \$83.44. Find the dealer's gain per cent.

99. A tailor buys cloth at \$1.75 a yard, which, in sponging, shrinks 5%. At what price per yard must he sell it to gain 20% on his outlay ?

100. An agent receives for renting a house 4 per cent. of the gross rental, but assumes responsibility for making the collection. He lets the house at \$35 a month, and, in the course of four years, loses the rent for one month. What sum did he realize from his commissions in the four years, and how much rent did the owner receive ?

101. A house and its contents are, together, insured for \$4,200 at  $\frac{1}{2}\%$  and  $\frac{2}{3}\%$  respectively. If the total premium paid is \$22.50, for how much is each insured ?

102. A merchant sold 40% of an invoice of goods at a profit of 25%, 50% of the remainder at a loss of 10%, and the balance of the invoice at 15% profit. If his net gain on the whole invoice was \$138, find the invoice price of the goods ?

103. Incomes of not less than \$1,200 are taxed for all in excess of \$500 ; incomes of less than \$1,200 are not rated. If the rate



of taxation is  $19\frac{1}{2}$  mills, how much better off is the man who receives an income of \$1,195 than one who receives \$1,200 ?

**104.** The terms on which certain goods are sold are : " Net, 60 days ; 3%, 10 days." A customer is invoiced on May 2nd with goods to the value of \$560.20, and on May 12th pays \$300 in cash. He desires to close the transaction by note, taking full time for the unpaid balance. What will be the amount of the note ?

**105.** The unclaimed dividends on a certain amount of stock, which pays 6% per annum, amounted in 3 years to \$1,152. The stock was sold at a discount of  $12\frac{1}{2}$ % on its par value. What sum was realized ?

**106.** A person exchanged 180 shares of 6% stock at 80 for 10% stock at 125. How much was his yearly income increased ?

**107.** A house and lot were offered for sale at an advance of 50% above their cost ; but the agent sold the house at 20% below the asking price, receiving \$500 more than the cost. What is the owner's per cent. gain, after allowing the agent 2% for his services ?

**108.** A manufacturer sells goods to a merchant at a profit of  $62\frac{1}{2}$ %, but the merchant fails and pays  $62\frac{1}{2}$ c on the dollar. What per cent. will the manufacturer gain or lose ?

**109.** A grocer, by selling 6 lbs. of tea for a certain price, gained 15%. Afterwards he increased the price, giving  $5\frac{1}{4}$  lbs. for same price. What per cent. did he gain ?

**110.** How many pounds of tea worth 35 cents a pound must be mixed with 14 lbs. at 40 cents a pound, so that 20% may be made by selling the mixture at \$4.41 for 10 lbs. ?

**111.** An insurance agent takes a risk of \$20,000 on a vessel at  $2\frac{1}{4}$ %, and immediately re-insures  $\frac{1}{2}$  of the risk at  $2\frac{3}{4}$ %. What net premium does he receive ?

**112.** Two brothers receive by bequest \$3,600 each. One's money increases at the rate of 12% per annum for 5 years, while the other's increases at the rate of 10% per annum for the same time. How much more has the one than the other ?

113. Fred White sold tobacco at 5% commission; with the proceeds he bought wheat at 2% commission; his whole commission amounted to \$315. Find the value of the tobacco and wheat.

114. A merchant marked cloth to make a profit, as he supposed, of 20 per cent., but the cloth had cost \$3 a yard more than he had supposed, and he therefore lost 25%. What did the cloth cost per yard?

115. A speculator bought stock at a discount of  $7\frac{1}{8}$ , and sold  $\frac{1}{2}$  at a premium of  $3\frac{1}{2}\%$  and the remainder at par. Allowing brokerage of  $\frac{1}{8}\%$  each way, how many shares did he buy if he netted \$6,900?

116. A dealer marked his goods at an advance of 25% on cost, and, in selling them, he used a yard measure  $\frac{1}{2}$  in. too short. His entire gain was \$26.50. Find cost of goods.

117. An agent's commission was  $4\frac{1}{8}\%$  of the amount remitted to the consignor of the goods. What was his rate of commission?

118. A farmer took the following insurance in the Perth Mutual: House, valued at \$6,000, for  $\frac{3}{4}$  valuation at  $1\frac{1}{4}\%$ ; barn, valued at \$4,500, for  $\frac{2}{3}$  valuation at  $1\frac{1}{2}\%$ ; live stock for \$5,500 at  $\frac{3}{4}\%$ ; grain for \$3,000 at  $\frac{1}{2}\%$ . What was his total premium?

119. I bought two articles for \$150. I sold both, and lost 4% on what one cost me but gained 6% on what the other cost me. I gained  $1\frac{1}{3}\%$  on the whole. What was the price of each?

120. The whole stock of a company is \$200,000; the net gain is sufficient to pay 4% on the whole amount. The company pays 2% to ordinary shareholders, the balance giving 7% on the preference stock. What was the amount of the preference stock?

121. In the purchase of oats, wheat, and barley, a merchant expended equal sums. In the sales he gained 7% on the oats, 9% on the wheat, but on the barley he lost 21%; the total sales were \$2,212.50. What sum did he invest in each kind of grain?

122. A commission merchant bought 40,000 bushels of wheat at 75 cents per bushel, which he insured at \$20,000, taking a policy containing the "average clause." How much does he receive, the wheat being damaged to the extent of \$7,500?

**123.** A township assessor reports as follows : Personal property valued at \$120,000, real estate \$2,400,000 ; the personal property is taxed on  $\frac{2}{3}$  valuation, and real estate on  $\frac{1}{2}$  valuation ; there are 364 polls taxed \$1.75 each, the rate of taxation \$1.14 per \$100. Find amount of taxes.

**124.** An agent sold a consignment of sugar, charging  $2\frac{1}{2}\%$  commission. He invested part of the proceeds in 200 barrels of flour at \$6.50 per barrel, charging 2% commission ; and, after deducting \$114 for expenses other than his commission, he remitted to his principal the balance, which was \$900. For how much did he sell the sugar ?

**125.** The assignee of a bankrupt firm sold its realty for \$9,000, and its merchandise for \$7,000. He collected \$2,000 on bills receivable, and \$500 on personal accounts. The statement showed the liabilities to be \$27,000. If A's claim is \$7,200, how much would he receive, the expenses of the assignment being \$500 ?

**126.** A wine merchant mixes 8 gallons of wine worth \$1.12 $\frac{1}{2}$  a gallon, 12 gallons worth \$1.25 a gallon, and 14 gallons worth \$1.50 a gallon, with 16 gallons of water, and sells the mixture at \$1 a gallon. Find his gain per cent.

**127.** A merchant's sales for one year were \$50,000,  $\frac{1}{4}$  of the sales being at a profit of 25%,  $\frac{1}{3}$  at a loss of  $33\frac{1}{3}\%$ , and the balance at a gain of 50%. What did the goods cost ?

**128.** A man who gives a 6% discount for cash, sold \$235 worth of goods. He gave a horse worth \$85 and \$140 cash, and booked the balance. How much did he book ?

**129.** What sum invested in a 4% stock at 94 will yield a net income of \$1,980.80, after paying an income tax of 16 mills on the dollar on all over \$800 ?

**130.** An agent receives a consignment of 5,000 lbs. of tea, with instructions to sell and to invest the proceeds in flour, having deducted his commissions for the two transactions. The rate of commission for selling is 5%, and for buying  $2\frac{1}{2}\%$ . He sells the tea at 41 cents a pound, and buys the flour at \$4 a barrel. Find his total commission and the amount of flour bought.

131. A commission merchant has goods consigned to him to sell, and, after deducting 2% for both selling and investing, he finds that his commission for selling exceeds his commission for buying by \$6. Find the value of the goods remitted to him.

132. A person invested in 3% stock, and received  $5\frac{1}{4}\%$  clear on his investment, after paying an income tax of 2%. What was the market price of the stock, brokerage  $\frac{1}{2}\%$ ?

133. I mix 60 gallons of Madeira wine, costing \$3.50 a gallon, with 40 gallons of a superior quality, and sell the mixture at \$4.44 per gallon, thereby gaining 20%. Find the cost per gallon of the superior quality.

134. A merchant bought cloth at \$2 per yard, and sold the whole at a profit of \$120. Had he sold it at 20% less, he would have lost \$96. How many yards did he buy?

135. A piano dealer instructed his clerk to mark a piano, so that by allowing a discount of 25% he would realize a profit of  $33\frac{1}{3}\%$ . By mistake the clerk marked the piano at \$300, thereby producing a loss of  $16\frac{2}{3}\%$  on the sale. What was the intended marking price and what was the loss?

136. Find the total duty on the following at the rate of \$7 per ton:

45 Steel angles,  $3'' \times 3'' \times \frac{1}{4}''$ , at 4.9 lbs. per foot, 32' long.

29 Steel angles,  $2'' \times 2'' \times \frac{1}{4}''$ , at 3.2 lbs. per foot, 32' long.

30 Steel beams, 4'', at  $7\frac{1}{2}$  lbs. per foot, 38' long.

137. I hold some 3% stock. On receiving my first half-year's dividend, I invest it in the same stock at  $93\frac{3}{4}$ , and my next half-year's dividend is \$1,905. What amount of stock had I at first?

138. A farm cost  $3\frac{3}{4}$  times as much as a house. By selling the house at 10% loss and the farm at  $7\frac{1}{7}\%$  gain, \$3,993.30 is received. Find the cost of each.

139. A merchant bought sugar at \$3.75 per cwt., and paid for freight and other charges  $\frac{1}{4}$  of a cent. per pound. How many pounds can he sell for a dollar to make a clear gain of 25%?



140. My house cost \$6,000, which was insured at  $\frac{2}{3}$  valuation in the Sun, at  $1\frac{1}{2}\%$  per annum for 3 years. The house was totally destroyed by fire. What was my loss, and what was the loss of the company?

141. When milk is sold at the rate of 16 quarts for \$1, there is a gain of 25%. What would be the gain per cent. if 15 quarts were sold for \$1?

142. A person invested \$8,341 in 8% stock on the 7th day of January, at  $109\frac{1}{2}$ , and on the 12th day of February of the same year sold it out at  $117\frac{1}{2}$ , paying  $\frac{1}{4}\%$  brokerage on each transaction. Find his gain per cent. on what the stock cost him.

143. A manufacturing plant carries insurance as follows: On building \$12,000, on machinery \$20,000, and on stock \$30,000, paying  $1\frac{3}{4}\%$  premium. What is the net loss of the insurance company if they pay the following losses: On machinery  $\frac{1}{2}$  of valuation, on building  $\frac{2}{3}$  of valuation, and on stock a total loss?

144. A merchant buys a quantity of tea, and sells it again so as to gain 10%. Had he bought it at 10% less and sold it for \$5 less, he would have gained 20% on supposed cost. Find what the tea cost.

145. A man buys stock at  $90\frac{1}{2}$  and sells out at 90, thereby losing \$206; he then invests in stock at 3% premium, and sells again when it has reached 5% premium. With the proceeds he invests in 3% stock at 81. Find his yearly income from the last investment.

146. A merchant buys 4 gal. 2 qt. of vinegar for \$1, and sells 2 gal. 3 qt. for the same sum. What is his gain per cent.?

147. At what price shall an agent be ordered to buy potatoes at 2% commission that, after paying 7 cents per barrel for transportation, they can be sold at \$1.76 per barrel and net 10% profit?

148. A merchant invested \$42,500 in dry goods. The first year he gained 20%, the second year he gained  $12\frac{1}{2}\%$ , the third year he lost  $16\frac{2}{3}\%$ , and the fourth year he gained 5%. Find the amount invested in business at the end of the fourth year, the gain being added or the loss subtracted each year?



149. A fruit dealer bought an invoice of oranges for \$735, sold 80% of the invoice at 75% of the entire cost, and the remainder at 20% profit. What was his net loss?

150. I sold tea at a gain of 25%; if it had cost 10 cents a pound less, the same selling price would have realized 50% gain. Find the cost of the tea per pound.

151. A man invested \$5,500, a part in the 4 per cents. at  $83\frac{3}{4}$ , and the rest in the 5 per cents. at  $102\frac{1}{4}$ , brokerage  $\frac{1}{4}\%$  in each case. His total income being \$266 $\frac{3}{8}$ , find the sum invested in each stock.

152. At what price must goods which cost \$66.69 be marked so that, after giving a discount of  $2\frac{1}{2}\%$ , they may still be sold at a profit of  $12\frac{1}{2}\%$ ?

153. T. H. Smith, Toronto, buys of C. M. McCollum & Co., Brampton, 10 cases of eggs, 30 doz. each, at 14 cents per doz., f.o.b. Brampton. Messrs. McCollum & Co. ship with the eggs 500 lbs. of butter. 300 lbs. of the butter sell at 16 cents, and the balance at  $15\frac{1}{2}$  cents per pound. The total weight of the shipment is 1,100 lbs., and the freight rate is 14 cents per cwt., and the cartage 22 cents. Smith's commission for selling butter is 5%. What are the net proceeds of the entire shipment?

154. By selling a lot for \$89.75 above cost, I realized a profit of 5%. I sold another lot, which cost the same, for \$1,848.85. What was my gain per cent. on the second sale?

155. A certain school section is assessed for \$150,000. The trustees have built a school-house costing \$1,800.

(a) What will the school-house cost a ratepayer whose property is assessed for \$4,500?

(b) What would be the rate of taxation per annum on the whole section if the house were paid for in six equal annual payments, without interest?

156. I invested in 7% stock at  $78\frac{1}{8}$ , and, having received a half-year's dividend, I sold out at  $79\frac{3}{8}$ , paying  $\frac{1}{8}\%$  brokerage on each transaction, and increased my capital altogether by \$292.50. How much did I invest?

157. A grocer, retailing sugar at the rate of 22 lbs. for \$1, makes a profit of  $11\frac{1}{5}\%$ . If a barrel of sugar costs \$11.25 and contains 290 lbs., what per cent. of the weight is lost in retailing?

158. My house is valued at  $\frac{1}{3}$  more than my brother's; my house is insured at  $\frac{5}{8}\%$  on  $\frac{3}{4}$  of its value, my brother's at  $\frac{7}{8}\%$  on  $\frac{2}{3}$  of its value. I pay \$1.75 more premium than my brother. Find the value of each house.

159. If \$5,400 stock, paying 5%, be sold at  $89\frac{1}{8}$ , and the proceeds invested in a 6% stock at  $107\frac{7}{8}$ , what is the change in income, brokerage each way  $\frac{1}{8}\%$ ?

160. A sells goods to B at a gain of 12%, and B sells the same goods to C at a gain of  $7\frac{1}{2}\%$ ; C paid \$3,762.50 for the goods. How much did A pay for them?

161. What rate per cent. do I gain on my money by investing in stock at  $83\frac{1}{3}$ , which pays a 5% annual dividend?

162. Goods were sold at 10% gain, but if they had cost \$60 more, there would, at the same selling price, have been 10% loss. Find the cost of the goods?

163. Find the duty on 50 cases of tobacco, each weighing 60 lbs., worth \$1 per pound, and 50,000 Havana cigars, weighing 550 lbs., invoiced at \$75, per thousand, the duty being 50c per pound specific on the tobacco and \$3 per pound specific on the cigars, and 25% ad valorem on cigars.

164. What is the price of a 6% stock which pays  $4\frac{1}{2}\%$  on the money invested?

165. A merchant marks his cloth at  $3\frac{7}{11}\%$  profit. After selling  $\frac{3}{5}$  of his stock at this rate, he is forced by competition to reduce the price 2 cents per yard, and in the end gains only  $\frac{11}{13}$  of what he had intended. Find the cost price per yard.

166. A certain school is supported by popular subscription to defray expenses, as follows: Salaries \$5,000, rent \$1,650, fuel \$150, gas \$125, janitor \$750, incidentals \$225. The enrolment numbered 325 pupils, of whom 20% missed 20% of the time, and 24% missed 5% of the time. The school year was for 40 weeks of 5 days each. What was A's tax, who sent 5 pupils, one of them missing 20% of the time?

167. A man sold his 5 per cents. at 78, and invested the proceeds in 6 per cents. at 104. His change in income being \$385, find how much 5% stock he had.

168. If the cost of an article had been 10% less, the same selling price would have brought me 12% more. What was the gain per cent. ?

169. A house that cost \$15,500 rents for \$155 a month. It is insured for \$10,850 at  $\frac{4}{5}$ % yearly ; the taxes are 15 mills on an assessment of \$12,450, and \$346.45 is spent each year on repairs. What rate of interest does the investment pay ?

170. A retired farmer invested 40% of his capital in  $3\frac{1}{2}$ % stock at 90, and the remainder in 4% stock at 95. His income was \$698 per year. What capital had he invested ?

171. How many pounds of sugar, at  $4\frac{1}{4}$  cents per pound, can be bought by an agent for \$897.75, after deducting \$5 paid for drayage and a purchasing commission of 5% ?

172. A man sold 54 yards at a profit of 10% and 165 yards at a profit of 20%, and found that had he sold it all at a uniform profit of 15%, he would have realized \$2.77 $\frac{1}{2}$  less than he did. What was the cost price of the cloth ?

173. A corporation, having net earnings of \$5,665, wishes to declare a dividend after reserving a working fund of \$2,000. The corporation organized with a capital stock of 250 shares, but has since issued 90 shares of preferred stock, secured at 6%. What rate of dividend can they declare ?

174. I bought a case of prints, containing 4,500 yards, at 4 cents per yard, less 5%, and by paying cash in 10 days an additional discount of 2% was allowed. I sold at 5 cents per yard ;  $\frac{1}{2}$  of the amount, being in jobbing lots, was discounted at 4%. What per cent. profit did I average on the net cost, after allowing for freight and drayage \$12.35, and taking advantage of the additional discount ?

175. Bought of J. S. Davidson & Co., wholesale merchants : 15 bbl A Sugar, each 327-36, at 6c ; 10 sk. Rio Coffee, each 155-3, at 12c ; 9 ch. Young Hyson Tea, each 95-8, at  $37\frac{1}{2}$ c ; 12 ch. Japan Tea, each 76-12, at 40c ; 14 bx. Laundry Soap, each 74-14, at  $4\frac{1}{2}$ c ; 12 bbl. Kerosene, each 45 gal., at 14c. I am allowed a list discount of 20%, and a second discount of 5% on the first three items. What is the net amount of the bill ?

**176.** A firm receives an invoice from England as follows: Cost of goods £67 2s. 3d., freight \$18.92, insurance \$1.02, and duty  $23\frac{1}{3}\%$ . What would the selling price be per shilling at Toronto in order to clear 20%, exchange at rate of 10% premium, cash discount  $2\frac{1}{2}\%$  60 days, no prepayment.

**177.** A commission merchant received a consignment of 2,000 barrels of flour, which he sold at \$8 a barrel, on a commission of  $1\frac{1}{2}\%$ ; the expenses for freight, paid by the agent out of the proceeds, amounted to \$150; he bought cotton at 15 cents a pound with the net proceeds, charging  $\frac{3}{4}\%$  commission for buying. How many pounds of cotton did he buy?

**178.** I sent my agent at Collingwood \$1,000 to invest in apples at \$1.75 per barrel, after deducting charges as follows; Commission 4%, drayage 5 cents per barrel. How many barrels did he buy, and what was the unexpended balance on hand?

**179.** The profits of a loan company for a year were sufficient to enable the directors to add \$20,000 to their reserve fund, to pay \$5,965 for cost of management, to pay two half-yearly dividends of  $3\frac{1}{2}\%$  on a paid-up capital stock of \$309,056, and to have still on hand \$4,236. Find the profits for the year.

**180.** (a) By Act of Parliament, in 1898, the number of pounds in a bushel of onions was changed from 60 to 50. What per cent. gain does this mean to the grower if the price per bushel remains the same? (b) If, in consequence, the price per bushel falls from \$1 to 85 cents, what per cent. does the grower gain by the changes?

**181.** The assessed value of property in a municipality is  $26\frac{1}{2}$  million dollars; the rate of taxation is 20 mills on the dollar, distributed as follows: City Debt,  $4\frac{7}{10}$  mills; Public Schools,  $4\frac{3}{10}$  mills; Collegiate Institute,  $\frac{9}{10}$  mill; Free Library,  $\frac{5}{10}$  mill; Park Fund,  $\frac{3\frac{1}{2}}{10}$  mills; and the rest to General Expenses. How much does each of these receive from the taxes collected?

**182.** A commission merchant bought for his principal 12,000 bushels of wheat at \$1.20 per bushel, and immediately sold it at \$1.30, investing the proceeds in the purchase of oats in car load



lots of 1,000 bushels each, at 35 cents per bushel, his commission being 2% for buying and  $2\frac{1}{2}\%$  for selling. Find the following: number of car loads, unexpended balance, and total commission.

183. Bank of Commerce stock sells at 120, and pays an 8% dividend; Bank of Toronto pays a 10% dividend. If I invest \$4,800 in Bank of Toronto stock, my income will be \$20 less than if I invest in Bank of Commerce stock. At what premium is Bank of Toronto stock selling?

184. My agent in Montreal charges 3% for buying and 2% for selling, and an additional commission of 2% for guaranteeing quality of goods purchased. I make him a shipment of wheat, with instructions to sell and invest in a shipment of cotton to Liverpool. What was the selling price of the wheat, and what was the cost of the cotton, his total commission being \$1,031.25?

185. I purchased 12 shares of C.P.R. stock at  $112\frac{1}{2}$ , and, after receiving a semi-annual dividend thereon of  $3\frac{1}{2}\%$ , sold again at 135, brokerage in each case  $\frac{1}{4}\%$ . What rate per cent. per annum did I realize on my investment if the shares were in my possession just six months?

186. An agent charged me 5% for selling corn, and 2% for investing the proceeds in cotton. His commission amounted to \$280. What was the selling price of the corn?

187. A cargo, valued at \$45,000, was insured for \$10,000 in the Continental Insurance Company, \$6,000 in the Liverpool Insurance Company, and \$14,000 in the Hamburg Insurance Company. The cargo was injured during a storm to the extent of \$9,600. What sum did each company pay according to "average clause" insurance?

188. A commission merchant received 4,000 bushels of wheat and 24,000 lbs. of beef, with instructions to sell and invest \$2,500 of the proceeds in cotton, and remit the balance after deducting the charges. He sold the wheat at  $62\frac{1}{2}$  cents per bushel, and the beef at  $9\frac{1}{2}$  cents per pound. He paid \$375 for freight, \$24 for drayage, and charged a commission of  $\frac{7}{8}$  cents per bushel on the wheat,  $2\frac{1}{2}\%$  on the beef, and  $2\frac{1}{4}\%$  on the cotton. How much did he remit?



# SHORT METHODS

## SHORT METHODS IN MULTIPLICATION

### Aliquot Parts

The learner should commit to memory the following tables so thoroughly that the aliquot parts can be named without the least hesitation when the fraction is given, and vice versa.

#### ALICQUOT PARTS OF 10

$$\begin{array}{llll} 1\frac{1}{9} = \frac{1}{9} & 1\frac{3}{7} = \frac{1}{7} & 2\frac{1}{2} = \frac{1}{4} & 5 = \frac{1}{2} \\ 1\frac{1}{4} = \frac{1}{8} & 1\frac{2}{3} = \frac{1}{6} & 3\frac{1}{3} = \frac{1}{3} & \end{array}$$

#### ALICQUOT PARTS OF 100

$$\begin{array}{lll} 6\frac{1}{4} = \frac{1}{16} & 25 = \frac{1}{4} & 62\frac{1}{2} = \frac{5}{8} \text{ or} \\ 8\frac{1}{3} = \frac{1}{12} & 31\frac{1}{4} = \frac{5}{16} \text{ or} & \frac{1}{2} + (\frac{1}{4} \text{ of } \frac{1}{2}) \\ 12\frac{1}{2} = \frac{1}{8} & \frac{1}{4} + (\frac{1}{4} \text{ of } \frac{1}{4}) & 75 = \frac{3}{4} \text{ or} \\ 14\frac{2}{7} = \frac{1}{7} & 33\frac{1}{3} = \frac{1}{3} & \frac{1}{2} + (\frac{1}{2} \text{ of } \frac{1}{2}) \\ 16\frac{2}{3} = \frac{1}{6} & 37\frac{1}{2} = \frac{3}{8} \text{ or} & 87\frac{1}{2} = \frac{7}{8} \text{ or} \\ 18\frac{3}{4} = \frac{3}{16} \text{ or} & \frac{1}{4} + (\frac{1}{2} \text{ of } \frac{1}{4}) & \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \\ \frac{1}{8} + (\frac{1}{2} \text{ of } \frac{1}{8}) & 50 = \frac{1}{2} & \end{array}$$

#### ALICQUOT PARTS OF 1000

$$\begin{array}{lll} 83\frac{1}{3} = \frac{1}{12} & 375 = \frac{3}{8} \text{ or} & 833\frac{1}{3} = \frac{5}{6} \text{ or} \\ 125 = \frac{1}{8} & \frac{1}{4} + (\frac{1}{2} \text{ of } \frac{1}{4}) & \frac{1}{2} + \frac{1}{3} \\ 166\frac{2}{3} = \frac{1}{6} & 625 = \frac{5}{8} \text{ or} & 875 = \frac{7}{8} \text{ or} \\ 250 = \frac{1}{4} & \frac{1}{2} + (\frac{1}{2} \text{ of } \frac{1}{4}) & \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \\ 333\frac{1}{3} = \frac{1}{3} & & \end{array}$$

### Multiplication by Aliquots

To multiply any number by

10 annex a cipher.

1000 annex three ciphers.

100000 annex five ciphers.

$1\frac{1}{4}$  annex a cipher and divide by 8.

$1\frac{2}{3}$  annex a cipher and divide by 6.

100 annex two ciphers.

10000 annex four ciphers.

- $2\frac{1}{2}$  annex a cipher and divide by 4.  
 $3\frac{1}{3}$  annex a cipher and divide by 3.  
 $6\frac{1}{4}$  annex two ciphers and divide by 16.  
 $8\frac{1}{3}$  annex two ciphers and divide by 12.  
 $12\frac{1}{2}$  annex two ciphers and divide by 8.  
 $14\frac{2}{7}$  annex two ciphers and divide by 7.  
 $16\frac{2}{3}$  annex two ciphers and divide by 6.  
25 annex two ciphers and divide by 4.  
 $33\frac{1}{3}$  annex two ciphers and divide by 3.  
50 annex two ciphers and divide by 2.  
 $66\frac{2}{3}$  annex two ciphers and subtract  $\frac{1}{3}$  of the product.  
75 annex two ciphers and deduct  $\frac{1}{4}$  of the product.  
 $87\frac{1}{2}$  annex two ciphers and deduct  $\frac{1}{8}$  of the product.  
 $37\frac{1}{2}$  annex two ciphers and take  $\frac{3}{8}$  of the product.  
 $62\frac{1}{2}$  annex two ciphers and take  $\frac{5}{8}$  of the product.  
 $112\frac{1}{2}$  annex two ciphers and add  $\frac{1}{8}$  of the product.  
125 annex three ciphers and divide by 8.  
 $133\frac{1}{3}$  annex two ciphers and add  $\frac{1}{3}$  of the product.  
 $137\frac{1}{2}$  annex two ciphers and add  $\frac{3}{8}$  of the product.  
150 annex two ciphers and add  $\frac{1}{2}$  of the product.  
 $162\frac{1}{2}$  annex two ciphers and add  $\frac{5}{8}$  of the product.  
 $166\frac{2}{3}$  annex three ciphers and divide by 6.  
175 annex two ciphers and multiply by 2 and deduct  $\frac{1}{8}$  of the product.  
 $187\frac{1}{2}$  annex two ciphers and multiply by 2 and deduct  $\frac{1}{16}$  of the product.  
250 annex three ciphers and divide by 4.

### SERIES 61

Multiply mentally and add products :

1. 480 by $1\frac{1}{4}$	2. 375 by $1\frac{2}{3}$	3. 432 by $6\frac{2}{3}$
490 by $2\frac{1}{2}$	870 by $3\frac{1}{3}$	528 by 50
144 by $8\frac{1}{3}$	732 by 25	588 by $33\frac{1}{3}$
680 by $12\frac{1}{2}$	512 by $6\frac{1}{4}$	340 by $2\frac{1}{2}$
660 by $16\frac{2}{3}$	434 by $14\frac{2}{7}$	784 by 25

- |                       |                           |                          |
|-----------------------|---------------------------|--------------------------|
| 4. 464 by 125         | 5. 494 by $12\frac{1}{2}$ | 6. 384 by $6\frac{1}{4}$ |
| 584 by $1\frac{1}{4}$ | 381 by $16\frac{2}{3}$    | 532 by $14\frac{2}{7}$   |
| 564 by $8\frac{1}{3}$ | 482 by 50                 | 326 by 125               |
| 462 by $1\frac{2}{3}$ | 864 by $33\frac{1}{3}$    | 498 by $6\frac{2}{3}$    |
| 240 by $6\frac{2}{3}$ | 165 by $3\frac{1}{3}$     | 342 by $11\frac{1}{9}$   |
- 
- |                          |                            |                        |                             |
|--------------------------|----------------------------|------------------------|-----------------------------|
| 7. 360 by $7\frac{1}{2}$ | 8. 1095 by $33\frac{1}{3}$ | 9. 840 by 250          | 10. 420 by $133\frac{1}{3}$ |
| 125 by 64                | 250 by 44                  | 232 by $62\frac{1}{2}$ | 544 by 125                  |
| 584 by 75                | 2163 by $3\frac{1}{3}$     | 320 by $18\frac{3}{4}$ | 384 by $37\frac{1}{2}$      |
| 240 by $13\frac{1}{3}$   | 432 by $6\frac{1}{4}$      | 360 by $87\frac{1}{2}$ | 468 by $11\frac{1}{9}$      |
| 440 by $37\frac{1}{2}$   | 351 by $66\frac{2}{3}$     | 450 by $83\frac{1}{3}$ | 364 by $14\frac{2}{7}$      |

To Multiply by Numbers from 13 to 19 inclusive

ILLUSTRATION.—Multiply 485 by 13.

(a) Ordinary Method.—

$$\begin{array}{r}
 485 \\
 13 \\
 \hline
 1455 \\
 485 \\
 \hline
 6305
 \end{array}$$

(b) Short Method.—

$$\begin{array}{rcl}
 485 & 3 \times 5 & = 15 \text{ carry } 1 \\
 13 & 3 \times 8 + 1 \text{ (carried)} + 5 & = 30 \text{ carry } 3 \\
 \text{—} & 3 \times 4 + 3 \text{ (carried)} + 8 & = 23 \text{ carry } 2 \\
 6305 & 4 + 2 \text{ (carried)} & = 6
 \end{array}$$

Multiply each figure by 3, and to each product add the carrying figure (if any) and the figure to the right of the one multiplied. After all the figures have been multiplied by 3, complete the result by adding the carrying figure to the last figure of the multiplicand.

# SERIES 62

Multiply the following mentally :

- |                 |                  |
|-----------------|------------------|
| 1. 346725 by 13 | 5. 435827 by 17  |
| 2. 647386 by 14 | 6. 210349 by 18  |
| 3. 582630 by 15 | 7. 536274 by 19. |
| 4. 138629 by 16 | 8. 742976 by 19  |

Multiply the following mentally and total the results :

9. 347 by 16	10. 158 by 14	11. 158 by 16	12. 329 by 15
392 by 19	376 by 16	431 by 18	176 by 17
157 by 18	457 by 11	413 by 13	384 by 12
469 by 12	386 by 19	415 by 19	910 by 18

### To Multiply by the Factors of a Number

ILLUSTRATION.—Multiply 95 by 32.

$$\begin{array}{r}
 95 \\
 8 \\
 \hline
 760 \\
 4 \\
 \hline
 3040
 \end{array}$$

SOLUTION.—The factors of 32 are 8 and 4. First multiply by 8 and then multiply that product by 4.

### SERIES 63

1. $784 \times 36$	7. $2956 \times 35$	13. $21754 \times 96$
2. $891 \times 72$	8. $2179 \times 44$	14. $17845 \times 420$
3. $794 \times 77$	9. $4754 \times 108$	15. $78941 \times 144$
4. $485 \times 99$	10. $2816 \times 256$	16. $29715 \times 196$
5. $284 \times 56$	11. $4712 \times 324$	17. $49165 \times 98$
6. $187 \times 64$	12. $2175 \times 192$	18. $97142 \times 625$

### To Multiply by means of Cross Multiplication

NOTE.—To see the reason for any of the following solutions, work the question by the ordinary method, putting down every line of the solution. When you do the same work by cross multiplication, you will see that it is simply a matter of carrying in your head the work you ordinarily do on paper.

ILLUSTRATION 1.—Find the product of  $74 \times 33$ .

$$\begin{array}{r}
 74 \\
 33 \\
 \hline
 2442
 \end{array}$$

SOLUTION.— $4 \times 3 = 12$ . Write 2 as the first figure of the product and carry 1.  $7 \times 3 + 1$  (carried)  $+ 12$  ( $4 \times 3$ ) = 34. Write 4 as the second figure of the product and carry 3.  $7 \times 3 + 3$  (carried) = 24. Write 24 to the left of the figures already written in the product, thus completing the multiplication and obtaining a product of 2442.

ILLUSTRATION 2.—Find the product of  $124 \times 62$ .

SOLUTION.— $4 \times 2 = 8$ . Write 8 as the first figure of the product.  $2 \times 2 + 24$  ( $4 \times 6$ ) = 28. Write 8 as the second figure of the product and carry 2.  $1 \times 2 + 12$  ( $2 \times 6$ ) + 2 (carried) = 16. Write 6 as the third figure of the product and carry 1.  $1 \times 6 + 1$  (carried) = 7. Write 7 as the fourth figure of the product, thus completing the multiplication and obtaining a product of 7688.

ILLUSTRATION 3.—Find the product of  $2146 \times 32$ .

SOLUTION.— $6 \times 2 = 12$ . Write 2 and carry 1.  $4 \times 2 + 1$  (carried) + 18 ( $6 \times 3$ ) = 27. Write 7 and carry 2.  $1 \times 2 + 2$  (carried) + 12 ( $4 \times 3$ ) = 16. Write 6 and carry 1.  $2 \times 2 + 1$  (carried) + 3 ( $1 \times 3$ ) = 8. Write 8.  $2 \times 3 = 6$ . Write 6, thus completing the multiplication and obtaining a product of 68672.

# SERIES 64

- |                   |                     |                      |
|-------------------|---------------------|----------------------|
| 1. $74 \times 26$ | 10. $75 \times 35$  | 18. $278 \times 87$  |
| 2. $97 \times 44$ | 11. $175 \times 24$ | 19. $224 \times 58$  |
| 3. $33 \times 27$ | 12. $261 \times 73$ | 20. $976 \times 47$  |
| 4. $48 \times 46$ | 13. $485 \times 56$ | 21. $7172 \times 26$ |
| 5. $79 \times 52$ | 14. $697 \times 28$ | 22. $1478 \times 55$ |
| 6. $15 \times 75$ | 15. $441 \times 56$ | 23. $4196 \times 74$ |
| 7. $27 \times 68$ | 16. $247 \times 87$ | 24. $2198 \times 57$ |
| 8. $79 \times 47$ | 17. $478 \times 56$ | 25. $5164 \times 26$ |
| 9. $28 \times 54$ |                     |                      |

## To Divide by the Factors of a Divisor

ILLUSTRATION.—Divide 9128 by 126.

SOLUTION.—Since the divisor is equal to  $3 \times 7 \times 6$ , the division of 9128 by 126 may be accomplished by dividing successively by these factors.

Dividing 9128 by 3 (or one forty-second of the true divisor, 126) produces 3042 (or 42 times the true quotient) and a remainder of 2. Since this remainder is left from the true dividend, it must be a part of the true remainder.

Dividing 3042 (one forty-second of the true quotient) by the second factor, 7, produces 434 (which must be one seventh of 42, the true quotient) and a remainder of 4. Since 4 is left from

$$126 = 3 \times 7 \times 6$$

$$3)9128$$

$$7)3042 + 2 =$$

$$6)434 + 4 \times 3 =$$

$$72 + 2 \times 3 \times 7 = 42$$

$$\text{Quotient, } 72 \frac{56}{126}$$

$$2$$

$$12$$

$$56$$



dividing one-third of the true dividend, this remainder must be one-third of the true remainder ( $4 \times 3 = 12$ ), second part of true remainder.

Dividing 434 (one-sixth of the true quotient) by the remaining factor, 6, produces 72 (the true quotient) and a remainder of 2. Since 2 is the remainder from dividing 434 (one-third of one-seventh of the true dividend), this remainder must be one-third of one-seventh of the true remainder ( $2 \times 3 \times 7 = 42$ ), third part of the true remainder.

Add the several parts of the true remainder, obtaining 56 as the total true remainder.

### SERIES 65

1. $25380 \div 36$	6. $31279 \div 72$	11. $43716 \div 168$
2. $178584 \div 48$	7. $43827 \div 84$	12. $29373 \div 81$
3. $23741 \div 42$	8. $19375 \div 125$	13. $41658 \div 45$
4. $43165 \div 64$	9. $41643 \div 135$	14. $23725 \div 96$
5. $41765 \div 63$	10. $17496 \div 147$	15. $47916 \div 648$

## SHORT METHODS IN DECIMALS

### Approximations

Suppose that the exact result of an operation is 27.47186. For ordinary business purposes three places may be sufficient. Reading our result to the nearest figure, and retaining but three places of decimals, it becomes 27.472, which is an approximate value of 27.47186 correct to three places of decimals. Short methods in decimals are, therefore, attempts at getting approximate values to a certain number of places.

### Addition

		SOLUTION (a) shows the addition
(a) $72.142756$	(b) $72.143$	carried out complete.
$15.2176$	$15.218$	SOLUTION (b) shows the solution
$42.71594$	$42.716$	correct to three decimal places.
<hr/>	<hr/>	This is done by writing each
$130.076296$	$130.077$	addend, retaining only three places
		of decimals.

The addition is then performed in the usual way.

Subtraction may be handled in the same way where an approximation is sufficient.

### Multiplication

ILLUSTRATION.—Multiply 171.2478 by 8.4712, retaining only 4 decimal places.

(a)	171.2478	(b)	171.24780
	8.4712		21748
	342 4956		136998240
	1712 478		6849912
	119873 46		1198729
	684991 2		17124
	13699824		3424
	1450.6743 6336		1450.67429
	Ans.		1450.6743

SOLUTION (a) shows the work carried out in full.

SOLUTION (b) shows the work contracted, so as to give an approximation to four decimal places.

### RULE

*Reverse the multiplier, placing the units figure thereof directly under the decimal to which it is intended to extend the work (which should be one place further than an accurate answer is required). If the multiplier does not contain a whole number, place the tenths figure directly under the next figure to the left of the decimal to which it is intended to extend the work. Multiply as in ordinary multiplication, ignoring all figures in the multiplicand to the right of the figure which is used as multiplier. Arrange the several products so that the figures on the extreme right are in line vertically. Add and point off the number of places to which the work is extended.*

### SERIES 66

1. 36.275 × 4.3678 retaining 2 decimal places.
2. 41.3075 × 467.32 retaining 3 decimal places.
3. 17.0036 × .08245 retaining 4 decimal places.
4. .43261 × .73158 retaining 3 decimal places.
5. .003647 × .12739 retaining 4 decimal places.
6. 700.375 × .02736 retaining 3 decimal places.
7. .374825 × .693847 retaining 5 decimal places.

### Division

In dividing decimals the quotient will take one of three different forms. It will be

- (a) A whole number, with or without a decimal part.
- (b) A decimal in which the first figure is a significant figure ; or
- (c) A decimal in which there are one or more noughts between the decimal point and the first significant figure.

These three forms may be illustrated thus :

45.6 or 456 ; .456 ; and .00456.

In dividing by the contracted method, the first thing to be decided is, in the quotient, how many figures will there be in the whole number, or how many noughts will there be to the right of the decimal point.

Thus, in dividing 3754.26 by 13.254, it will be seen that there will be three figures in the whole number of the quotient. For, disregarding the decimals, 13 is contained in 37, and there will be one figure in the quotient for the 5 and one for the 4.

In dividing 32.4986 by 18376.25, there will be two noughts to the right of the decimal point in the quotient. For 18376 is not contained in 32, and, after reaching the decimal point, there must be a nought inserted after it in the quotient for every figure until the divisor is contained in the dividend, and 18376 is not contained in 324 nor 3249, but is contained in 32498.

In dividing 367.52 by 485.346, there will be no whole number in the quotient, and, at the same time, no noughts between the decimal point and first significant figure, since 485 is not contained in 367, but is contained in 3675.

In testing the quotient of .00375 by .2864, it will be necessary to shift the decimal point three figures to the right in both dividend and divisor, thus multiplying them both by the same number, 1000, which will not affect the quotient. The question then becomes  $3.75 \div 286.4$ , in which it is seen that the quotient takes the form in which there is one nought between the decimal point and the first significant figure.

Having decided how many figures there will be in the whole number in the quotient, or how many noughts there will be to the right of the decimal point, the next step is to decide how many figures there will be in the whole quotient. To decide this, if the quotient contains a whole number, we will add to the number of figures to which we want the answer to be correct the number of figures in the whole number. If the quotient contains no whole number, but takes the form in which there are noughts between the decimal point and the first significant figure, we will subtract from the number of figures to which we want the answer correct the number of noughts between the decimal point and the first significant figure. If there is no whole number in the quotient and there are no noughts between the decimal point and the first significant figure, the number of figures in the quotient will simply be the number of figures to which we want the answer correct.

Having decided how many figures there will be in the quotient, we will arrange our divisor to contain the same number of figures. Beginning at the left of the divisor, we will count off the number of figures desired in the quotient, and strike out the remainder. If there are not a sufficient number of figures in the divisor, we will add noughts to make up the number.

Having adjusted the divisor, the next step is to adjust the dividend. To do this we will count off from the left side enough figures in the dividend to make it contain the divisor, rejecting the others, or supplying noughts to make up the deficiency.

Having adjusted both divisor and dividend, we will divide as in ordinary division, except that, instead of bringing down a figure from the dividend

each time we divide, we will drop one figure from the right of the divisor, continuing to do so until the divisor is all eliminated.

NOTE 1.—It is not necessary to remove the decimal point from the divisor, as in ordinary division of decimals, since we know before commencing the division what form the quotient is to take, and can place the decimal point in the quotient in its proper place. So the decimal points may be disregarded.

NOTE 2.—It is always well to take the result one place farther than that really wanted, so as to be sure the last figure of the contracted quotient is correct; since, if the figure coming after the one to which we want the answer correct is 5 or more, 1 should be added to the last figure of the contracted quotient.

ILLUSTRATION.—Divide 714.2965 by 14.367 correct to four places of decimals.

SOLUTION 1.—Ordinary Method

14.367)714.2965(

14367)714296.5(49.71786

57468

139616

129303

103135

100569

25660

14367

112930

100569

123610

114936

86740

86202

Ans. = 49.7179

SOLUTION 2.—Contracted Method

~~1436700~~)7142965(4971793

5746800

1396165

1293030

103135

100569

2566

1436

1130

1001

129

126

3

Ans. = 49.7179

On examining the numbers, we see that there will be two figures in the whole number of the quotient. Since we wish the result correct to four places of decimals, we will take it out to five places, so we will have seven figures in our quotient. We will therefore need seven figures in the divisor. Since there are only five figures in the divisor, we will add two noughts to the right of it. On examining the dividend, we find that it will contain our divisor. So we proceed with our division, discarding one figure of the divisor



each time. The quotient obtained is 4971793; and, since there are to be two figures in the whole number of the answer, we count off from the left side two figures and place the decimal point. Then, since we were to have the answer correct to four decimal places, we drop the last figure of the quotient, since it is less than 5. Had it been 5 or more, we would have made our quotient 49.7180.

### SERIES 67

Divide

1.	27.3782	by	4.3267	correct to 3 decimal places.
2.	487.24	by	1.003675	correct to 2 decimal places.
3.	8.47326	by	75.43	correct to 5 decimal places.
4.	.8487564	by	.075637	correct to 3 decimal places.
5.	478.325	by	1.43 $\frac{2}{3}$	correct to 3 decimal places.
6.	8972.436	by	756.3452	correct to 4 decimal places.
7.	1	by	1.007633	correct to 6 decimal places.
8.	.953728	by	44.73654	correct to 3 decimal places.

### To Reduce British Currency to Canadian Currency at Commercial Par

SHORT METHOD.—£1 at par is equal to \$4.86 $\frac{2}{3}$ . Analyze this by taking as a base the old Halifax currency of \$4 for £1, 20 cents ( $\frac{1}{5}$  of \$1) for 1 shilling, and 1 $\frac{3}{4}$ c ( $\frac{1}{12}$  of 20c) for 1 penny.

£1 at par calculated in this way gives :

	\$4.00
+ $\frac{1}{5}$ of \$4, or	.80
+ $\frac{1}{12}$ of 80c or	.06 $\frac{2}{3}$
	<hr/>
	\$4.86 $\frac{2}{3}$

To find the value of £15 3s. 7d. we proceed as follows :

£15, at \$4 each	.....	\$60.00
3s., at 20c each	.....	.60
7d., at 1 $\frac{3}{4}$ c each	.....	.12
		<hr/>
		\$60.72
+ $\frac{1}{5}$ of \$60.72 =		12.14
+ $\frac{1}{12}$ of 12.14 =		1.01
		<hr/>
		\$73.87

### SERIES 68

Change to Canadian currency :

	£	s.	d.		£	s.	d.
1.	72	5	7	6.	475	18	10
2.	47	15	4	7.	547	8	5
3.	57	12	6	8.	297	19	11
4.	195	17	8	9.	547	10	2
5.	240	6	9	10.	679	13	3



## SHORT METHODS IN PERCENTAGE

The principles of aliquot parts may be used to advantage in many operations in percentage, and the percentages of the following table with their equivalent fractions should be thoroughly learned.

PER CENT.	DECIMAL VALUE	FRACT'NAL VALUE	FRACT'NAL VALUE IN LOWEST TERMS	PER CENT.	DECIMAL VALUE	FRACT'NAL VALUE	FRACT'NAL VALUE IN LOWEST TERMS
1%	.01	$\frac{1}{100}$	$\frac{1}{100}$	22 $\frac{2}{3}$ %	.22 $\frac{2}{3}$	$\frac{22\frac{2}{3}}{100}$	$\frac{2}{9}$
1 $\frac{1}{4}$ %	.01 $\frac{1}{4}$	$\frac{1\frac{1}{4}}{100}$	$\frac{1}{80}$	28 $\frac{1}{4}$ %	.28 $\frac{1}{4}$	$\frac{28\frac{1}{4}}{100}$	$\frac{2}{7}$
1 $\frac{2}{3}$ %	.01 $\frac{2}{3}$	$\frac{1\frac{2}{3}}{100}$	$\frac{1}{60}$	31 $\frac{1}{4}$ %	.31 $\frac{1}{4}$	$\frac{31\frac{1}{4}}{100}$	$\frac{5}{16}$
2 $\frac{1}{2}$ %	.02 $\frac{1}{2}$	$\frac{2\frac{1}{2}}{100}$	$\frac{1}{40}$	33 $\frac{1}{3}$ %	.33 $\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}$	$\frac{1}{3}$
3 $\frac{1}{3}$ %	.03 $\frac{1}{3}$	$\frac{3\frac{1}{3}}{100}$	$\frac{1}{30}$	37 $\frac{1}{2}$ %	.37 $\frac{1}{2}$	$\frac{37\frac{1}{2}}{100}$	$\frac{3}{8}$
6 $\frac{1}{4}$ %	.06 $\frac{1}{4}$	$\frac{6\frac{1}{4}}{100}$	$\frac{1}{16}$	42 $\frac{6}{7}$ %	.42 $\frac{6}{7}$	$\frac{42\frac{6}{7}}{100}$	$\frac{3}{7}$
6 $\frac{2}{3}$ %	.06 $\frac{2}{3}$	$\frac{6\frac{2}{3}}{100}$	$\frac{1}{15}$	43 $\frac{3}{4}$ %	.43 $\frac{3}{4}$	$\frac{43\frac{3}{4}}{100}$	$\frac{7}{16}$
8 $\frac{1}{3}$ %	.08 $\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$	$\frac{1}{12}$	50%	.50	$\frac{50}{100}$	$\frac{1}{2}$
9 $\frac{1}{11}$ %	.09 $\frac{1}{11}$	$\frac{9\frac{1}{11}}{100}$	$\frac{1}{11}$	56 $\frac{1}{4}$ %	.56 $\frac{1}{4}$	$\frac{56\frac{1}{4}}{100}$	$\frac{9}{16}$
10%	.10	$\frac{10}{100}$	$\frac{1}{10}$	62 $\frac{1}{2}$ %	.62 $\frac{1}{2}$	$\frac{62\frac{1}{2}}{100}$	$\frac{5}{8}$
11 $\frac{1}{9}$ %	.11 $\frac{1}{9}$	$\frac{11\frac{1}{9}}{100}$	$\frac{1}{9}$	66 $\frac{2}{3}$ %	.66 $\frac{2}{3}$	$\frac{66\frac{2}{3}}{100}$	$\frac{2}{3}$
12 $\frac{1}{2}$ %	.12 $\frac{1}{2}$	$\frac{12\frac{1}{2}}{100}$	$\frac{1}{8}$	68 $\frac{3}{4}$ %	.68 $\frac{3}{4}$	$\frac{68\frac{3}{4}}{100}$	$\frac{11}{16}$
14 $\frac{2}{7}$ %	.14 $\frac{2}{7}$	$\frac{14\frac{2}{7}}{100}$	$\frac{1}{7}$	75%	.75	$\frac{75}{100}$	$\frac{3}{4}$
16 $\frac{2}{3}$ %	.16 $\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$	$\frac{1}{6}$	81 $\frac{1}{4}$ %	.81 $\frac{1}{4}$	$\frac{81\frac{1}{4}}{100}$	$\frac{13}{16}$
18 $\frac{3}{4}$ %	.18 $\frac{3}{4}$	$\frac{18\frac{3}{4}}{100}$	$\frac{3}{16}$	83 $\frac{1}{3}$ %	.83 $\frac{1}{3}$	$\frac{83\frac{1}{3}}{100}$	$\frac{5}{6}$
20%	.20	$\frac{20}{100}$	$\frac{1}{5}$	87 $\frac{1}{2}$ %	.87 $\frac{1}{2}$	$\frac{87\frac{1}{2}}{100}$	$\frac{7}{8}$
25%	.25	$\frac{25}{100}$	$\frac{1}{4}$	93 $\frac{3}{4}$ %	.93 $\frac{3}{4}$	$\frac{93\frac{3}{4}}{100}$	$\frac{15}{16}$

## SERIES 69

Find

		\$	\$	\$
1.	20% profit on	5,	25,	45
2.	25% loss on	4,	36,	76
3.	4% commission on	25,	75,	125
4.	$12\frac{1}{2}\%$ interest on	64,	96,	160
5.	$16\frac{2}{3}\%$ duty on	6,	36,	72
6.	$8\frac{1}{3}\%$ discount on	12,	72,	60
7.	$37\frac{1}{2}\%$ premium on	80,	32,	48
8.	$66\frac{2}{3}\%$ advance on	9,	27,	75
9.	$6\frac{1}{4}\%$ brokerage on	32,	64,	256
10.	$31\frac{1}{4}\%$ assessment on	48,	80,	144
11.	$87\frac{1}{2}\%$ dividend on	16,	72,	108
12.	$22\frac{2}{9}\%$ tax on	27,	45,	63
13.	$28\frac{4}{7}\%$ rebate on	21,	35,	56
14.	$7\frac{9}{13}\%$ allowance on	26,	39,	78
15.	75% of the value of	24,	32,	28
16.	90% of the value of	70,	110,	40
17.	$31\frac{1}{4}\%$ of the value of	86,	475,	373
18.	$43\frac{3}{4}\%$ of the value of	374,	228,	937
19.	50% of the value of	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{9}$
20.	125% of the value of	7.50	375	

In addition to the methods suggested by the table, such as dividing by 4 to get 25%, dividing by 3 to get  $33\frac{1}{3}\%$ , and so on, a number of percentages can be rapidly calculated from the 10% base. In billing, the student should cultivate the habit of writing the percentage on the paper direct, without carrying the work to a scratch pad and back to the bill.

## ILLUSTRATIONS

1. 10% of \$747.25 = \$74.73.

Move the decimal point one place to the left, getting \$74.725. If the mills are 5 or over, add one cent; if less than 5 mills, drop them altogether. Thus, to the nearest cent, the answer is \$74.73.

2. 20% of \$747.25 = \$149.45.

See first what 10% would be, \$74.725. Taking twice this figure to the nearest cent, we have \$149.45.

3. 30% of \$747.25 = \$224.18.

Again, 10% = \$74.725 ; 30% is three times the amount, or \$224.18. In this way 40%, 50%, 60%, 70%, 80%, or 90% of any number may be immediately written.

4.  $2\frac{1}{2}\%$  of \$747.25 = \$18.68.

As before, 10% = \$74.725 ;  $2\frac{1}{2}$  is  $\frac{1}{4}$  of 10%,  $\frac{1}{4}$  of \$74.725 = \$18.68.

In other words, see 10% first and divide by 4.

NOTE.—In placing the first figure see if the division, if carried out another place, would give 5 mills or over. If so, follow the rule of adding an extra cent.

5.  $3\frac{1}{3}\%$  of \$747.25 = \$24.91.

See 10% first and divide by 3.

6. What is 36% of \$2,500 ?

SOLUTION.—Since 36 times 25 will give the same product as 25 times 36, 36% of \$2,500 will give the same result as 25% of \$3,600. 25% is  $\frac{1}{4}$  of a number ; therefore,  $\frac{1}{4}$  of \$3,600, or \$900, is the required result.

7. What is 16% of \$12,500 ?

SOLUTION.—16 times  $12\frac{1}{2}$  will give the same product as  $12\frac{1}{2}$  times 16 ; hence, 16% of \$12,500 is equivalent to  $12\frac{1}{2}\%$  of \$16,000.  $12\frac{1}{2}\%$  is  $\frac{1}{8}$ .  $\frac{1}{8}$  of \$16,000 is \$2,000, or the required result.

8. What is 24% of \$37,500 ?

SOLUTION.—24 times  $37\frac{1}{2}$  will give the same product as  $37\frac{1}{2}$  times 24 ; hence, 24% of \$37,500 is equivalent to  $37\frac{1}{2}\%$  of \$24,000.  $37\frac{1}{2}\%$  is  $\frac{3}{8}$ .  $\frac{3}{8}$  of \$24,000 is \$9,000, or the required result.

### SERIES 70

Calculate the following mentally, giving result correct to nearest cent :

1. 10 % of \$ 358.78

2. 10 % of 182.34

3. 10 % of 526.45

4. 20 % of 346.28

5. 30 % of 156.97

6. 40 % of 358.79

7. 5 % of 342.87

8.  $2\frac{1}{2}\%$  of 1,368.74

9.  $3\frac{1}{3}\%$  of \$ 438.73

10.  $1\frac{1}{4}\%$  of 8,326.75

11. 25 % of 3,827.49

12. 3 % of 1,437.52

13. 26 % of 2,500.00

14. 18 % of 12,500.00

15.  $\frac{1}{4}\%$  of 3,742.85

16.  $33\frac{1}{3}\%$  of 3,284.95

In the following questions calculate discounts mentally :

17. What is \$928.65 less 10%, 10%, and 5% ?
18. What is \$396.17 less 20%, 10%, and 2% ?
19. What is \$432.50 less 15%,  $3\frac{1}{3}\%$ , and  $2\frac{1}{2}\%$  ?
20. What is \$342.16 less 30%, 3%, and  $\frac{1}{4}\%$  ?
21. What is \$1,754.64 less  $33\frac{1}{3}\%$ ,  $12\frac{1}{2}\%$ , and 16% ?
22. What is \$5,438.79 less 25%, 40%, and  $2\frac{1}{2}\%$  ?
23. What is \$1,927.46 less 20%, 14%, and  $\frac{1}{3}\%$  ?
24. What is \$398.79 less 30%, 6%, and  $1\frac{1}{4}\%$  ?
25. What is \$1,827.54 less  $12\frac{1}{2}\%$ ,  $6\frac{1}{4}\%$ , and  $\frac{1}{2}\%$  ?

### The Six per Cent Method of Calculating Interest

This method is called the six per cent. method from the fact that, no matter what the given rate is, the interest is computed first at six per cent., and then changed to what it would be at the given rate.

Also, in computing interest by the six per cent. method, the amount is calculated first on the basis of a 360-day year, and then changed to the basis of a 365-day year.

Then, the interest for 360 days being 6% of the sum, the interest for 60 days is 1% of the sum, and is got by dividing by 100, which simply shifts the decimal point two places to the left.

The interest for 60 days is taken as the basis of the operation.

ILLUSTRATION.—Find the interest on \$540 for 49 days at 6%, calculated on the basis of 360 days to the year.

The interest for 60 days = \$5.40

The interest for 30 days = \$2.70 (Half as much, since time is halved).

The interest for 10 days = \$0.90 (One-third as much as for 30 days).

The interest for 6 days = \$0.54 (One-fifth as much as for 30 days).

The interest for 3 days = \$0.27 (One-half as much as for 6 days).

---

The interest for 49 days = \$4.41 (Sum of amount for 30 days, 10 days, 6 days, and 3 days).

It will be noted that in the illustration above we start with the interest for 60 days, and from it build up the interest for 49 days by taking, in each

case, a number of days which is an exact factor of some preceding number of days. Thus, 30 is  $\frac{1}{2}$  of 60, 10 is  $\frac{1}{3}$  of 30, 6 is  $\frac{1}{5}$  of 30, and 3 is  $\frac{1}{2}$  of 6.

The number 60 has so many factors that any number whatever can be built up in this way. Thus, had the time been 28 days, we would have got the sum of the interest for 20 days ( $\frac{1}{3}$  of 60 days), 6 days ( $\frac{1}{10}$  of 60 days), and 2 days ( $\frac{1}{3}$  of 6 days). Had it been 57 days, we would have got the sum of the interest for 30 days ( $\frac{1}{2}$  of 60 days), 15 days ( $\frac{1}{2}$  of 30 days), and 12 days ( $\frac{1}{5}$  of 60 days), and so on.

Of course, in building up the amount, a great many different groupings of the number of days may be formed. Thus, in the last case we might have taken 30 days ( $\frac{1}{2}$  of 60 days), 20 days ( $\frac{1}{3}$  of 60 days), 6 days ( $\frac{1}{5}$  of 30 days or  $\frac{1}{6}$  of 60 days), and 1 day ( $\frac{1}{6}$  of 6 days); or we might have taken 30 days, 20 days, 5 days ( $\frac{1}{4}$  of 20 days), and 2 days ( $\frac{1}{5}$  of 10 days), and so on.

The facility in performing the operation lies in the wisdom displayed in making the groupings. This is got only from practice.

If the time is over 60 days, we would first take out the largest multiple of 60 days possible. Thus, had the time in our illustration been 267 days, we would have taken the sum of the interest for 240 days (4 times 60 days), 20 days ( $\frac{1}{3}$  of 60 days), 6 days ( $\frac{1}{10}$  of 60 days), and 1 day ( $\frac{1}{6}$  of 6 days).

Having got the interest at 6%, the next step is to change it to what it would be at the given rate. Thus, in our illustration, had the rate been 4%, we would proceed as follows :

Interest at 6% =	\$4.41	
Interest at 3% =	2.205	( $\frac{1}{2}$ as much as at 6%).
Interest at 1% =	.735	( $\frac{1}{3}$ as much as at 3%).
Interest at 4% =	2.94	(Sum of interest at 3% and at 1%).

Or

Interest at 6% =	\$4.41	
Interest at 2% =	1.47	( $\frac{1}{3}$ as much as at 6%).
Interest at 4% =	2.94	(Interest at 6% - interest at 2%).

It will be noted that the interest at the given rate is built up from the interest at 6% in the same manner as the interest for the given number of days is built up from the interest for 60 days.

Having got the interest for the given time at the given rate, on a basis of 360 days to the year, it remains to change it to what it would be on the basis of a 365-day year.



Now, the amount, \$2.94, could be got by simplifying the following expression :

$$\$540 \times \frac{4}{100} \times \frac{49}{360}$$

The expression necessary to give the correct amount of interest reckoned at 365 days to the year, is

$$\$540 \times \frac{4}{100} \times \frac{49}{365}$$

The latter expression can be got from the former one by multiplying it by  $\frac{360}{365}$ . The 360 in the numerator will cancel the 360 in the denominator, throwing the 365 into the denominator. Thus :

$$\$540 \times \frac{4}{100} \times \frac{49}{360} \times \frac{360}{365}$$

From which we see that, to change the amount of interest reckoned on the basis of a 360-day year to the amount reckoned on the basis of a 365-day year, we must multiply the amount by  $\frac{360}{365}$ ; or, which is the same thing, multiply it by  $\frac{72}{73}$ .

Now, the easiest way to multiply any amount by  $\frac{72}{73}$  is to subtract  $\frac{1}{73}$  of the amount ( $\frac{73}{73} - \frac{1}{73} = \frac{72}{73}$ ).

ILLUSTRATION.—Find the interest on \$397.85 for 223 days at 7%.

SOLUTION.—It will be observed from the foregoing that there are three steps to the solution :

- (a) Get the interest on the amount for 223 days at 6%, reckoned on the basis of a 360-day year.
- (b) Change the amount of interest to what it would be at the given rate.
- (c) Change the amount of interest to what it would be reckoned on the basis of a 365-day year.

(a) Interest for 60 days at 6% = \$ 3.9785

Interest for 180 days at 6% = 11.9355 (3 times amt. for 60 days).

Interest for 30 days at 6% = 1.9892 + ( $\frac{1}{2}$  of amt. for 60 days).

Interest for 10 days at 6% = .6631 + ( $\frac{1}{3}$  of amt. for 30 days).

Interest for 3 days at 6% = .1989 + ( $\frac{1}{10}$  of amt. for 30 days).

Interest for 223 days at 6% = 14.7867 +

(b) Interest at 6% = 14.7867 +

Interest at 1% = 2.4644 + ( $\frac{1}{6}$  of amt. at 6%)

Interest at 7% = 17.2511 + (amt. at 6% + amt. at 1%).

(c) \$17.2511 -  $\frac{1}{73}$  of \$17.2511 = \$17.2511 - \$.2363  
= \$17.0148 = \$17.01.

In calculating the amount, where the numbers do not divide evenly, allow two decimal places past the cents' figures, but no further, as that is enough to give the amount correct to the nearest cent.

NOTE 1.—In reckoning interest for an exact number of months, or for a number of months and days where the particular months are not known, allow 30 days to the month, and do not take off the  $\frac{1}{73}$  of the amount; since, if a month is considered as 30 days, the year would be 360 days.

NOTE 2.—In reckoning interest for one or more years, together with an exact number of days, allow 365 days for the whole year or years, and deduct  $\frac{1}{73}$  from the whole amount.

### SERIES 71

By the six per cent. method find the interest on

Principal.	Time in dys.	Rate.	Principal.	Time in dys.	Rate.
1. \$ 450	33	6 %	15. \$ 385.60	32	8 %
2. 920	48	6 %	16. 438.15	206	6 %
3. 630	58	6 %	17. 926.50	111	4 %
4. 385	226	5 %	18. 457.29	87	3 %
5. 875	152	4 %	19. 837.45	289	3½%
6. 1,450	47	6 %	20. 187.28	156	9 %
7. 1,385	269	7 %	21. 1,368.19	43	5 %
8. 429	88	6 %	22. 436.73	251	4 %
9. 1,500	59	5 %	23. 15.28	117	8 %
10. 325	125	3 %	24. 187.65	17	5 %
11. 1,280	143	8 %	25. 348.27	46	3½%
12. 850	79	4 %	26. 1,425.36	325	5½%
13. 338	148	3 %	27. 129.37	293	6½%
14. 540	329	4½%			

28. \$650 from May 13, 1907, to Sept. 18, 1907, at 6%.

29. \$3,260 from Jan. 9, 1908, to Aug. 1, 1908, at 7%.

30. \$346.75 from Sept. 3, 1907, to May 10, 1908, at 4%.

31. \$293.26 from April 12, 1907, to May 28, 1908, at 5%.

32. \$914.68 from July 27, 1906, to April 13, 1908, at 3%.

33. \$536.27 from Nov. 23, 1903, to July 11, 1907, at 3½%.

34. \$1,368.45 from Dec. 14, 1904, to May 25, 1908, at 4%.

35. \$459.38 from Aug. 17, 1902, to Jan. 3, 1907, at 5%.

36. \$98.67 from Oct. 30, 1905, to July 14, 1908, at 6½%.

## BILLING

A **Bill** or **Invoice** is a detailed statement of merchandise sold or services rendered. A bill or invoice of goods sold usually gives the following information : Place and date of the sale ; the names of the buyer and the seller ; the terms of sale ; the identifying marks, if any, of case, package, barrel, etc., in which goods are shipped ; the quantity, name, and price of each article ; the extension of each item ; and the total amount of the whole bill.

A bill or invoice is receipted by writing at the bottom the words, "Received payment," followed by the signature of the seller of the goods or someone authorized to sign for him.

A **Credit Note** is a bill or invoice used as an offset to a previous bill or invoice. Thus, where goods are once billed and part of the goods are returned, the credit note is intended to show the items which were returned and for which credit is being given.

A **Statement of Account** is an exhibit of the dates and totals of the bills or invoices for a period, say of a month. A statement may be receipted as a bill or invoice is receipted.

The **Work of Billing** requires good writing and quick and accurate figuring. There is no commoner introduction to the general work of any office than this work of billing. Any young man or woman contemplating engaging in office work should, therefore, make certain that the requisites for success are thoroughly mastered. The rapid figuring will require not only a thorough knowledge of the simple rules, vulgar and decimal fractions, but also the application of many short rules, which will be found to both quicken and simplify the work. A bill clerk should also see to it that in every case possible extensions are made direct, without the necessity of working the extension on a scratch pad, and then transferring the result to the bill or invoice. We commend the preceding work on short rules and business methods to the most earnest attention of all students.

## Commercial Abbreviations

A 1 . . . . . First quality.	Ea. . . . . Each.	Net. Without discount.
Acct. . . . . Account.	E. & O.E. . . . . { Errors and omissions excepted.	N.B. . . . . { Take particular notice.
Ag't. . . . . Agent.	Exch. . . . . Exchange.	No. or # . . . . . Number.
Amt. . . . . Amount.	Ex. . . . . Without.	Nov. . . . . November
Apr. . . . . April.		
Arr. . . . . Arrived.		
Ass'd. . . . . Assorted.		
Aug. . . . . August.		
	Feb. . . . . February.	Oct. . . . . October.
Bal. . . . . Balance.	Fig'd. . . . . Figured.	Oz . . . . . Ounces.
Bbl. . . . . Barrel.	F.O.B. Free on Board.	
Bdls. . . . . Bundles.	Fol. . . . . Folio or page.	P. . . . . Page.
Bds. . . . . Boards.	Fr't. . . . . Freight.	Pp. . . . . Pages.
Bgs. . . . . Bags.	Ft. . . . . Foot or feet.	Pay't or } . . . . . Payment.
Bkts. . . . . Baskets.		Paym't } . . . . . Paid.
Blk. . . . . Black.	Gal. . . . . Gallon.	Pd. . . . . Paid.
Bls. . . . . Bales.	Gr. or gro. . . . . Gross.	Per, p., or ₤ { By, or by the.
Bot. . . . . Bought.	Guar. . . . . Guarantee.	Pkgs. . . . . Packages.
Brot. . . . . Brought.		P.O. . . . . Post Office.
Bu. . . . . Bushels.		Pr. . . . . Pair.
Bxs. . . . . Boxes.	Hlf. . . . . Half.	Prox. . . . . Next Month
	Hhd. . . . . Hogshead.	Ps. . . . . Pieces.
		Pts. . . . . Pints.
C. . . . . Hundred.	i.e. . . . . That is.	Qr. . . . . Quarter
c. or cts. . . . . Cents.	In. . . . . Inches.	Qts. . . . . Quarts.
Chgd. . . . . Charged.	Ins. . . . . Insurance.	
Ch. . . . . Chest.	Inst. . . . . Present month.	Rec'd } . { Received
Co. . . . . Company.	Int. . . . . Interest.	paym't } . { payment.
C.O.D. . . . . { Collection		Rec't. . . . . Receipt.
Col'd. . . . . Colored.	Jan. . . . . January.	R.R. . . . . Railroad
Com. . . . . Commission.	Jr. or Jun. . . . . Junior.	
Cr. . . . . Creditor.		s. . . . . Shilling.
Cs. . . . . Cases.	Lbs. . . . . Pounds.	Shipt. . . . . Shipment.
Cwt. . . . . Hundredweight.	Ledg. . . . . Ledger.	Stor. . . . . Storage.
		Str. . . . . Steamer.
d. . . . . Pence.	M. . . . . Thousand.	Sunds. . . . . Sundries.
Dec. . . . . December.	Mar. . . . . March.	Super. . . . . Superfine.
Dft. . . . . Draft.	Mdse. . . . . Merchandise.	
Disct. . . . . Discount.	Mem. . . . . Memorandum.	Trcs. . . . . Tierces.
Do., ditto, or " { The same.	Messrs. . . . . { Gentlemen or Sirs.	Ult. . . . . Last month.
Doz. . . . . Dozen.	Mo. . . . . Month.	
Dr. . . . . Debtor.	Mr. . . . . Master or Minister.	Wt. . . . . Weight.
Ds. or da. . . . . Days.	Mrs. . . . . Mistress.	
		Yds. . . . . Yards.
		Yr. . . . . Year.

## Commercial Characters

@.....At.	\$ ..... Dollars.	o/c..... Old account.
a/c..... Account.	"..... Ditto, or the same.	per..... Per, or by the.
c..... Cents.	%..... Hundredths.	£..... Pounds sterling.
c/o..... Care of.	n/c..... New account.	× .. By, as 7 × 8 inches.
✓..... Check mark.	# ..... Number.	

## Commercial Expressions

$7/4$  English shillings and pence are often written this way. This character stands for 7 shillings, 4 pence.

June 14/77. The nominal due date and the legal due date are often expressed as indicated.

Toronto, 4/11/08. A date is shortly expressed as indicated, the illustration meaning March 11, 1908. Some prefer to write it 11/4/08.

$6^1$  yds.,  $7^2$  yds.,  $8^3$  yds., respectively mean  $6\frac{1}{4}$ ,  $7\frac{1}{2}$ , and  $8\frac{3}{4}$  yards.

$15^{10}$  bushels means 15 bushels and 10 pounds.

$20^6$  pounds means 20 pounds 6 ounces.

15 doz.,  $\frac{5}{\$8}$ ,  $\frac{5}{\$9}$ ,  $\frac{5}{\$10}$  means 15 doz., 5 doz. at \$8 a dozen, 5 doz. at \$9 a dozen, and 5 doz. at \$10 a dozen.

2 bbls.  $\frac{240 - 18}{250 - 20}$  means 2 barrels, one of 240 lbs. gross weight, from which 18 lbs. is taken for weight of barrel, and one of 250 lbs., from which is taken 20 lbs. for barrel.  
or  
2 bbls.  $\frac{240}{18}$ ,  $\frac{250}{20}$

Terms: 5/30, 3/60, n/90. On an invoice this means that if paid in 30 days 5% is allowed; if paid in 60 days, 3% is allowed; and the bill must be paid, in any event, with no discount in 90 days.

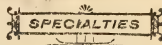


F. ROBBINS SMITH, PRESIDENT AND TREASURER

J. M. HUNGERFORD, VICE-PRESIDENT AND SECRETARY



*J. M. Hungerford Smith Co.*  
LIMITED



MANUFACTURERS OF SODA FOUNTAIN REQUISITES.

TRUE FRUIT FOUNTAIN SYRUPS  
TRUE FRUIT CRUSHED FRUITS  
TRUE FRUIT FLAVORING EXTRACTS

FACTORY: 21 AND 23 ALICE STREET

Toronto, Ont.,

April 30, 1908.

Sold to Mr. James Robins, 816 College St..

TERMS 30 DAYS NET. 2% 10 DAYS

BILLS NOT PAID IN THIRTY DAYS ARE SUBJECT TO SIGHT DRAFT.

City.

WE ARE NOT RESPONSIBLE FOR ANY DAMAGE OR LOSS IN TRANSPORTATION. ANY CLAIM FOR DAMAGE, BREAKAGE OR DELAY SHOULD BE MADE TO THE R. R. COMPANY. ALL OTHER CLAIMS MUST BE MADE WITHIN FIVE DAYS AFTER RECEIPT OF GOODS.

4	Gals. Pineapple	2.50	10 00	
1	" Raspberry		2 50	
7	" Claret	2.50	17 50	
3/12	Doz. Nut Frappe	24.00	6 00	
3/12	" Cherry, Cr.	16.00	4 00	
3/12	" Raspberry	14.00	3 50	
3/12	" Cherry, Whole	17.00	4 25	
30#	(1 Case) Ice Cream Powder	.32	9 60	
			\$57 35	
			11 47	\$45 88
1	Gal. Ext. Vanilla		8 00	
1	" " Lemon		10 00	
			18 00	
			1 80	16 20
				\$32 08

SPECIMEN INVOICE

## CANADA CYCLE AND MOTOR CO.,

TORONTO JUNCTION, ONT.

LIMITED.

DATE

May 1, 1908

SOLD TO

Mr. D. C. Andrews

ADDRESS

Halifax, N.S.

VIA

CPR Car 233546

Terms

Russell Model N 522 Engine 645			
Regular equipment	25 00	00	
Extras-			
Extension Cape Cart Top	1 03	00	
Special Side Curtains	15	00	
Tilting Glass Front	60	00	
Envelope for top	12	50	
Brass Foot Rail	5	50	
Brass Coat Rail	3	00	
Brass Double Set Tire Irons	12	50	
No 3 Gabriel Horn	30	00	
Type No 1 Smith Speed Indicator	75	00	28 16 50

SPECIMEN INVOICE

# THE DAVIDSON & HAY LIMITED

IMPORTERS AND WHOLESALE GROCERS

36 YONGE STREET

## CASH DISCOUNTS

3 MO'S GOODS

3 PER CENT IN TEN DAYS

30 DY'S GOODS

1 PER CENT. IN TEN DAYS

AFTER 15 DAYS, INTEREST  
ALLOWED AT RATE OF  
12 PER CENT. PER ANNUM

TORONTO, Sept. 16th 1907

SOLD TO

Mr. A. Miller

389 Church St.

INTEREST AT 7 PER CENT,  
CHARGED ON OVERDUE  
ACCOUNTS

S. R. NO 1094

O. S. NO 1094 CONVEYANCE

TELEPHONE CONNECTING ALL DEPARTMENTS: MAIN 7270

		Quantity	Price	3 Months Goods	30 Day Goods	Net Cash Goods
1	Roll Manilla paper 12"	17	.34	64		
1	sack McIntosh's Rolled oats 90	1	2.50			2.50
10	lbs Standard Oatmeal	10	.34			.35
1	Dz. Grape nuts	1	1.50			1.50
5	bags Redpaths Gran'l't'd sugar ea. 100 lbs	5	4.45		22.25	
				.64	22.25	4.35

## SPECIMEN INVOICE

## CREDIT NOTE

## CANADA CYCLE AND MOTOR CO.,

MANUFACTURERS OF

LIMITED

Bicycles, Motor Vehicles, Accessories and Skates

CREDITED TO Mr. R. M. Dunn,

CREDIT ORDER

NO A 368

Toronto, Ont.

PARTS AND ACCESSORIES DEPARTMENT

May 1st, 1908. 190

Quantity	DESCRIPTION	Price	Time	Parts	Accessories	TOTAL
11	Pcs Aluminum Castings, 5½ lbs. lb.	.80		4 20		4 20

## SPECIMEN CREDIT NOTE

315		Toronto,		Dec. 2, 1907		190.....	
The Elliott Stationery Co.							
485 Yonge St.							
In Account with		R. G. McLEAN					
HIGH GRADE PRINTING							
DATE		CHARGES		CREDIT		BALANCE	
Nov.	1	Acct. RENDERED	1235 40				
	5	By Cash		300 00		935 40	
	1	3/30	4 46				
	6	Net	4 87				
	7	"	2 31				
	9	"	68				
	12	"	2 45				
	19	3/30	9 25				
	28	Net	70 33				
	"	"	25 38				
	"	"	1 80				
	25	"	78 30				
	"	"	61 63				
	"	"	52 20				
	29	"	73 86			387 52	
						1322 92	

Received payment Dec. 5, 1907  
R. G. McLEAN, Per E. F. G.

SPECIMEN STATEMENT

## SERIES 72

SANDERSON'S LTD., 66 Wellington Street West, Toronto, Ont., sell the following goods:

1. To W. J. Ross, 98 Carlton Street, Toronto, Ont. Terms cash.

1 doz. Belts at \$2.25;  $\frac{1}{2}$  doz. Belts, Bro., at \$4.50;  $\frac{1}{2}$  doz. Belts, Tan, at \$4.50;  $\frac{1}{2}$  doz. Belts, Lt. Tan, at \$4.50; 3 doz. Turnovers at 75c.

2. To Duncan Ferguson, Stratford, Ont. Terms, 30 days net, 10 days 2%.

1 pc. Point d'Esprit, 21 yds., at 38c; 1 pc. Allover Lace, 11 $\frac{3}{4}$  yds., at 45c; 1 pc. Allover Lace, 4 $\frac{1}{2}$  yds., at 60c; 1 pc. Allover Lace, 5 yds., at 60c; 1 pc. Guipure, 23 $\frac{1}{4}$  yds., at 25c; 1 pc. Guipure, 29 $\frac{1}{4}$  yds., at 18c; 1 pc. Guipure, 23 $\frac{1}{4}$  yds., at 25c; 1 pc. Guipure, 23 $\frac{1}{4}$  yds., at 30c; 1 pc. Veiling, 21 $\frac{7}{8}$  yds., at 30c; 1 pc. Veiling, 24 $\frac{1}{8}$  yds., at 30c.

3. To Mickleborough & Co., Ltd., St. Thomas, Ont. Terms, net 30 days, 2% ten days.

1 pc. Insertion, 21 $\frac{1}{2}$  yds., at 16c; 1 pc. Insertion, 22 yds., at 25c; 1 pc. Insertion, 23 $\frac{1}{4}$  yds., at 25c; 1 pc. Insertion, 14 $\frac{1}{4}$  yds., at 72c; 1 pc. Insertion, 21 $\frac{1}{2}$  yds., at 16c; 1 pc. Insertion, 22 yds., at 25c; 1 pc. Insertion, 21 $\frac{1}{2}$  yds., at 35c; 1 pc. Insertion, 13 $\frac{3}{4}$  yds., at 35c; 1 pc. Insertion, 29 $\frac{1}{4}$  yds., at 35c; 1 pc. Insertion, 23 $\frac{1}{4}$  yds., at 12c; 1 pc. Insertion, 23 $\frac{1}{4}$  yds., at 38c; 1 pc. Insertion, 23 $\frac{1}{4}$  yds., at 45c.

4. To J. White & Co., Woodstock, Ont. Terms, 30 days net, 2% ten days.

1 pc. Guipure, 23 $\frac{1}{4}$  yds., at 35c; 1 pc. Guipure, 20 yds., at 12c; 1 pc. Guipure, 23 $\frac{1}{4}$  yds., at 30c; 1 pc. Guipure, 20 yds., at 16c; 1 pc. Frilling, 1 doz., at \$1.35; 1 pc. Lace, 3 doz., at 75c; 1 pc. Lace, 3 doz., at 90c; 1 pc. Lace, 3 doz., at 45c; 1 pc. Lace, 3 doz., at 75c; 1 pc. Lace, 3 doz., at 90c.

5. To Hugh M. Fraser & Co., Vancouver, B.C. Terms, net 30 days, 2% ten days.

1 pc. Veiling, 23 yds., at 18c; 1 pc. Veiling, 21 $\frac{7}{8}$  yds., at 18c; 1 pc. Veiling, 28 $\frac{1}{2}$  yds., at 18c; 1 pc. Veiling, 27 $\frac{1}{8}$  yds., at 18c; 1 pc. Veiling, 25 $\frac{7}{8}$  yds., at 25c; 1 pc. Veiling, 23 yds., at 25c; 1 pc. Flouncing, 9 yds., at 55c; 1 pc. Edging, 22 $\frac{1}{2}$  yds., at 15c; 1 pc. Insertion, 22 $\frac{1}{2}$  yds., at 15c; 1 pc. Edging, 27 yds., at 9c; 1 pc. Edging, 27 yds., at 10c; 1 pc. Edging, 25 $\frac{3}{4}$  yds., at 10 $\frac{1}{2}$ c; 1 pc. Edging, 27 yds., at 8c; 1 pc. Insertion, 27 yds., at 10 $\frac{1}{2}$ c; 1 pc. Beading, 30 yds. at 8c.

A. MILES, 536 Dundas Street, Toronto, Ont., sells the following bills of goods:

6. To J. J. Walsh, 45 Kendal Avenue, Toronto, Ont.

30 pr. Sash D. C. R., at \$1.50; 137 pr. 1 $\frac{3}{4}$  Sash, at \$1; 40 Cellar Sash, at 40c; 24 Casements, at 50c.

7. To F. Armstrong, 296 Berkeley Street, Toronto, Ont.

20 pcs.  $2 \times 6$ —18 Hem., at \$19; 18 pcs.  $2 \times 6$ —16 Hem., at \$17.50; 36 pcs.  $2 \times 4$ —10 Hem., at \$18; 26 pcs.  $2 \times 4$ —16 Hem., at \$18; 10 pcs.  $2 \times 4$ —12 Hem., at \$17.50; 800 ft. 1-in. Hem., at \$17.50; 10 8-in. Cedar Posts, at 30c.

8. To C. J. Powley, 26 Boustead Avenue, Toronto, Ont.

3 pcs.  $2 \times 6$ —16 Hem., at \$17.50; 12 pcs.  $2 \times 4$ —16 Hem., at \$18; 1 pc.  $2 \times 10$ —12 Hem., at \$17.50; 16 ft.  $3 \times 4$  Hand-rail Pine, at 8c; 16 ft. Bottom-rail Pine, at 5c; 24 ft.  $3 \times 4$  Hand-rail Birch, at 12c; 70 ft. Parting Stop, at 60c; I.M.B.C. Shingles, at \$3.40; 100 ft.  $\frac{7}{8} \times 1\frac{3}{8}$  Balusters, at \$1; 24 ft. Bottom-rail Birch, at 7c; 100 ft.  $\frac{7}{8}$  Cove, at 60c.

NOTE.—Balusters, Cove, and Parting Stop sold by the hundred.

CANADA PAINT CO., LTD., 572 William Street, Toronto, Ont., sell the following bills of goods :

9. To Messrs. Evans & Co., Oshawa, Ont.

100 lbs. Imp. Green, Lt. Dry, at 9c; 1 28-lb. box "D" Ultramarine Blue, Dry, at 11c; 100 lbs. Yell. Ochre, Dry, at 2c; 1 B.B.C. B.B. Ven. Red. 400 lbs., at 3c.

10. To Messrs. Johnstone & Co., Yonge Street, Toronto, Ont.

2 bbls. Boiled Oil, 468 lbs.—69, 504 lbs.—81, at 60c per gal.

Oil is figured at 9 lbs. to the gallon.

H. P. ECKARDT & Co., Wholesale Grocers, Cor. Front and Scott Streets, sell the following bills of goods :

11. To J. A. Hopkins, Main Street.

Quan. Price. 90 dys. 30 dys. Net.

2	Cases Raynor Lime Juice. Qts. Ea. 1 dz. ....	22.40	...	...	...	...	...	...	...
1	Case Napanee Red Cherries, 2's, not pitted ..	21.35	...	...	...	...	...	...	...
2	Hf. Cases Monogram Currants, 151-21 .....	130 .07½	...	...	...	...	...	...	...
4	Dz. C. & B. Malt Vinegar. Qts .....	100 1.85	...	...	...	...	...	...	...
3	Bags Rangoon Rice .....	100 .03½	...	...	...	...	...	...	...
5	Cases Corn Flakes .....	2.85	...	...	...	...	...	...	...
2	Dz. No. 27 Brooms .....	2.25	...	...	...	...	...	...	...
1	Dz. Victor Bamboo Handle Brooms .....	3.10	...	...	...	...	...	...	...



## 12. To J. A. Farewell, Cor. Parliament and Carlton Streets.

Quan. Price. 90 dys. 30 dys. Net.

15 Lb. Blue Label Ludella Tea, 1's, Black	....	.20						
10 Lb. Blue Label Ludella Tea, ½'s, Black	....	.21						
10 Lb. Blue Label Ludella Tea, 1's, Mixed	....	.20						
5 Lb. Blue Label Ludella Tea, ½'s, Mixed	....	.21						
8 Dz. Japanese Stove Pipe Varnish	.....	1.20	....					
4 Dz. Swiss Food, 10 cts. size	.....	.95						
6 Cases Challenge Milk	.....	4.05	....					
2 Kegs Ivory Gloss Starch, Large Crystal	.....	200 .07						
15 Lb. Roasted Peanuts Pineapple	.....	.12½						
3 Cases Lyle's Golden Syrup, 2's	.....	48 .14½						
5 Bk. Nonsuch Stove Enamel, No. 1	.....	15 .40	....					
5 Bk. " " " " 2	.....	15 .70	....					

## 13. To A. B. Woodley, 296 Avenue Road, Toronto, Ont.

Quan. Price. 90 dys. 30 dys. Net.

2 Bx. Cowan's Cocoa, 10 cts. size	.....	6 .90	....					
2 Bags McL. Rolled Oats	.....	3.00						
30 Lb. White Sago (bag 10 cts.)	.....	.07½	....					
15 Bags Natural Figs	.....	392 .03½	....					
3 Case Paraffin Candles, 12's	.....	108 .09½	....					
5 Lb. British Navy Tobacco, 10 ct. size	.....	.44						
2 Dz. Nestle's Milk	.....	1.25	....					
2 Cases Heinz New Style Tomato Soup, med. size	.....	4 1.80						

14. LYMAN BROS. & CO., LTD., Wholesale, Toronto, Ont.,  
sell to James Arbuckle, Grafton, Ont.

¼ doz. Calox Tooth Powder, at \$3; ¼ lb. tr. Connii, at 75c (bot. 4c); 1 lb. Acid Oxalic, at 20c; 1 doz. P. D. & Co. Capsules, 60 B., at \$1.70; 1 doz. Orange Wood Sticks, Hoof, at 60c; ¼ lb. Oil Peppermint, at \$4 (bot. 4c); ½ doz. R & G. Cosmetic Tubes, Brown, at \$1.50; 10 lbs. Moth Balls. at 10c; 1 doz. Lyman's Crushed Violet Talc. Powder, at \$2.40; 10 lbs. Alum Powder, at 5c; 3 doz. Tooth Brush, T 46, at \$2; ½ doz. Hair Brush, No. H 242, at \$12; 1/100 Wyeths, C. T., Bland, & Manganese Co., 20c; 1/12 doz. Hot Water Bottle. 3 qt., L. B. & Co., at \$20.

15. THE CORTICELLI SILK CO., LTD., sold to Bews Bros., Hamilton. Terms, 5% 30 days.

5 lb. Gilt Edge, 16 oz., M.T., at \$12.50; 1 lb. Gilt Edge, 12 oz., B.H.T., at \$9; 1½ lb. Gilt Edge, 16 oz. Sp. Sews, at \$13; 10 yds. 2504 Merv., 30", at \$2.50; 3 yds. 100 Satin, 21", at \$1.05.

16. To Gordon, McKay & Co., Ltd., Toronto, Ont. Terms, 3% 30 days.

382 doz. Cort., at 47½c; 5 doz. Cort., at 95c; 5 lbs. G. E. 16 oz. SPL SWS, at \$13; 5 lbs. G.E. 12 oz. B.H.T., at \$9; 12 doz. Bunch Braid, at 32½c; 6 gr. 81 Lama, at \$1.65; 6 gr. Worsted Skirt Prot., at \$3.25; 2 gr. 3 Twill Tape, at 60c.

17. To W. Kerns & Co., Burlington, Ont. Terms, 5% 30 days.

32 doz. Cort., at 47½c; 1 doz. Cort., at \$2; 1½ gr. Worsted Skirt Prot., at \$3.25; 2½ gr. Mohr. Skirt Prot., at \$5.25; 2 doz. 7 Twill Tape, at 40c; 2 doz. 8 Twill Tape, at 42½c; ½ gr. 3 Dress Cord, at \$4.20; ¼ gr. 5072/4 Braid, at \$4.20; ¼ gr. 5072/3 Braid, at \$3.20; 35⅔ yds. 201 Tamiline, at 34c.

18. THE CANADA CYCLE AND MOTOR CO., West Toronto, Ont., sold to R. J. Young, Fredericton, N.B. Terms, net 30 days, 2% cash in 15 days.

1 only 20th Century Gas Lamp, at \$3; 1 doz. No. 82 Bells, at \$3; 6 only Winner Covers, 28", at \$1.75 ea.; 1 doz. Foot Pumps, No. 2, at 50c ea.; 1 roll Patching Rubber, at \$5; 1 doz. No. 13361 Oilers, at 85c. a doz.; 12 sets No. 66 Front Axle Sets, at \$1.25; 1 No. 14 Left Crank, at \$2; 5 pr. Dun. Lam. Rims, S.P.D.R., at \$1.30.

19. To the Canada Cycle and Motor Co., Branch Winnipeg, Man.

21 Front Hubs, M/4, at 70c; 80 pr. Cranks & Sleeves, fitted R. & L., M/A 2, at \$2; 30 pr. R.T. 4" Exp. Pedals, at \$1.25; 26 Seat Post Exp. Sleeves, at 10c; 19 20" Front Locks (C.C.M. Blue), M/A, at \$1.50; 29 Rear Hubs, M/A 70, at \$1.25; 10 22" M/18 Front Forks, Blk., G.S., at \$1.50; 60 23 Tooth Sprockets, M/40, at 50c; 30 4" R. T. Pedals, Exp., at \$1.25; 10 Rear Hubs, M/70 A, at \$1; 600 Rim 1½" Rear Spokes (Racer), at 40c a hundred.

20. THE EBY, BLAIN CO., LTD., Toronto, Ont., sell to Messrs. J. A. Hopkins, North Dovercourt, Ont.

	Price.	30 dys.	Net Cash.	90 dys.
2 kegs Snowflake Baking Soda .....	2.25	.....		
5 doz. Corkscrew Can Openers .....	.85	.....		
4 doz. Imperial Pickles, Mixed .....	1.00	.....		
4 doz. Shino, 5 cts .....	.45	.....		
2 cs. Toasted Corn Flakes .....	2.85	.....		
6 bx. Taylor's Carbolic Soap .....	.40	.....		
10 cs. King Edward Matches .....	3.60	.....		
6 dz. Chloride of Lime, 1's .....	.85	.....		
10 cs. Quaker Puffed Rice .....	2.90	.....		
2 cs. Cowan's Cocoa, $\frac{1}{4}$ 's .....	1.30	.....		
4 dz. Cow Brand Baking Soda, 5 cts. ....	.45	.....		
3 cs. Gold Medal Table Syrup, 2's .....	2.25	.....		
3 dz. Magic Baking Powder, 5 cts .....	.75			.....
2 dz. Red Cross Baking Powder, 5 cts .....	.45			.....
2 dz. Nonsuch Stove Polish, 10 cts. ....	.90			.....
8 lb. Jumbo Roasted Peanuts .....	.15		.....	

CHRISTIE, BROWN & CO., LTD., Biscuit Manufacturers, sell the following bills of goods:

21. To George Porter, Corner Gerrard & Sackville Streets. Terms, 3% dis. for cash.

	Quan.	Price.	Amt.
1 tin      50      Rusks .....	3 $\frac{1}{2}$	16	
2 "      100      Gin. Nuts .....	14 $\frac{3}{4}$	11	
1 "      50      Asso. Sand .....	12	13	
1 "      50      Niagara .....	6 $\frac{3}{4}$	14	
1 "      50      Aby .....	8 $\frac{1}{2}$	10	
1 "      50      Map. Cream .....	5 $\frac{1}{4}$	13 $\frac{1}{2}$	
1 "      50      Rich Traveller .....	8	14	
1 "      50      Animal .....	6	13	
1 "      50      App. Bloss. ....	6 $\frac{1}{2}$	13 $\frac{1}{2}$	
1 "      50      Mol. Snap .....	7 $\frac{1}{2}$	6 $\frac{1}{2}$	
1 "      40      Sodas .....	16 $\frac{1}{2}$	7	

NOTE.—The figures in the second column from left represent cents to be added for cost of tin.

22. To W. Scott, Parkdale. Terms, 3% discount allowed for cash.

			Quan.	Price.	Amt.
1 tin	40	Sodas, $\frac{1}{2}$ lbs.....	2 $\frac{1}{2}$	48	
2 "	100	Fig Drops .....	17	11 $\frac{1}{2}$	
1 "	50	Map. Creams.....	5 $\frac{3}{4}$	13 $\frac{1}{2}$	
1 "	50	F. G. Bread.....	7 $\frac{3}{4}$	10	
1 "	100	Rich Traveller.....	15 $\frac{3}{4}$	14	
2 "	50	Gin. Nut .....	7 $\frac{1}{2}$	11	
1 "	50	Sultana .....	14	13 $\frac{1}{2}$	
1 "	50	Pilot .....	8	7	
2/40 "	80	Sodas .....	33 $\frac{1}{2}$	7	

JOHN SLOAN & Co., Wholesale Grocers, Winnipeg, Man., sell the following bills of goods :

23. To W. Turnbull, 540 Main Street, Winnipeg, Man.

		Quan.	Price.	10 dys.	Net Cash.
1 Cs. Robt. Scotch Marm. ....	4	1.55	.....		
1 " Shell Walnuts .....	55	.26	.....		
10 Lbs. " Almonds .....	10	.33	.....		
1 Hlf. Cs. Curr. 78-9 .....	69	.08	.....		
1 Bbl. Common Salt .....	1	1.40	.....		
1 Cs. E. Army Blkg. ....	3	.75	.....		
10 Lbs. Rape Seed. ....	10	.08	.....		
1 Cs. Def. Syrup, 2's .....	1	2.40	.....		
3 Cs. Magic Bak. Powder, less 5% .....	12	.75	.....		
2 " " " " less 5% .....	8	1.65	.....		
3 Bbls. No. 1 St. L. Gran. Sugar, 346-21, 322-19, 342-19, 1010-59 .....	951	4.50	.....		
1 Cs. Silent Matches, 500's .....	1	5.20	.....		
1 " Cr. of Wheat .....	1	5.75	.....		
1 Bag Pot Barley .....	100	.02 $\frac{1}{2}$	.....		
" Prl. " .....	100	.03 $\frac{1}{2}$	.....		
1 Bx. G. & S. 70/80 Prunes .....	25	.08	.....		
5 Cs. Def. Map. Syrup Mxt., Pts. ....	5	2.50	.....		
5 " " " " " 5's .....	5	3.90	.....		
5 " H. S. Salmon, $\frac{1}{2}$ 's .....	40	1.17 $\frac{1}{2}$	.....		
1 " Gusto .....	1	2.85	.....		
1 " Q. A. Root .....	3	.90	.....		
1 " King Ed. Matches .....	1	3.60	.....		
1 " Upton's H. M. Blk. C. Jam .....	2	2.00	.....		

## 24. To Johnston &amp; Co., Brandon, Man.

	Quan.	Price.	10 dys. Net Cash.
6 Bags Yellow Sugar .....	600	4.25	.....
6 Doz. Def. Marmalade .....	.....	1.15	.....
1 " K. Cust. Powder .....	.....	.90	.....
1 Box Baker's Cocoa, $\frac{1}{4}$ 's .....	12	.43	.....
1 " " " $\frac{1}{2}$ 's .....	6	.43	.....
1 Cs. Rob. Marmalade .....	4	1.55	.....
1 Doz. " Barley .....	.....	2.40	.....
1 Box Durham C. Starch .....	40	.06 $\frac{3}{4}$	.....
1 " Bens. Corn .....	40	.07 $\frac{1}{2}$	.....
5 Cs. Quaker Oats, Family .....	.....	4.50	.....
1 " Comfort Soap .....	.....	3.85	.....
1 " Upt. Marmalade, 1's. ....	2	1.00	.....
2 " Sift Peas, Quaker .....	4	1.27 $\frac{1}{2}$	.....
1 Bbl. Common Salt .....	.....	1.40	.....
1 Cs. Bee Syrup, 5's .....	.....	2.75	.....
1 " " " 2's .....	.....	2.40	.....
1 " Def. Syrup, ,, .....	.....	2.40	.....
3 " Pearline, 1's .....	.....	3.80	.....
2 " " 100's .....	.....	3.70	.....
1 Doz. Cell. Starch .....	.....	1.00	.....
2 Cs. Shred. Wheat .....	.....	5.15	.....
1 " Gusto .....	.....	2.85	.....
1 Brl. Wind. Salt, 100's .....	.....	2.85	.....
1 Cs. King Edward Matches .....	.....	3.60	.....
1 Box Reck. Blue .....	12	.16	.....
1 Cs. Candles, 12's .....	36	.08 $\frac{1}{2}$	.....
1 Box Keen's Blue .....	12	.16	.....
1 Cs. Olives, Stuffed .....	4	1.40	.....
1 Box Clay Pipes .....	.....	.80	.....
1 Cs. Wetley's Mincemeat .....	3	1.00	.....



25. ROBERTSON BROS., LTD., Wholesale Confectioners, Calgary, Alta., sell to J. Gibson. Terms, 3% discount for cash.

	Weight.	Price.		
1 T. T. Mix .....	5	.12		
1 Coco. Taffy .....	18 $\frac{1}{4}$	.07 $\frac{1}{2}$		
2 Valentines .....	....	.50		
1 Peanut Taffy .....	5 $\frac{3}{4}$	.07 $\frac{1}{2}$		
1 Ruby Mandarines .....	5	.22		
1 " Pept. ....	5	.22		
1 Sc. Mints .....	5	.15		
1 Ell. Almonds .....	5	.20		
1 " Crisp .....	5	.20		
1 Ic. Jujubes .....	....	.70		
1 Quakers .....		.55		
1 Grab Bags .....		.35		
1 Drum No. 1 Choc. Culls .....	18	.14		
1 " Cry. Mix. ....	20	.07 $\frac{1}{2}$		
1 Swiss Milk Patties .....		.50		
6 Cameo Bon-bons .....		.35		

J. J. McLAUGHLIN CO., LTD., sell.

26. To George Harris, 346 St. Paul Street. St. Catharines, Ont. Terms, 30 days net, 2% 10 days.

3 gal. Pineapple Syrup, at \$2.50; 3 gal. Lemon Syrup, at \$2.50; 2 gal. Orange Syrup, at \$2.50; 5 gal. Ginger Syrup, at \$2; 3 gal. Strawberry Syrup, at \$2.50; 1/12 doz. Crushed Strawberry, at \$15; 1/12 doz. Crushed Cherry, at \$16; 3/12 doz. Crushed Pineapple, at \$14; 3/12 doz. Crushed Peach, at \$14; 5 lbs. Ext. Vanilla, at \$1.50; 25 lbs. Powdered Chocolate, at 38c. Less 15%.

27. To Messrs. Heath & Robertson, Halifax, N.S. Terms, 30 days net, 2% 10 days.

1/12 doz. Apricot, at \$14; 1/12 doz. Strawberry, at \$15; 1/12 doz. Chop Suey, at \$16; 1/12 doz. Cherry, Whole, at \$17; 5 lbs. (1 tin) Ice Cream Powder, at 35c; 10 lbs. Powdered Chocolate, at 42c; 1 1/2 pt. Fruit Acid, at 50c; 1 M. Straws, at 50c. Less 5%.

28. ISAAC PITMAN & SONS, 31 Union Square West, New York, sell to Copp, Clark & Co., Toronto, Ont. Terms, net cash.

588	Cumulative Speller .....	.40	Less 25%	
3	Bible, roan.....	3.50	" 20%	
3	Prayer Book, roan.....	1.50	" "	
3	German Shorthand .....	.50	" "	
3	" " cloth .....	.60	" "	
1	Church Service, morocco.....	4.00	" 25%	
144	Shorthand Gradus .....	.06	" "	
2	Insurance Office Organ .....	1.50	" "	
6	Office Work In Shorthand .....	.40	" "	
6	" " " cloth ....	.50	" "	
4	" " & Key, 1 vol.....	.60	" "	
3	Church Service, roan .....	3.00	" "	
6	Sel. Am. Authors, cloth .....	.50	" "	
2	Shorthand Writer.....	1.00	" "	
6	Key to Office Work, cloth .....	.40	" "	
12	Cumulative Spel., C.S. Ed.....	.40	" "	
6	Self-Culture .....	.40	" "	

NOTE.—It is the custom of this firm to show every item at net figure, notwithstanding that so many items have the same discount.

29. J G. McILWRAITH & Co., Hamilton, bought of Gordon, McKay & Co., Ltd., Toronto. Terms, 3 mos. 30 ds., 5%.

10 pcs. R. Gingham, 61<sup>2</sup>, 50<sup>1</sup>, 51, 61<sup>3</sup>, 50, 55, 51<sup>3</sup>, 50, 55, 60, at 8c; 5 doz. C. Silk, at 80c; 15 gr. G. Buttons, at \$1.12½; 10 pcs. L. Cotton, 56, 61, 60<sup>1</sup>, 65<sup>3</sup>, 55, 52, 62, 55<sup>2</sup>, 65, 51<sup>3</sup>, at 8¼c; 500 lbs. S. Warp, at 18c; 10 pcs. B. Checks, 41, 45, 40<sup>2</sup>, 45<sup>2</sup>, 50, 55, 42, 55, 55<sup>1</sup>, 45<sup>2</sup>, at 25c; 6 pcs. E. Lining, 50<sup>1</sup>, 52, 40, 55<sup>1</sup>, 52, 41, at 5c; 5 doz. L. L. Gloves, at \$2.95; 5 pcs. A. Cashmere, 62<sup>2</sup>, 60, 51<sup>3</sup>, 55, 60, at 18¾c; 6 pcs. A. L. Cotton, 40, 46<sup>2</sup>, 55, 51<sup>2</sup>, 42<sup>1</sup>, 40, at 4c; 2 pcs. G. Flannel, 60, 65, at 30c.

30. JOSEPH MICKELBOROUGH, St. Thomas, bought of W. R. Brock & Co., Toronto. Terms, draft 30 ds.

10 pcs. B. Checks, 52<sup>1</sup>, 41<sup>2</sup>, 40, 55<sup>3</sup>, 50<sup>2</sup>, 45, 40, 50<sup>3</sup>, 51<sup>1</sup>, 50<sup>2</sup>, at 25c; 15 pcs. A. Flannel, 65<sup>1</sup>, 62<sup>2</sup>, 58, 60<sup>1</sup>, 56<sup>3</sup>, 63, 60<sup>2</sup>, 62, 65<sup>3</sup>, 63<sup>1</sup>, 62<sup>3</sup>, 65<sup>1</sup>, 61, 58<sup>2</sup>, 55, at 33½c; 10 pcs. B. Velvet, 29<sup>3</sup>, 26<sup>2</sup>, 25, 27<sup>3</sup>, 21<sup>2</sup>, 23<sup>2</sup>, 20<sup>3</sup>, 21, 24<sup>3</sup>, 22<sup>2</sup>, at \$6.50; 20 pcs. B. Sheeting, 41<sup>2</sup>, 35, 38<sup>2</sup>, 35, 37<sup>3</sup>, 39<sup>2</sup>, 40<sup>2</sup>, 51, 44<sup>3</sup>, 44<sup>1</sup>, 40, 37<sup>2</sup>, 32<sup>1</sup>, 32<sup>3</sup>, 46<sup>3</sup>, 49<sup>2</sup>, 38<sup>1</sup>, 41<sup>3</sup>, 38<sup>2</sup>, 36<sup>1</sup>, at 6¼c.

31. J. A. DUGGAN, Stratford, bought of J. G. McKenzie & Co., Montreal. Terms, sight draft.

20 pcs. M. Gingham,  $39^3$ ,  $49^1$ ,  $41^3$ , 36,  $47^2$ , 50, 45,  $38^1$ ,  $35^3$ , 46, 42,  $36^1$ , 34,  $32^3$ , 41,  $48^3$ ,  $33^2$ ,  $39^1$ , 36, 34, at  $12\frac{1}{2}c$ ; 20 pcs. R. Gingham,  $55^3$ , 51,  $53^3$ ,  $56^1$ ,  $55^2$ , 58,  $61^2$ ,  $62^1$ ,  $60^3$ , 50,  $52^1$ , 51,  $51^2$ , 55,  $61^2$ , 61,  $58^1$ , 56,  $54^2$ ,  $51^1$ , at  $6\frac{1}{4}c$ ; 10 pcs. Sateen, 50,  $51^3$ , 53,  $52^2$ ,  $35^2$ , 51,  $50^3$ ,  $54^1$ , 56, 55, at  $5\frac{3}{4}c$ ; 4 pcs. C. Denims,  $39^2$ ,  $36^2$ ,  $48^3$ ,  $61^1$ , at  $12\frac{1}{2}c$ .

## CASH STORAGE BILLS

**Storage** is a charge made for storing goods in a warehouse.

**Cash Storage** is that paid or estimated at the time of each withdrawal of goods from store.

**Credit** or **Average Storage** is that paid or estimated at the time of last withdrawal from store.

The **Term of Storage** is the period of time for which storage charges are made.

Rates of storage may be arranged between the parties to the contract, but are frequently regulated by boards of trade or by associations of warehousemen.

Only cash storage bills will be considered here. Average storage will be dealt with under its appropriate heading, as an application of the average principle.

**ILLUSTRATION.**—At a warehouse there was received and delivered merchandise as follows :

RECEIVED.	DELIVERED.
May 6, 300 bbls. flour.	May 23, 200 bbls. flour.
“ 27, 250 “ “	June 3, 80 “ “
June 12, 180 “ “	“ 7, 220 “ “
	“ 25, 230 “ “

How much must be paid for storage on the above at the rate of 5 cents per barrel for the first 10 days or part thereof, and 3 cents per barrel for each subsequent 10 days or part thereof ?

## SOLUTION

<i>Date.</i>	<i>Time of Storage.</i>	<i>Receipts and Deliveries.</i>	<i>Rate.</i>	<i>Storage.</i>
May 6,		received 300 bbl.		
" 23 (17 da., or 2 terms),		delivered 200	" × 8c. =	\$16.00
		Bal. of 1st receipt = 100	"	
June 3 (28 da., or 3 terms),		delivered 80	" × 11c. =	8.80
		Bal. of 1st receipt = 20	"	
" 7 (32 da., or 4 terms),		Of 220, delivered 20	" × 14c. =	2.80
May 27,		received 250	"	
June 7 (11 da., or 2 terms),		Of 220, delivered 200	" × 8c. =	16.00
		Bal. of 2d receipt 50	"	
" 25 (29 da., or 3 terms),		Of 230, delivered 50	" × 11c. =	5.50
" 12,		received 180	"	
" 25 (13 da., or 2 terms),		Of 230, delivered 180	" × 8c. =	14.40
			Storage due,	\$63.50

EXPLANATION.—The first delivery of 200 bbl. was made May 23, or 17 days after the date of the first receipt, constituting one term of 10 days and part of another, or practically 2 storage terms, which, at the given rate of storage (5 cents for first term + 3 cents for second) produces 200 times 8 cents, or \$16 storage. Of the remainder of the first receipt (100 bbl.), 80 bbl. were delivered June 3, or 28 days after the date of their receipt, constituting 2 terms of 10 days and part of another, or 3 storage terms; hence the storage thereon must be 80 times (5 + 3 + 3 =) 11 cents, or \$8.80. The remainder of the first receipt (20 bbl.) was included in the delivery of 220 bbl. on June 7, and hence was stored 32 days, or 4 terms, producing 20 times (5 + 3 + 3 + 3 =) 14 cents, or \$2.80 storage.

The second receipt (250 bbl.) was on May 27, and the next delivery (of 220 bbl.) on June 7, on 20 bbl. of which storage has already been computed, leaving 200 bbl. which were in store 11 days, or 2 terms, producing 200 times 8 cents, or \$16 storage. The remaining 50 bbl. of the second receipt were included in the delivery of 230 bbl. on June 25, after being stored 29 days, or 3 terms, producing 50 times 11 cents, or \$5.50 storage.

The third receipt (180 bbl.) was on June 12, all of which were delivered on June 25 (being included in the delivery of 230 bbl.), after being stored 13 days, or 2 terms, producing 180 times 8 cents, or \$14.40 storage. Hence, the total storage is the sum of the storage on all the deliveries, or \$63.50.

## RULE

1. Multiply each delivery of the first receipt by the rate of storage for the number of terms which such delivery has been in store, considering any fraction of a term as an entire term.

2. Similarly compute the storage on each delivery of the second receipt, and so continue in regular order with subsequent receipts until the storage on all the deliveries has been found.

3. The sum of the storage on all the deliveries will be the required cash storage.

## SERIES 73

1. The receipts and deliveries at a certain warehouse were :

RECEIVED.		DELIVERED.	
Sept. 25,	350 bbls.	Oct. 11,	85 bbls.
Oct. 14,	250 "	" 19,	140 "
" 29,	80 "	" 28,	160 "
Nov. 10,	50 "	Nov. 8,	210 "
		" 16,	40 "
		" 30,	95 "

What was the entire cash storage due Nov. 30, the rate being 5 cents per barrel for the first 10 days or part of 10 days, and 3 cents per barrel for each subsequent 10 days or part of 10 days ?

2. Find the cash storage on the following account, the rate of storage being 6 cents per barrel for the first 10 days or part thereof, and 3 cents per barrel for each subsequent 10 days or part thereof :

RECEIVED.		DELIVERED.	
May 15,	350 bbls.	May 20,	200 bbls.
" 22,	70 "	" 25,	30 "
June 6,	30 "	June 16,	60 "
" 14,	50 "	" 28,	210 "

3. Compute the storage on the following account, the rate being 1 cent per bushel for the first 20 days or part thereof, and  $\frac{1}{2}$  cent per bushel for each subsequent 20 days or part thereof :

RECEIVED.		DELIVERED.	
July 1,	1,600 bu.	July 10,	1,000 bu.
" 15,	3,000 "	" 24,	2,500 "
" 28,	2,500 "	Aug. 7,	1,700 "
Aug. 12,	6,000 "	" 11,	1,300 "
" 27,	1,200 "	" 30,	4,500 "
Sept. 14,	3,400 "	Sept. 25,	6,700 "



# APPLICATIONS OF SIMPLE INTEREST

## NEGOTIABLE PAPERS

**Notes, Cheques, and Drafts** are known as negotiable papers. In business they are handled a great deal. A good knowledge of the law concerning them is absolutely necessary to any one who has anything to do with them from the standpoint of either bookkeeping or arithmetic.

In Canada the law concerning such papers is the same for all Provinces, and is embodied in a Dominion statute, "The Bills of Exchange Act."

### Bills of Exchange

A **Bill of Exchange** is an unconditional order in writing addressed by one person (the drawer) to another (the drawee), signed by the person giving it, requiring the person to whom it is addressed to pay, on demand or at a fixed or determinable future time, a sum certain in money to or to the order of a specified person (the payee) or to bearer.

An **Inland Bill** is a bill which is, or on the face of it purports to be, (a) both drawn and payable within Canada, or (b) drawn within Canada upon some person resident therein. Any other bill is a foreign bill.

### Inland Bill or Draft

YONGE & COLLEGE BRANCH	
\$ 500 <sup>00</sup> / <sub>100</sub>	Toronto Ont. Nov. 21, 1908.
One month after date pay to the order of	
The Canadian Bank of Commerce	
Five Hundred <sup>00</sup> / <sub>100</sub> Dollars	
value received, and charge to the account of	
To D. Roberts & Co. 142 James St. Hamilton, Ont.	R. A. Smith & Co.

The **Acceptance** of a bill is the signification by the drawee of his assent to the order of the drawer. It is written on the bill (usually across the face), and for the bill shown would read :

“Accepted.

D. ROBERTS & Co.”

Bills may be drawn with respect to time in a number of ways, as the following section will show :

A bill is payable on demand :

- (a) Which is expressed to be payable on demand, or on presentation.
- (b) In which no time for payment is expressed.

A bill is payable at a determinable future time, within the meaning of this Act, which is expressed to be payable :

At sight or at a fixed period after date or sight.

Where a bill is not payable on demand, the day on which it falls due is determined as follows :

Three days, called days of grace, are, in every case where the bill itself does not otherwise provide, added to the time of payment as fixed by the bill, and the bill is due and payable on the last day of grace.

Provided that :

(1) Whenever the last day of grace falls on a legal holiday or non-judicial day in the Province where any such bill is payable, then the day next following, not being a legal holiday or non-judicial day in such Province, shall be the last day of grace.

(2) Where a bill is payable at sight, or at a fixed period after date or after sight, the time of payment is determined by *excluding* the day from which the time is to begin to run and by *including* the day of payment.

(3) Where a bill is payable at sight, or at a fixed period after sight, the time begins to run from the date of acceptance. The acceptance of all such bills must, therefore, show date.

(4) The term “ Month ” in a bill means the calendar month.

(5) Every bill which is made payable at a month or months after date becomes due on the same numbered day of the month in which it is made payable as the day on which it is dated—unless there is no such day in the month in which it is made payable, in which case it becomes due on the last day of that *month*—with the addition, in all cases, of the days of grace.

## Cheques

A **Cheque** is a bill of exchange drawn on a bank, payable on demand.

YONGE & COLLEGE BRANCH  
 No. 41 Toronto, Ont. Nov. 21, 1908.  
 To The Canadian Bank of Commerce  
 COR YONGE AND COLLEGE STS.  
 Pay W. R. Brock & Co. or <sup>order</sup> Bearer  
 Twenty <sup>25</sup>/<sub>100</sub> Dollars  
 \$ 20 <sup>25</sup>/<sub>100</sub>  
 A. M. Gough

## Promissory Notes

A **Promissory Note** is an unconditional promise in writing made by one person (the maker) to another (the payee), signed by the maker, engaging to pay, on demand or at a fixed or determinable future time, a sum certain in money, to, or to the order of, a specified person to bearer :

THE BANK OF TORONTO.  
 INCORPORATED 1855.  
 PAID-UP CAPITAL \$ 2000,000.  
 \$ 600 <sup>00</sup>/<sub>100</sub> Due Toronto, Nov. 21, 1908.  
 Three months after date for value received, I promise  
 to pay to A. F. Sproth or order  
 at the office of The Bank of Toronto here, the sum of  
 Six Hundred <sup>00</sup>/<sub>100</sub> Dollars  
 A. M. Gough

### Interest-Bearing Notes

The legal rate of interest in Canada is five per cent. By this it is not meant that a higher rate cannot be agreed upon or collected, but that this is the rate that may be collected where interest is considered by law to be collectible, and still the parties have not fixed upon a definite rate.

Where nothing is said about interest in the note, the note does not bear interest unless it should happen that it is not paid at maturity. In such a case it begins at maturity to bear the legal rate, five per cent., and will continue to do so until paid. This point is worth remembering, in view of the fact that a great many seem to think that a note must necessarily bear interest from the date of its issue, which is manifestly a mistake. In the note that we have just shown, the amount that is payable on the due date (Feb. 24, 1909) is just the face amount, \$600.00. If it were desired to make this an interest-bearing note it would have to read as follows :

\$600 $\frac{00}{100}$

Toronto, Nov. 21, 1908.  
Canada.

*Three months after date for value received ....I.....promise  
to pay to A. F. Sprott .....or order  
at the office of The Bank of Toronto here, the sum of.....  
Six Hundred ..... $\frac{00}{100}$  Dollars  
with interest at six per cent. per annum.*

A. M. GOUGH.

In the case of this note, the amount payable at maturity is \$600.00, plus the interest on \$600.00 for ninety-five days at six per cent. Should this note not be paid at maturity, the rate of interest for any time after maturity would be five per cent., the legal rate. That is, suppose the note were not paid until March 24th, the amount that could be collected would be \$600.00, plus the interest on \$600.00 for ninety-five days at six per cent., plus the interest on \$600.00 for twenty-nine days at five per cent.

Where an interest-bearing note has a rate which is less than five per cent.; and such a note is not paid at maturity, the interest would rise to five per cent. for any time after maturity.

If the rate is over or under five per cent., and it is desired that it should remain at that rate after maturity, the interest clause should be written as shown below :

\$500.00	Toronto, May 29, 1907.
<i>Thirty days after date .....I.....promise to pay to.....</i> <i>A. G. McKay .....or order</i> <i>at the Bank of Montreal, Toronto .....</i> <i>Five Hundred .....<sup>00</sup>/<sub>100</sub> Dollars</i> <i>for value received, with interest at six per cent. per annum until maturity,</i> <i>and thereafter at the same rate until paid.</i>	
No. 3.	Due July 2, 1907. R. BROWN.

Simply inserting the clause, " With interest at six per cent. until paid," would not answer, as the six per cent. would not be collectible beyond the date of maturity.

The following section from our Canadian law should also be carefully noted : " Whenever any interest is by the terms of any contract, whether under seal or not, made payable at a rate per day, week, month, or for any period less than a year, no interest exceeding the rate of five per cent. per annum shall be recoverable, unless the contract contains the express statement of the yearly rate of interest to which such other rate is equivalent."

By this we understand that, if a note were drawn with interest at twelve per cent. per annum, it could be collected. If it read " one per cent. per month," only five per cent. per annum could be collected.

In the last-mentioned case, where it is desired to charge one per cent. per month, a note should, in addition to this statement, give the rate per annum to which one per cent. per month is equivalent.

This section does not apply to mortgages on real estate.

Compound interest cannot be collected, unless it is agreed upon in the contract.

Book accounts differ from notes. A book account overdue will not draw interest unless it is expressly stated on invoices and statements that interest will be charged after a certain date. Even then only five per cent. can be collected, unless the debtor has agreed to pay more. Simply having



eight or ten per cent. printed on the invoice and statement forms does not make the charge binding, and the debtor may refuse to pay anything over five per cent.

### Calculating Interest Periods

Notes are generally given at a certain number of days or months after date. Drafts may be given at a certain number of days or months after date or after sight. This may give rise to difference in the method of getting the due date. There is no difference, however, in any case, in getting the period of time for which interest or discount is to be calculated. It is always a question of fixing two dates, and then calculating the time between these dates in actual days.

For instance, an interest-bearing note is given June 15th, at thirty days. It falls due July 18th. The term for which the interest is reckoned is the term from June 15th to July 18th, reckoned by the actual counting of the days. In other words, the interest will be for thirty-three days.

Again, an interest-bearing note is given July 8th, at two months. It will fall due September 11th. This due date is reckoned by taking two calendar months from July 8th, and adding the three days of grace. The interest on this note will be for the term between July 8th and September 11th, reckoned in actual days. In other words, the interest term will be sixty-five days.

Note that, having once fixed the date of the note and the due date, the interest term in each case is computed in exactly the same way. It is always the exact number of days between the two dates. Let us caution the student against such mistakes as saying that the interest on the first note will be for thirty days and on the second note for two months.

### Dealing with Banks

Business men do a great deal of business with banks in the way of buying and selling negotiable papers. In buying from the bank they generally purchase what are known as bank drafts, which enable them to pay debts in distant places without transmitting the actual money.

For instance, should John White, a merchant of Woodstock, desire to remit \$300.00 to Toronto, he would purchase from his bank a paper of the following nature :

<i>THE CANADIAN BANK OF COMMERCE.</i>	
\$300.	No. 56
<i>Woodstock, May 28, 1903.</i>	
<i>Pay to John White.....or order</i> <i>Three Hundred.....<math>\frac{00}{100}</math> Dollars</i> <i>and charge to this Bank.</i>	
<i>To Canadian Bank of Commerce,</i> <i>Toronto, Ont.</i>	{ <i>J. F. BROWN, Manager.</i> { <i>H. F. SOMERS, Accountant.</i>

This paper he could endorse to anyone to whom he owed the money in Toronto, and remit the same by mail.

Of course, it is to be expected that the bank, in thus accommodating its customers, will charge for the accommodation. This charge is known in business as exchange, and is reckoned as a percentage of the face of the draft. Thus, should the bank charge John White one-quarter of one per cent. exchange, it means that it would cost him seventy-five cents in addition to the \$300.00 to make his remittance to Toronto.

In practice these bank drafts are always drawn payable on demand, and exchange is always an additional item in the cost. In other words, exchange is at a premium.

In theory, it may be considered possible to buy time drafts from a bank, and it may be considered possible to have exchange at a discount or at par. These, however, are purely theoretical considerations, as the ordinary draft bought at the bank is always payable on demand and bought at a premium of exchange.

In selling to a bank, business men generally dispose of notes or acceptances of other people or notes of their own. This is called discounting negotiable papers, and, of course, is done for the purpose of raising money on the papers thus sold. In this connection the following sections from our "Bank Act" regarding collection and agency fees should be of interest :

"The bank may, in discounting, at any of its places of business, branches, agencies, or offices of discount and deposit, any note, bill, or other negotiable security or paper payable at any other of its own places or seats of business, branches, agencies, or offices of discount and deposit in Canada, receive or retain, in addition to the discount, any amount not exceeding the following

rates per cent., according to the time it has to run, on the amount of such note, bill, or other negotiable security or paper, to defray the expenses attending the collection thereof, that is to say: under thirty days, one-eighth of one per cent.; thirty days or over, but under sixty days, one-fourth of one per cent.; sixty days and over, but under ninety days, three-eighths of one per cent.; ninety days and over, one-half of one per cent."

"Agency Fees: The bank may, in discounting any note, bill, or other negotiable security or paper, *bona fide* payable at any place in Canada different from that at which it is discounted, and other than one of its own places or seats of business, branches, agencies, or offices of discount and deposit in Canada, receive and retain, in addition to the discount thereon, a sum not exceeding one-half of one per cent. on the amount thereof, to defray the expenses of agency and charges in collecting the same."

### SERIES 74

Find the date of maturity of the following notes. Find, also, the number of days between date and due date.

DATE OF NOTE.	TIME.	DATE OF NOTE.	TIME.
1. Jan. 4, 1907....	30 dys.	9. Dec. 29, 1906....	2 mos.
2. Jan. 4, 1907....	1 mo.	10. Dec. 30, 1906....	2 mos.
3. Feb. 9, 1907....	30 dys.	11. Dec. 31, 1906....	2 mos.
4. Feb. 9, 1907....	1 mo.	12. July 31, 1907....	1 mo.
5. Feb. 9, 1908....	30 dys.	13. July 31, 1907....	2 mos.
6. Feb. 9, 1908....	1 mo.	14. Dec. 1, 1907....	1 mo.
7. Dec. 28, 1906....	60 days.	15. Nov. 29, 1907....	1 mo.
8. Dec. 28, 1906....	2 mos.		

\$500<sup>00</sup>/<sub>100</sub>

Toronto, April 9, 1907.

Two months after date.....I.....promise to pay.....  
 J. F. Nunn.....or Bearer  
 at the Canadian Bank of Commerce, Yonge and College Branch,.....  
 Five Hundred.....<sup>00</sup>/<sub>100</sub> Dollars  
 for value received.

No..... Due.....

H. DENTON.

16. How much would it require to redeem the above note on June 12th, 1907; on July 4th, 1907; on Sept. 9th, 1908?

17. Adding the following interest clause to the note, “with interest at seven per cent. per annum,” how much would it require to redeem the note on June 12th, 1907 ; on July 4th, 1907 ; on Sept. 9th, 1908 ?

18. Adding the following interest clause to the note, “with interest at four per cent. per annum,” how much would it require to redeem the note on June 12th, 1907 ; on July 4th, 1907 ; on Sept. 9th, 1908 ?

19. Adding the following interest clause to the note, “with interest at seven per cent. per annum, both before and after maturity until paid,” how much would it require to redeem the note on June 12th, 1907 ; on July 4th, 1907 ; on Sept. 9th, 1908 ?

20. Adding the following interest clause to the note, “with interest at one per cent. per month,” how much would it require to redeem the note on June 12th, 1907 ; on July 4th, 1907 ; on Sept. 9th, 1908 ?

\$425<sup>00</sup>/<sub>100</sub>

Toronto, Jan. 14, 1907.

Ninety days after date.....pay to the Order of

..... John Henderson .....

Four Hundred and Twenty-five.....<sup>00</sup>/<sub>100</sub> Dollars,

value received and charge the same to the account of

To Herman Long.....}

No..... Due.....}

JAMES BALSDON.

21. The above draft is accepted as follows :

“ Accepted.

Jan. 17, 1907.

Payable at the Bank of Toronto, Toronto.  
HERMAN LONG.”

When is it due ? What amount will redeem it on its due date ?  
On June 18th, 1907 ?

22. Suppose the above draft were drawn at ninety days after sight and accepted as above, when would it fall due ? What amount would redeem it on its due date ? On June 18th, 1907 ?

## BANK DISCOUNT

### Questions of the First Aspect

**Discounting** a negotiable paper, such as a note or acceptance, simply means selling it or realizing on it before its maturity.

Banks, naturally, do the greater part of this business of buying such paper, and the term "bank discount" is given to the deduction from the maturity value that is made by the bank in finding the present value it places on a paper offered to it.

**Bank Discount** is figured as the simple interest on the maturity value for the time the paper has to run before the date of maturity is reached.

The **Present Worth** or **Present Value** or **Proceeds** of the paper is the difference between the maturity value and the discount.

In finding bank discount these questions must be answered :

1. When does the paper fall due ?
2. What is it worth at maturity ?
3. What is the number of days from the proposed date of discount till the date of maturity ?

If we can answer these and can work a question in simple interest, we know how to find bank discount.

### ILLUSTRATION 1

<p><b>\$200<sup>00</sup>/<sub>100</sub></b></p> <p><i>Three months after date.....I.....promise to pay to</i>  <i>John Wildfong ..... or order</i>  <i>at the Bank of Montreal, Stratford.....</i>  <i>Two Hundred ..... <sup>00</sup>/<sub>100</sub> Dollars</i>  <i>for value received.</i></p> <p>No..... Due.....</p>	<p><i>Toronto, July 7, 1907.</i></p> <p><b>GEO. CAMPBELL.</b></p>
---	---

This note is discounted at the Bank of Montreal, Toronto, on July 29th. If the rate of discount is 6%, what are the proceeds of the note ?

### SOLUTION

1. The due date is Oct. 10th.
2. The value of the note on Oct. 10th is \$200.
3. The number of days from July 29th to Oct. 10th is 73 days.
4. The interest on \$200 for 73 days at 6% is \$2.40.
5. The proceeds of the note are \$200 - \$2.40, or \$197.60.



## ILLUSTRATION 2

\$300 $\frac{00}{100}$ 

Toronto, May 27, 1907.

Sixty days after date.....I.....promise to pay to  
 Wm. Colville ..... or order  
 at the Bank of Hamilton, Queen and Spadina Branch .....  
 Three Hundred ..... $\frac{00}{100}$  Dollars  
 for value received, with interest at five per cent. per annum.

No..... Due.....

W. MILLER.

This note is discounted June 4th, at the Bank of Commerce, Stratford. If the rate of discount is 6%, what are the proceeds?

## SOLUTION

1. The due date is July 29th.
2. The value of the note on July 29th is \$302.59 (\$300 + interest on \$300 for 63 days at 5%).
3. The number of days from June 4th to July 29th is 55 days.
4. The interest on \$302.59 for 55 days at 6% is \$2.74.
5. The proceeds of the note are \$302.59 - \$2.74, or \$299.85.

## SERIES 75

Find the date of maturity, term of discount, bank discount, and proceeds of the following ten notes and acceptances:

1. \$367 $\frac{50}{100}$  St. Thomas, Ont, Jan. 5, 1908.

Three months after date I promise to pay to Clare Crawford, or order, at the Imperial Bank, here, Three Hundred and Sixty-seven and  $\frac{50}{100}$  Dollars, value received.

Discounted Jan. 5, 1908, at 6%.

JOHN SMITH.

2. \$423 $\frac{70}{100}$  Toronto, Ont., Dec., 30, 1907.

Two months after date I promise to pay to Fred. L. Sanders, or order, at the Merchants Bank, here, Four Hundred and Twenty-three and  $\frac{70}{100}$  Dollars, value received.

Discounted Feb. 1, 1908, at 5%.

R. LEARN.

3. \$126 $\frac{45}{100}$  Toronto, Ont., Dec., 30, 1907.

Sixty days after date I promise to pay to P. McIntosh, or order, One Hundred and Twenty-six and  $\frac{45}{100}$  Dollars, value received.

Discounted Jan. 10, 1908, at 5 $\frac{1}{2}$ %.

WM. OSTRANDER.

4. \$537 $\frac{25}{100}$ 

Montreal, Que., Feb. 29, 1908.

Six months after date I promise to pay to W. H. Shaw, or order, Five Hundred and Thirty-seven and  $\frac{25}{100}$  Dollars, value received, with interest at six per cent. per annum.

Discounted Feb. 29, 1908, at 6%.

T. MEDCRAFT.

5. \$162 $\frac{90}{100}$ 

Aylmer, Ont., Dec. 13, 1907.

Four months after date I promise to pay to F. D. White, or order, One Hundred and Sixty-two and  $\frac{90}{100}$  Dollars, value received, with interest at five per cent. per annum.

Discounted Feb. 16, 1908, at 5%.

G. P. BROWN.

6. \$75 $\frac{30}{100}$ 

Windsor, Ont., Nov. 15, 1907.

One month after date I promise to pay to J. E. Norman, or order, Seventy-five and  $\frac{30}{100}$  Dollars, value received, with interest at four per cent. per annum.

Discounted Nov. 15, 1907, at 6%.

JOHN GLOIN.

7. \$240 $\frac{00}{100}$ 

Aylmer, Ont., Oct. 31, 1907

Four months after date I promise to pay to D. H. Price, or order, Two Hundred and Forty and  $\frac{00}{100}$  Dollars, value received, with interest at seven per cent. per annum.

Discounted Dec. 5, 1907, at 5%.

A. A. LESLIE.

8. \$346 $\frac{75}{100}$ 

Ingersoll, Ont., Dec. 18, 1907.

One year after date I promise to pay to Emerson McTaggart, or order, Three Hundred and Forty-six and  $\frac{75}{100}$  Dollars, value received, with interest at five per cent. per annum.

Discounted May 16, 1908, at 6%.

P. GILLETT.

9. \$168 $\frac{75}{100}$ 

Toronto, Ont., Mar. 15, 1907.

Ninety days after sight pay to the order of T. Tanton, One Hundred and Sixty-eight and  $\frac{75}{100}$  Dollars, value received, and charge to the account of

A. F. SPOTT.

To W. W. AUSTIN, Toronto, Ont.

Accepted Mar. 16, 1907; discounted May 1, 1907, at 6%.

10. \$243 $\frac{25}{100}$ 

Ottawa, Ont., Aug. 1, 1907.

Thirty days after date pay to the order of ourselves, Two Hundred and Forty-three and  $\frac{25}{100}$  Dollars, value received, and charge to the account of

BENNER &amp; LINDSAY.

To R. ANGER, Toronto, Ont.

Discounted Aug. 1, 1907, at 6%.

11. A merchant bought \$3,500 worth of goods for cash, and sold them on 3 months' time at 15% advance, and got the note discounted at 5% to pay the bill. How much did he gain?

- 12. Find the proceeds of a draft for \$580 at 60 days, discounted at 6%.
- 13. A note for \$1,500, with interest, dated May 1, 1907, at 3 months, was discounted June 3rd. Find the proceeds.
- 14. I bought a lot for \$1,200 cash, and sold it at an advance of 12½%, on a 90-day note, which I immediately discounted at 6%. Find my gain or loss.
- 15. C. H. Good & Co.'s bank account is overdrawn \$7,564.19. They discount at 6%: a 90-day note for \$3,975.21, a 60-day note for \$1,546.19, and a 20-day note for \$2,546.85; proceeds of all to their credit at the bank. What is the condition of their bank account after they receive credit as above?
- 16. A dealer bought 100 brls. apples at \$4 per brl. for cash. He sold 25 brls. for cash at \$5 per brl., 25 brls. at \$6 per brl. for a 30-day note without interest, and the remaining 50 brls. at \$5.75 per brl. for a 60-day note drawing interest at 5%. He discounted the notes at 6% the same day as drawn, getting cash for the proceeds. How much did he make on the transaction?
- 17. A farmer purchases a machine for \$62, and gives in settlement two notes, one for \$31, due 1st October following (1907), and one for \$31, due 1st October one year later. On July 15th the company proposes, and farmer accepts, a proposition to pay off both notes on basis of discount at 6% per annum. What is the net amount paid by the farmer to retire his notes?

Questions of the Second Aspect

ILLUSTRATION

\$1,000.00

Toronto, January 7, 1907.

Three months after date.....I.....promise to pay to the Bank of Nova Scotia.....or order at their office, Toronto, Ont.....  
One Thousand.....100 Dollars for value received.

No..... Due..... 17

J. P. MURRAY.

It is desired to fill in the above note for such a sum that, when discounted at 5%, the note will realize \$500 in cash. What should be the face value of the note?

## SOLUTION

1. The due date is April 10th.
2. Assume a value of \$1 for the face of the note on April 10th.
3. The number of days from Jan. 7th to April 10th is 93 days.
4. The interest on \$1 for 93 days at 5% is \$.0127.
5. The proceeds of a \$1 note are \$1.00 - .0127, or \$.9873.
6. If \$.9873 is proceeds of note for \$1.

Then \$500 is proceeds of note for  $\frac{\$500}{.9873}$ , or \$516.56.

The note should have a face value of \$516.56.

## SERIES 76

1. I wish to borrow \$900 at a bank. For what sum must I issue a 90-day note to obtain the amount, discount being at 6%?

2. A note, dated Mar. 15, 1908, payable in 3 months, with interest at 5%, was discounted April 10, 1908, at 6%. If the proceeds were \$1,342.27, what must the face have been?

3. The proceeds of a 3-months' note, dated Sept. 20, 1907, and discounted on Oct. 15. were \$426.89. What was the face of the note?

4. A 30-day, 5% interest-bearing note was discounted 10 days after it was drawn up. If the rate of discount was 6% and the bank discount \$13.40, what was the face of the note?

5. Owing a man \$615, I gave him my 60-day note. What should be the face of the note to pay him the exact debt, if discounted at 7%?

6. You have \$650.80 to your credit at a bank; you give your cheque for \$1,872.40, after which you discount a 30-day note for \$850.80, proceeds to your credit at the bank. You then discount a 90-day note, made by F. D. White, proceeds to your credit, when you find yourself indebted to the bank \$24.74. If discount be at 6%, what must have been the face of the note made by White?

## TRUE DISCOUNT

The **Present Worth** of a debt, payable at some time in the future, is its value at the present time, and is such a sum that, if put out at interest, it will amount to the given debt when it becomes due.

The **True Discount** is the difference between the amount of the debt and its present worth.



ILLUSTRATION.—Jones owes Smith \$525, to be paid at the end of one year. If Smith can get 5% interest for his money, what sum should he be willing to accept from Jones to-day in payment of the debt? What discount would Smith be allowing Jones by accepting this amount?

#### SOLUTION

If Jones were to pay Smith \$100 to-day, Smith would put the \$100 out at interest, and by the end of the year he would have \$105.

So \$100 paid to-day is equivalent to \$105 paid at the end of the year.

Or \$105 paid at the end of the year is equivalent to \$100 paid to-day.

And \$525 paid at the end of the year is equivalent to \$500 paid to-day.

Smith should be willing to accept \$500 to-day in payment of the debt, and he would by so doing allow Jones a discount of  $\$525 - \$500 = \$25$ .

The present worth of the debt would be, therefore, \$500, and the true discount on it would be \$25.

It will be well right here to note the difference between bank discount and true discount.

Suppose Jones gives Smith his note at one year for the amount of the debt of \$525, and Smith takes the note to a bank and gets it discounted. The bank would charge him a discount of 5% of the face of the note, which would be \$26.25, and Smith would receive  $\$525 - \$26.25 = \$498.75$ . The true discount is really the interest on the present worth of the debt; while the bank discount is really the interest on the amount of the debt.

The bank discount in this case amounts to \$26.25, and the true discount to \$25. The difference between the bank discount and the true discount is \$1.25, which is the interest on \$25 for the given time (one year) at the given rate (5%).

This may be stated generally thus:

The difference between the bank discount and the true discount is the interest on the true discount.

This may be proved generally as follows:

Bank Discount = Interest on the debt.

True Discount = Interest on the present worth of the debt.

True Discount = Interest on (the debt - true discount).

True Discount = Interest on the debt - interest on true discount.

True Discount = Bank discount - interest on true discount.

∴ Bank discount is greater than true discount by the interest on the true discount.

Or

The bank discount = the given percentage for the given time of the debt.



The true discount = the given percentage for the given time of the present worth of the debt.

But the debt = the present worth of the debt + the true discount.

So the bank discount exceeds the true discount by the given percentage for the given time of the true discount.

In our illustration the bank discount is  $\frac{5}{100}$ , or  $\frac{1}{20}$ , of the debt, and the true discount is  $\frac{3}{25}$ , or  $\frac{1}{25}$ , of the debt. By comparing these fractions it will be seen that the numerators are the same, but that the denominator of the fraction representing the true discount is made up of the sum of the two terms of the fraction representing the bank discount.

This relation is always true, and may be proved as follows :

Suppose, say, the bank discount is  $\frac{3}{100}$  of the debt.

Bank discount is really the interest on the debt.

∴ \$16 in the given time at the given rate will gather \$3 interest.

∴ \$16 in the given time at the given rate will amount to \$19.

∴ \$19 will have a present worth of \$16.

Or \$19 will have a true discount of \$3.

And the true discount will be  $\frac{3}{19}$  of the debt.

Comparing  $\frac{3}{100}$  and  $\frac{3}{19}$ , it will be seen that the numerators are the same, but that the denominator of the fraction  $\frac{3}{19}$  is made up of the sum of the two terms of the fraction  $\frac{3}{100}$ .

Similarly, having given the fraction which the true discount is of the debt, the fraction representing the bank discount can be derived from it. Thus, if the true discount is  $\frac{4}{19}$  of the sum, we know that the fraction representing the bank discount must be  $\frac{4}{15}$  of it.

From the foregoing it will be gathered that the bank discount is always greater than the true discount.

Bank discount is theoretically incorrect, but practically it is the only discount used where the question of present worth of any negotiable paper is under consideration. Business men have also grown accustomed to using this style of discount, even in figuring the present worth of any amount due on account.

True discount is theoretically correct, but for ordinary business transactions is not practical or valuable. It is used, however, where comparisons are instituted, and, as we shall see later, the principle is also applied in all work in finding the present worth of bonds or debentures and annuities.

## SERIES 77

Find the present worth and true discount of

1. \$503.36 for 1 year at 4%.
2. \$752.40 for 9 months at 6%.
3. \$109.89 for 3 months at 7%.
4. \$129.01 for 90 days at 4%.
5. \$588.80 for 60 days at 5%.
6. \$75.85 for 6 months at 5%.
7. \$918.50 for 48 days at 5%.
8. \$2,585.87 for 63 days at 7%.
9. \$1,500 for 90 days at 6%.

Find the true discount on

10. \$1,317.24 for 18 days at 5%.
11. \$1,250 for 7 months at 4%.
12. \$800 for 5 months at 7%.

13. Which is better, and how much, to buy flour at \$7.25 per barrel on 9 months' time, or to pay \$6.75 cash, money being worth 6%?

14. If money is worth 5%, what cash offer will be equivalent to an offer of \$1,385 for a bill of goods on 90 days' credit?

15. A offers flour at \$8 per barrel on 8 months' credit, and B offers the same grade of flour at \$7.50 per barrel on 6 months' credit. Which offer is the better and how much, money being worth 6%?

16. The terms on which a bill of goods amounting to \$478.25 was bought are 2% 10 days, net 60 days. If money is worth 7%, which would be better, to pay the bill on the 10th day or on the 60th, and how much better?

17. I bought a bill of goods amounting to \$583.60 on 60 days' credit, and immediately sold them for \$615.75 on 90 days' credit. Money being worth 6%, find my per cent. of profit or loss.

18. I carried a quantity of tweed in stock for 3 months, and then sold it at an advance of 25% on first cost, extending to the purchaser a credit of 6 months, without interest. If money is worth 5% per annum, what was my per cent. of profit or loss?

19. Which is the better, and how much, to buy property worth \$2,600, at 6% discount for cash, or on 8 months' time without interest, when money is worth 6%?

20. How large a note due in 90 days, with interest at 6%, will pay a debt of \$685.24 due in 90 days without interest?

**21.** How much will I gain if I borrow money at 8% to pay a debt of \$4,500, due in 6 months, if 6% discount is deducted from the face of the debt for cash ?

**22.** What is the difference between the interest and true discount on \$1,345 for 9 months at 7% ?

**23.** Sold a bill of goods on 8 months' credit for \$387.20. If the selling price was \$24.60 less than the goods cost, and money is worth 5% per annum, how much was the loss and the loss per cent. ?

**24.** A merchant bought a bill of goods amounting to \$937.60 on 5 months' credit, and the seller offered a discount of 5% for cash. If money is worth 7% per annum, how much would the merchant gain by accepting the seller's offer ?

**25.** If the interest is  $\frac{5}{12}$  of the principal, what fraction of the principal is the true discount for the same time and at the same rate ?

**26.** If the true discount is  $\frac{3}{11}$  of the sum, what fraction of the sum is the bank discount for the same time and at the same rate ?

**27.** The interest is \$9 and the true discount for the same time at the same rate is \$8. Find the sum.

**28.** The difference between the interest and the true discount for 9 months at 7% is \$.98. Find the sum.

**29.** The interest is  $\frac{3}{7}$  of the principal and the difference between the interest and true discount is \$15.30. Find the principal.

**30.** The bank discount on a certain sum of money is \$30 and the true discount on the same sum for the same time is \$25. Find the sum.

**31.** The true discount on \$1,378 for 6 months is \$53. Find the true discount on the same sum for 9 months.

**32.** The interest is \$3 and the difference between the interest and true discount for the same time and rate is 25 cents. Find the principal.

**33.** The true discount on a sum of money for 8 months at 9% is \$44.46. Find the sum.

**34.** The simple interest on a certain sum of money for 2 years is \$50, and the true discount for the same time and rate is \$45. Find the sum and the rate per cent. per annum.

**35.** If \$20 is the true discount on \$420 for 6 months, how much is the true discount on the same sum for 12 months ?

36. If \$10 is the true discount on \$110 for 8 months, on what sum would \$10 be the true discount for 4 months ?

37. If \$20 is the interest on \$180 for a certain time, what is the true discount on \$180 for the same time at the same rate ?

38. If \$105 be accepted in present payment of \$665 due some time hence, what should be a proper discount off a bill of \$665, which has only half the time to run ?

39. The bank discount on a certain sum for 8 months at 6% is \$5.98. What would be the true discount on the same sum for the same time and rate ?

40. If \$11.10 is the true discount on \$196.10 for 9 months, for how long would \$24.50 be the true discount on \$269.50 at the same rate ?

41. I have two notes, both drawn for 8 months, together amounting to \$112. I have them both discounted at 9%, one at bank discount and one at true discount. If the total discount is \$6.45, find the face of each note.

42. The difference between the true and bank discounts on a certain sum of money for 3 years at 8% is \$30. Find the sum.

43. The true discount on a certain sum of money at 8% for 60 days is \$82.24. What is the bank discount on the same sum for the same time at the same rate ?

44. A certain sum of money loaned at simple interest amounts to \$334.40 in 9 months, and in 7 months more to \$345.60. Find the sum and rate.

45. The present worth of \$359.04, due a certain number of days hence, the rate of interest being 6%, is \$350.40. Find the number of days.

46. The present worth of \$405.60, due a certain number of months hence, is \$390. If the rate of interest is 6%, find the number of months.

47. Find the difference between the true and bank discounts on \$350 or 48 days at 5%.

48. What must be the amount of a note which matures in 93 days for the difference between the true and bank discounts on it to be 1 cent ?

49. A tradesman marks his goods with two prices, one for cash and one for credit of 6 months. What relation should the prices bear to each other, allowing interest at  $7\frac{1}{2}\%$  ? If the credit price of an article be \$33.20, what is the cash price ?



50. Find the sum of money whose true discount for one year at 10% is greater by  $\$3\frac{58}{189}$  than the sum of the true discounts of one-half of it at 8% and the other half at 12% for one year.

51. Bought a farm for \$10,000, payable one-half cash, the remainder in 1 year, with interest at 6%. I sell immediately for \$12,000, payable in 3 months, with interest at 4%. What is my present gain, money being worth 5% per annum?

### Comparison of Rates of Interest and Discount

#### ILLUSTRATION 1

*Toronto, May 30, 1908.*

*Seventy days after date I promise to pay to .....*

*F. D. White ..... or order at the  
Bank of Montreal, Yonge and Carlton Branch, Toronto, .....*

*One Hundred .....  $\frac{00}{100}$  Dollars,  
value received. W. BENNER.*

Suppose that F. D. White got E. Bowlby to discount the above note at 6% on May 30, what rate of interest would Mr. Bowlby make on the money he so invested?

#### SOLUTION

The note will be legally due 73 days after the date of discount.

The amount of discount for 73 days at 6% on \$100 = \$1.20.

Mr. Bowlby would give Mr. White \$98.80 for the note. He would then wait 73 days until the note matured, when he would receive \$100 from Mr. Benner. The investment of \$98.80 for 73 days produces \$100 - \$98.80 = \$1.20. interest.

Interest on \$98.80 for 73 days = \$1.20.

Interest on \$98.80 for 1 year would be  $\$1.20 \times \frac{365}{73} = \$6.00$ .

Interest on \$100.00 for 1 year would be  $\$6.00 \times \frac{10000}{9880} = \$6\frac{18}{247}$ .

Therefore, Mr. Bowlby would really make  $6\frac{18}{247}\%$  interest on his investment.

#### ILLUSTRATION 2

Suppose that Mr. Bowlby had wanted to make 6% interest on his money, what rate of discount would he have charged Mr. White?



## SOLUTION

The interest on \$100 for 73 days at 6% = \$1.20.

Then the amount of \$100 for 73 days at 6% = \$101.20.

Therefore, to make 6% interest on his money, he would have to charge discount at the rate of \$1.20 on \$101.20 for 73 days.

Discount on \$101.20 for 73 days = \$1.20.

Discount on \$101.20 for 1 year would be  $\$1.20 \times \frac{365}{73} = \$6$ .

Discount on \$100.00 for 1 year would be  $\$6 \times \frac{100}{101.20} = \$5\frac{2}{3}\frac{5}{3}\%$ .

Therefore, to make 6% interest on the money he invested in discounting the note, Mr. Bowlby would have to charge discount at the rate of  $5\frac{2}{3}\frac{5}{3}\%$ .

## SERIES 78

1. What rate of interest is made when a note nominally due in 60 days is discounted at 5%?

2. A bank discounted a note legally due in 90 days at 8% per annum. What was the actual rate of interest made by the bank on the transaction?

3. At what rate should a note legally maturing in 45 days be discounted to produce 5% interest per annum on the investment?

4. A moneylender wishes to make 7% on the money he invests. What rate of discount should he charge on notes nominally maturing in 30 days?

5. A note of \$700, dated Aug. 12, 1907, and payable 90 days after date, was discounted Sept. 26, 1907. What was the rate per cent. of discount if the proceeds were \$694.40?

6. In question 5, what rate of interest did the purchaser make on his investment?

7. What rate of interest was made when a note legally due in 57 days was discounted at 4%?

8. The true discount on \$922.10 for 48 days is \$9.60. Find the rate. What rate of bank discount would give the same amount of discount?

9. A note legally maturing in 85 days was discounted at 9%. What rate of interest did the purchaser make on his investment?

10. A note dated 1st January, 1908, at 90 days, for \$730, with interest at 5%, was discounted at a bank on January 21st at 7%. Find the rate of interest made by the bank on the amount advanced?

## PARTIAL PAYMENTS

**Partial Payments** are simply part payments made from time to time on interest-bearing instruments, such as notes or mortgages.

Sometimes, as in the case of an instalment note, the paper itself specifies the time at which payments are to be made. A mortgage, also, is often drawn with the proviso that the principal is to be paid in definite instalments.

### Instalment Note

\$60<sup>00</sup>/<sub>100</sub>

Stratford, July 2, 1906.

*On the first day of each month hereafter, for four months consecutively, I promise to pay to .....*

*A. Kilgour ..... or order  
Fifteen Dollars, the whole amounting to Sixty Dollars, the first of such payments to be made on the second day of August next, interest, both before and after maturity, and until paid, at the rate of six per cent. per annum.*

J. ROBERTS.

Unless thus specified that partial payments are to be made, it is quite at the option of the creditor as to whether partial payments shall be accepted or not.

The problem presented, where partial payments are made and accepted, is to find at any given date the amount remaining due.

### Law of Application of Payments

It is a principle of law that, in making a payment on an interest-bearing paper, the payment is first to be applied to the reduction of the interest and then to the reduction of the principal. If the payment is not sufficient to at least cancel the interest, it might better be withheld, as the creditor is simply getting the use of the money without giving any return for it. All he can do is to hold such inadequate payments until such time as the sum of them will at least cancel the accrued interest. Then he may proceed to apply the payments to the reduction of the interest; and, if there is more than sufficient for the reduction of the interest, the balance may be applied to the principal. If this point be kept in mind, the work of partial payments presents a very easy application of simple interest.

## ILLUSTRATION 1

Where every payment is large enough to at least cancel the accrued interest.

\$500  $\frac{00}{100}$ 

Hamilton, Mar. 24, 1908.

*Six months after date I promise to pay* .....  
*J. E. Emerson* ..... *or order*  
*Five Hundred* .....  $\frac{00}{100}$  *Dollars*  
*value received, with interest at six per cent. per annum.*

ROBERT BROWNING.

On the back of this note there are the following endorsements of partial payments :

## BACK OF NOTE.

*Received on the within note, May 20, 1908.*

*One Hundred and Fifty Dollars (\$150).*

J. E. EMERSON.

*July 15, One Hundred Dollars (\$100).*

J. E. EMERSON.

What remains to be paid on the due date of the note, Sept. 27, 1908 ?

## SOLUTION

Face of note Mar. 24, 1908 .....	\$500.00
Interest on \$500 from Mar. 24, 1908, to date of first payment, May 20, 1908 .....	4.68
Amount due May 20, 1908 .....	\$504.68
First payment. ....	150.00
Balance .....	\$354.68
Interest on \$354.68 from May 20, 1908, to date of second payment, July 15.....	3.26
Amount due July 15 .....	\$357.94
Second payment.....	100.00
Balance .....	\$257.94
Interest on \$257.94 from July 15 to Sept. 27.....	3.14
Balance due Sept. 27, 1908 .....	\$261.08

## ILLUSTRATION 2

Where payments are not in all cases large enough to cancel accrued interest.

\$2,750 $\frac{00}{100}$ 

Hamilton, Aug. 18, 1908.

*Two years after date I promise to pay to.....  
 Roderick Bethune .....or order  
 Twenty-seven Hundred and Fifty..... $\frac{00}{100}$  Dollars  
 value received, with interest at seven per cent. per annum.*

GEORGE CAMPBELL.

On the above note are endorsed the following payments :

April 9, 1909, \$ 75.

July 3, 1909, 400.

Dec. 5, 1909, 150.

What remains to be paid on the due date of the note ?

## SOLUTION

Face of note .....	\$2,750.00
Interest on \$2,750 from Aug. 18 to Apr. 9, at 7%.....	123.41
Since the first payment is less than this interest, compute the interest on \$2,750 from Apr. 9th to July 3rd.....	44.83
Amount due July 3rd .....	\$2,918.24
First and second payments (\$75 + \$400).....	475.00
Balance due .....	\$2,443.24
Interest on \$2,443.24 from July 3rd to Dec. 5.....	72.63
Amount due Dec. 5.....	\$2,515.87
Third payment .....	150.00
Balance due .....	\$2,365.87
Interest on \$2,365.87 from Dec. 5 to due date of note.....	117.52
Balance due Aug. 21, 1910.....	\$2,483.39

NOTE.—Where notes are not paid at maturity careful attention must be paid to the wording of the interest clause. A reference to the chapter giving a synopsis of the law concerning interest-bearing documents will aid the student here.

## SERIES 79

1. \$850.00. Belleville, Ont., April 24, 1908.

Six months after date I promise to pay to W. G. Harwood or Order.....Eight Hundred and Fifty..... $\frac{00}{100}$  Dollars, value received, with interest. W. W. GREEN.

The above note has the following payments endorsed upon it: June 17th, \$125; Sept. 3rd, \$200. Find the value of the note at maturity.

2. \$1,200.00. Aylmer (West), Ont., June 6, 1908.

One year after date I promise to pay to Elmond Bowlby.... or Order.....Twelve Hundred..... $\frac{00}{100}$  Dollars, value received, with interest at seven per cent. per annum.

J. J. GOULD.

On the above note were endorsed these payments: Oct. 20th, \$120; January 30th, \$10; May 1st, \$350. Find the maturity value of the note.

3. \$900.00. Chatham, Ont., Mar. 3, 1908.

Nine months after date I promise to pay to George House or Order.....Nine Hundred..... $\frac{00}{100}$  Dollars, value received, with interest at four per cent. per annum.

GEORGE PARTLOW.

On the above note the following payments were made: June 12th, 1908, \$150; Nov. 9th, 1908, \$10. Find the amount necessary to pay the note on Mar. 3rd, 1909.

4. \$2,500.00. Halifax, N.S., Jan. 1, 1906.

Two years after date I promise to pay to W. W. Ingram.... or Order.....Twenty-Five Hundred..... $\frac{00}{100}$  Dollars, value received, with interest at six per cent. per annum.

J. D. BROWN.

The above note has endorsed upon it the following payments: Dec. 14th, 1906, \$100; Aug. 6th, 1907, \$500; Mar. 11th, 1908, \$1,000. Find the value of the note on Sept. 9th, 1908.

5. \$875.00. Winnipeg, Man., Oct. 8, 1907.

Nine months after date I promise to pay to John Gloin.... or Order.....Eight Hundred and Seventy-five.... $\frac{00}{100}$  Dollars, value received, with interest at six per cent. per annum until maturity, and thereafter at the same rate until paid.

J. SMALL.

The following payments were endorsed upon the above note: Jan. 3, 1908, \$125; Mar. 24, 1908, \$75; Sept. 12th, 1908, \$200. Find the amount necessary to pay the note on Jan. 1, 1909.



6. \$1,500.00. Owen Sound, Ont., April 2, 1908.

One year after date I promise to pay to W. H. Weir.....  
or Order.....Fifteen Hundred..... $\frac{00}{100}$  Dollars,,  
value received, with interest at four per cent. per annum until  
maturity, and thereafter at the rate of seven per cent. per annum  
until paid. J. G. DAVIDSON.

The following payments were made on the above note : Oct.  
11, 1908, \$25 ; Dec. 20, 1908, \$300 ; July 9, 1909, \$450. Find  
the amount necessary to pay the note on Sept. 10, 1909.

7. \$1,000.00 Vancouver, B.C., Nov. 3, 1907.

Six months after date I promise to pay to Frank Kent....  
or Order.....One Thousand..... $\frac{00}{100}$  Dollars.  
value received, with interest at four per cent. per annum until  
paid. J. R. GREEN.

On the above note the following payments were made : April  
9, 1908, \$250 ; July 30, 1908, \$300. What was the balance due  
Sept. 15, 1908 ?

8. \$750.00. Kingston, Ont., Mar. 8, 1908.

Fifteen months after date I promise to pay to Fred White  
or Order.....Seven Hundred and Fifty..... $\frac{00}{100}$  Dollars,  
value received, with interest at eight per cent. per annum until  
maturity, and thereafter at the rate of four per cent. per annum  
until paid. C. COWAN.

On the above note were endorsed the following payments :  
Oct. 10, 1908, \$25 ; Jan. 14th, 1909, \$280 ; Sept. 20, 1909, \$300.  
Find the balance due on the note Mar. 8, 1910.

9. \$500.00. Peterborough, Ont., July 2, 1907.

On demand I promise to pay to C. S. Cutler.....or Order,  
Five Hundred..... $\frac{00}{100}$  Dollars,  
value received, with interest at nine per cent. per annum from  
maturity until paid. F. THAYER.

The following payments were made on the above note : Dec.  
11, 1907, \$10 ; Mar. 15, \$150 ; Aug. 27, 1908, \$200. What was  
the balance of the note on Dec. 31, 1908 ?

10. \$1,450.00. Brantford, Ont., May 12, 1908.

Nine months after date I promise to pay to Albert Williams  
or Order.....Fourteen Hundred and Fifty..... $\frac{00}{100}$  Dollars,  
value received, with interest at  $4\frac{1}{2}$  per cent. per annum until  
maturity, and thereafter at the same rate until paid. J. DOUGLAS.

The following payments were endorsed on the above note :  
Aug. 31, 1908, \$400 ; Dec. 3, 1908, \$15 ; April 11, 1909, \$275.  
What remained due on the note on June 30, 1909 ?

11. \$450.00.

Woodstock, Ont., Aug. 20, 1907.

One year after date I promise to pay to John J. Doane....  
 or Order.....Four Hundred and Fifty..... $\frac{00}{100}$  Dollars,  
 value received, with interest at four per cent. per annum until  
 maturity, and thereafter at the rate of six per cent. per annum  
 until paid.

J. D. TODD.

The following payments were endorsed on the above note :  
 Dec. 24, 1907, \$50 ; April 10, 1908, \$4 ; Sept. 11, 1908, \$100.  
 What was the balance due on the note on Dec. 15, 1908 ?

12. \$1,800.00.

London, Ont., July 2, 1908.

Six months after date I promise to pay to W. J. Long.....  
 or Order.....Eighteen Hundred..... $\frac{00}{100}$  Dollars,  
 value received, with interest at ten per cent. per annum until  
 maturity, and thereafter at the same rate until paid.

C. BAILY.

The following payments were made on the above note : July  
 26, 1908, \$300 ; Oct. 11, 1908, \$400 ; Mar. 11, 1909, \$500. Find  
 the value of the note on July 9, 1909.

13. Required the balance due May 1, 1889, on a demand note  
 for \$4,119.82, at 6% from maturity till paid, dated June 25, 1888,  
 on which a payment of \$450.25 was made Aug. 1, 1888, and a  
 payment of \$21.19 on the 15th of each subsequent month.

14. On Oct. 15, 1907, I borrowed a certain sum of money  
 secured by mortgage at 6% interest. The following partial pay-  
 ments have been made : Nov. 24, 1907, \$200 ; April 15, 1908,  
 \$300 ; Aug. 18, 1908, \$750. On Jan. 31, 1909, I still owe \$4,099.739.  
 Find the sum borrowed.

15. Jones purchases a house from Smith, and gives a demand  
 note bearing interest at 5% on Jan. 1, 1901. Subsequently the  
 following payments were made and endorsed on the note : Mar.  
 15, \$59 ; July 28, \$10 ; Aug. 8, \$157 ; Jan. 1, \$214. On the  
 last-mentioned date a renewal note is given for the balance then  
 due, to run for one year, and bear interest at 7%. Smith imme-  
 diately discounts this note at 5 per cent., and realizes \$508.25.  
 Find the purchase price of the house. (In reckoning for  
 last-mentioned note, omit days of grace.)

16. D bought a piano, catalogued at \$600, at a reduced price.  
 In payment he gave his note dated Jan. 1, 1894, payable on demand,  
 and bearing interest at 5% per annum. During the year the  
 following payments were made on the note : Mar. 15, \$100 ;  
 July 31, \$7.10 ; Aug. 8, \$206. No payments to be applied to  
 reducing the debt until their sum is greater than the then due

interest. On Jan. 1, 1895, D renewed the note for one year, the rate of interest being changed to 8%. The holder discounted the new note on Aug. 8, 1895, at the Bank of Toronto, at 7 per cent. per annum, the proceeds being  $\$214.15\frac{13}{125}$ . By what percentage was the catalogue price reduced? (Allow no days of grace.)

### CASH BALANCE

The settlement of accounts between debtor and creditor often involves the calculation of simple interest.

A book account is different from a note, inasmuch as it does not bear interest when overdue, unless the merchant has it printed on his invoices and bills that interest will be charged after a certain date. Even then it can only be 5 per cent., unless the debtor consents to pay more. Simply having 7 or 8 or 10 per cent., as the case may be, printed on an invoice or statement does not make the charge binding, and the debtor, unless he has contracted to do so, may refuse to pay more than the legal rate of 5 per cent.

The time when an account becomes subject to interest is a matter of the facts of the particular case. A retail dealer may establish a practice with his customer that goods may be bought on account at any time during the month, and are to be paid for when the statement of account is rendered at the end of the month. His bills naturally bear a statement such as this: "Accounts rendered monthly. Interest at the rate of 5 per cent. charged on overdue accounts."

Even wholesale dealers very often make the same arrangements with retail dealers, especially in the case of city sales. In the case of the ordinary sales by wholesaler to retailer, however, each sale is made on a definite term, and it is just as easy to find when each item of the account matures as it is to find when a note matures.

### Questions of the First Aspect

#### ILLUSTRATION 1

May 1st.—J. Brown & Co., of Toronto, sell to R. Young, of Woodstock, a bill of goods amounting to \$500 on the following terms: 5/30, N/90.

Young's account, as it appears in Brown & Co.'s books, would show as follows:

<i>Dr.</i>		R. YOUNG.	<i>Cr.</i>
May 1st	5/30, N/90	\$500	

According to the terms, Young can pay in 30 days, and earn a 5 per cent. discount. In other words, he can settle the account for \$475 in cash on any date up to May 31st.

If he does not settle in that time, he will have to pay the full amount of \$500, and it must be paid in 90 days from May 1st, or on July 30th.

If not paid on July 30th, and the interest notice has been duly given, the account will bear interest from July 30th until paid. If paid on August 31st, for instance, it will require \$500 plus the interest on \$500 for 32 days, or, altogether, \$502.19 in cash to settle the account.

The **Merchandise or Commercial Balance** is the difference between the debit and credit items—\$500.

The **Cash Balance** is the sum required to settle the account on a given date—\$475 on May 31st, or \$502.19 on August 31st.

An **Account Current** is simply a detailed statement of an open account showing its cash balance on a given date.

The account current rendered by J. Brown & Co., to R. Young, on August 31st, will appear as follows :

R. YOUNG,

TORONTO, Aug. 31, 1908.

Woodstock, Ont.

In account with

J. BROWN & Co.

May 1	To Mdse. 5/30 N/90.....	\$500.00
	“ Interest from July 30 to Aug. 31 .....	2.19
	Balance due .....	\$502.19

NOTE.—Interest cannot be compounded in this case. If this account is not settled on August 31st, further interest is always reckoned on the \$500, not on the \$502.19.

### ILLUSTRATION 2

Find the cash balance of the following account on July 15, 1908, reckoning interest at 5 per cent. :

Dr				E. Jordan				Cr			
1908				1908							
Mar. 10	Mdse. 21/30		650 00	Apr. 20	Cash		500 00				
Apr. 1	" 21		1000 00	May 13	Draft 90 days		900 00				
May 26	" 21/60		12 00 00	June 1	Cash		1000 00				



## SOLUTION

DUE DATE.	DAYS.	ITEMS.	INTEREST.	DUE DATE.	DAYS.	ITEMS.	INTEREST.
1908.				1908.			
Apr. 9	97	\$ 650.00	\$ 8.64	Apr. 20	86	\$ 500.00	\$ 5.89
" 1	105	1,000.00	14.38	Aug. 14	30	900.00	
July 25	10	1,200.00		June 1	44	1,000.00	6.03
			3.70				1.64
		\$2,850.00	\$ 26.72			\$2,400.00	\$ 13.56
			2,850.00				2,400.00
Total Debit...		\$2,876.72		Total Credit..		\$2,413.56	
			2,413.56				
Cash Balance		\$	463.16				

## EXPLANATION

The steps in the above solution may be set down as follows :

1. Find due date of each item. This is a general direction that applies to all work on accounts. No figuring is done with any dates but due dates.
2. Find the interest of each item from its due date to the date on which the cash balance is being found.
3. Add debit interests to debit items in order to get the total debit, and add credit interests to credit items in order to get the total credit.
4. Compare total credit with the total debit, and the difference gives cash balance.

NOTE.—There is only one possible point of exception to this general plan. It is illustrated in item 3 on the debit side and item 2 on the credit side. In both these cases the due date is *after* the date on which we are asked to find cash balance. The interest in each case is carried to the reverse side of the account. In thus subtracting the interest from the item, we do not get the absolutely correct present worth of the item, but the result is considered to be near enough for business purposes, and the plan is generally followed in business calculations.

## Questions of the Second Aspect

The student who can find the cash balance of an account should have little difficulty in solving another question of prime importance in business. Let us illustrate.

The cash balance of the account just dealt with is \$463.16, calculated to July 15th. The merchandise balance of this



account—that is, the difference between its debit and credit items—is \$450 (\$2,850 – \$2,400). Using these figures, our question may be stated thus :

## ILLUSTRATION

If \$463.16 will settle an account on July 15th, on what date will \$450 settle it ?

Putting this in the more familiar form of one of our simple interest questions, it may be stated thus : In what time will \$450 amount to \$463.16 simple interest ?

## SOLUTION

\$450, to amount to \$463.16, must accumulate \$13.16 interest.

Interest on \$450 for 1 year at 5% = \$22.50.

\$22.50 is interest for 1 year, or 365 days.

\$13.16 is interest for  $\frac{13.16}{22.50}$  of 365 days = 213 days.

If \$463.16 will settle an account on July 15th, \$450 (a smaller sum of money) will settle it some time before July 15th, because we know that the longer an account runs, the more interest it will accumulate, and the greater will be the amount needed to settle it. This time we have just calculated at 213 days. 213 days before July 15, 1908, gives us Dec. 15, 1907. Therefore, \$450 would settle the account on Dec. 15, 1907.

NOTE.—It may be said that there is no sense in speaking of \$450 settling this account on Dec. 15, 1907, as the first transaction recorded took place on March 10, 1908. It is quite true that the account cannot actually be settled on Dec. 15, 1907, but that fixes a date from which to reckon interest on the account, whenever the debtor comes forward and wishes to settle. We can illustrate this, and prove our previous work, by finding what would settle the account on July 15, 1908. It would require \$450 plus the interest on \$450 from Dec. 15, 1907, to July 15, 1908 (213 days), or, altogether \$463.16.

## SERIES 80

1. (a) What is the cash balance of the following account on Mar. 15, 1908, reckoning interest at 6% ?

(b) On what date would the commercial balance settle it ?

Dr.		YOUELL & WRONG.				Cr.	
1908.							
Jan.	3	To Mdse., net	30	dys.	250		
Feb.	6	“ “ net	30	dys.	300		

2. (a) What is the cash balance of the following account on Dec. 3, 1907, reckoning at 5%?

(b) On what date would the commercial balance settle it?

Dr.				A. WHITE & SONS.				Cr.			
1907.								1907.			
Sept.	12	To Mdse., 30 dys.	450					Oct.	1	By Cash	400
Oct.	7	" " 30 "	310								

3. (a) What amount would settle the following account on May 10, 1908, interest 5%?

(b) When would the commercial balance of the account be an equitable settlement?

Dr.				INGRAM & DAVEY.				Cr.			
1908.								1908.			
Mar.	9	To Mdse., 60 dys.	275					Apr.	10	By Cash	240
Apr.	17	" " 30 "	340					May	1	" "	300

4. (a) What is due on settlement of following account, Dec. 20, 1908, interest 7%?

(b) When should interest start on the account?

Dr.				J. F. FOSTER & Co.				Cr.			
1908.								1908.			
July	10	To Mdse., 2 mos.	560					Aug.	11	By Cash	450
Aug.	1	" " 30 dys.	270					20	" Note, 30 dys.	300	

5. (a) What sum would be required to pay the following account on Sept. 19, 1908, interest at 5%?

(b) Find when the balance of the account should be considered an equitable settlement.

Dr.				C. CHUTE.				Cr.			
1908.								1908.			
May	30	To Sundries, 60 dys	370					June	20	By Cash	450
June	15	" " 30 "	145					"	30	" Draft, 60 dys.	360
"	23	" " 90 "	420								

6. Of the following account find :

(a) The amount due in settlement Aug. 23, 1908, interest at 7%.

(b) When a three months' note given in payment of the account for the commercial balance should be dated so that the note would fall due on the equitable date.

Dr.				GEO YOUNG.				Cr.			
1908.								1908.			
Jan.	10	To Mdse.	468 70					Mar.	28	By Cash	400
Apr.	5	" "	392 60					June	7	" "	145

7. (a) What amount would settle this account on June 20, 1908, interest at 5%?

(b) When would the commercial balance of the account pay it?

Dr.				W. H. HAYSLIP.				Cr.			
1908.				1908.							
Feb.	19	To Mdse., 60 dys. . .	387 25	Mar.	22	By Cash . . . . .	276 70				
Mar.	25	" " . . . . .	175	Apr.	11	" " . . . . .	200				
Apr.	30	" " 60 " . . . .	297 30	June	7	" Note, 1 mo. . . . .	185				

8. (a) What amount would be necessary to pay the following account on Dec. 2, 1908, reckoning interest at 4%?

(b) When would the balance of the account be accepted in settlement?

Dr.		F. L. SANDERS.		Cr.			
1907.				1907.			
July	7	To Mdse., 3 mos. . .	179 30	Aug.	17	By Cash . . . . .	150
Sept.	12	" " 60 dys. . .	210	Sept.	19	" Draft . . . . .	162 75
Nov.	30	" " 60 " . . . .	87 20				

9. (a) What is the cash balance of the following account on July 3, 1908, interest at 5%?

(b) When should a note, at 60 days, for the commercial balance of the account, be dated so that by the prompt payment of same Mr. Boyle would avoid paying interest on the account?

Dr.		H. J. BOYLE.				Cr.		
1908.				1908.				
Apr.	9	To Mdse., 90 dys.	..	327 50	May	3	By Cash . . . . .	268 30
"	12	" " 30 "	..	169 20	"	15	" Note, at 15 dys...	200
May	15	" " 30 "	..	126 35	May	31	" Cash . . . . .	150

10. (a) What amount would settle the following account on Jan. 10, 1909, interest at 5%?

(b) On what date should a note, at 3 months, for the balance of the account be dated so as to fall due on the equitable date of payment?

Dr.		G. P. BROWN.		Cr.			
1908.				1908.			
June	10	To Mdse., 4 mos. . .	850	Aug.	10	By Cash . . . . .	560
July	23	" " 3 " . . . .	600	Sept.	29	" " . . . . .	470
Sept.	15	" " 4 " . .	730	Oct.	7	" Note, 10 dys. . .	350

## EQUATION OF PAYMENTS AND AVERAGING OF ACCOUNTS

To **Average** an account is to find the date on which the account can be settled by the payment of a sum in cash equal to the amount of the merchandise or commercial balance (see work on cash balance).

The **Equated** or **Average Date** for the payment of an account is the date on which the merchandise balance may, with fairness to both debtor and creditor, be proffered in payment.

Sometimes an account has items on one side only. Sometimes there are both debit and credit items. In the first case the work of averaging is spoken of as Simple Average, or Equation of Payments. In the second case it is spoken of as Compound Average. These distinctions, however, cause no particular difference in the plan of working.

### Working Plan

Those who have carefully followed the work on cash balance know already how to average an account. The questions of the second aspect under the head of cash balance are all of them questions in averaging accounts. There is just one point to be noted by the student. In asking for the cash balance of an account, it is natural that the question should give the date on which it is required to find the cash balance. In finding the average date of an account, the student may select his own date up to which he will find the cash balance. He may also select his own rate of interest, as this work of finding the cash balance is merely a means to an end.

There are but two simple steps in the work.

**STEP 1.**—Find the cash balance of the account to any date (preferably the latest due date) and at any rate of interest.

**NOTE.**—This date selected is known as the focal date.

**STEP 2.**—Compare the cash balance with the merchandise balance, and find the date on which the merchandise balance will settle the account.

ILLUSTRATION 1.—Find the average date for the payment of the following account :

<i>Dr.</i>	ANDREW ADAMS.	<i>Cr.</i>
1908.		
May 15	Mdse. @ 30 days \$500.00	
June 9	" @ 60 " 350.00	
July 18	" @ net.... 250.00	
Aug. 5	" @ 30 days 470.00	
	<hr/>	
	\$1,570.00	

## SOLUTION

1st Step: Cash Balance to Sept. 4, at 5%.

<i>Dr.</i>	ANDREW ADAMS.	<i>Cr.</i>
DUE DATES.	ITEMS.	INT.
June 14....	\$500.00	\$5.62
Aug. 8....	350.00	1.29
July 18....	250.00	1.64
Sept. 4....	470.00	0
	<hr/>	<hr/>
	\$1,570.00	\$8.55
		1,570.00
		<hr/>
Cash Balance....	\$1,578.55	

2nd Step: Knowing that \$1,578.55 will settle the account on Sept. 4 find when \$1,570 will settle it.

In other words, find in what time \$1,570 will amount to \$1,578.55 at 5 per cent.

\$78.50 is interest on \$1,570 for 1 yr., or 365 days.

8.55 is interest on \$1,570 for 365 days  $\times \frac{8.55}{78.50}$ , or 40 days.

If \$1,578.55 will settle the account Sept. 4, \$1,570, a *smaller* amount, will settle it 40 days *before* Sept. 4, or July 26, 1908.

ILLUSTRATION 2.—Find the average date of the following account :

<i>Dr.</i>				CHARLES B. HEATH.		<i>Cr.</i>	
1908.				1908.			
May	21	Mdse., 3 mos...	\$500.00	May	24	Cash.....	\$300.00
"	28	" 3 " ..	250.00	June	5	Note, 60 dys....	400.00
June	9	" 30 days	160.00	July	21	Cash.....	100.00



## SOLUTION

1st Step : Cash Balance to Aug. 28 at 5 per cent.

Dr.			CHARLES B. HEATH.			Cr.		
DUE DATES.	ITEMS.	INT.	DUE DATES.	ITEMS.	INT.			
1908.			1908.					
Aug. 21....	\$500 ..	\$ 0.48	May 24....	\$300 ..	\$ 3.95			
" 28....	250 ..	.00	Aug. 7....	400 ..	1.15			
July 9....	160 ..	1.10	July 21....	100 ..	.52			
	<hr/>	<hr/>		<hr/>	<hr/>			
	\$910 ..	\$ 1.58		\$800 ..	\$ 5.62			
		910.00			800.00			
		<hr/>			<hr/>			
		\$911.58			\$805.62			
				Cash Balance....	105.96			
					<hr/>			
					\$911.58			

2nd Step : If \$105.96 will settle the account on Aug. 28, find when \$110 (merchandise balance) will settle it. In other words, find in what time the interest on \$110 will be \$4.04 (\$110.00 - \$105.96).

\$5.50 is the interest on \$110 for 1 year, or 365 days.

4.04 is the interest on \$110 for  $\frac{4.04}{5.50} \times 365$  days, or 268 days.

If \$105.96 will settle the account on Aug. 28, 1908, \$110, which is a *greater* amount, will settle it 268 days *after* Aug. 28th, or May 23rd, 1909.

## The Interest v. Product Method of Averaging Accounts

We have chosen to make the work in averaging accounts an outgrowth of finding cash balance. This we have done only after mature deliberation, as the result of many years' experience in presenting this subject of averaging accounts.

Naturally, the interest method of handling the work has been followed. Our first reason for using this method will be found in the fact that finding cash balance requires the finding of interests, and, if we use the cash balance work as a means of averaging accounts, we therefore naturally use the interest method. There are several other reasons, however. In the first place, it will be found that in many offices where there is a great deal of work along the



2nd Step: If \$107.05 will settle the account on Aug. 28, on what date will \$110 settle it? In other words, how long will it take \$110 to accumulate \$2.95 interest?

\$4.015 is interest on \$110 for 1 year, or 365 days.

2.95 is interest on \$110 for  $\frac{2.95}{4.015} \times 365$  days, or 268 days.

If \$107.05 will settle the account on Aug. 28, 1908, \$110, which is a *greater* amount, will settle it 268 days *after* Aug. 28, or on May 23rd, 1909.

## SOLUTION BY PRODUCT METHOD

Focal date, Aug. 28, 1908

<i>Dr.</i>		CHARLES B. HEATH.		<i>Cr.</i>	
DUE DATES.	ITEMS.	HEATH GAINS	DUE DATES.	ITEMS.	HEATH LOSSES
1908.		Use of \$1 for	1908.		Use of \$1 for
Aug. 21....	\$500	3,500 days.	May 24....	\$300	28,800 days.
“ 28....	250	0 “	Aug. 7....	400	8,400 “
July 9....	160	8,000 “	July 21....	100	3,800 “
		<hr/>			<hr/>
		11,500 days.			41,000 days.

Heath loses use of \$1 for 41,000 days.

Heath gains use of \$1 for 11,500 days.

Heath's net loss = use of \$1 for 29,500 days.

Expressed in terms of \$110, which is the amount owing by Heath, this is equal to a loss of \$110 for 268 days.

Now, an average date should be such that payment on that date should not occasion loss to either person. Therefore, if Heath, who is the one owing the money, is a loser by our date being selected as Aug. 28, 1908, that date is wrong, in that it is too early. In other words, Heath should be given 268 days after Aug. 28, 1908, or until May 23rd, 1909, in which to pay.

## EXPLANATION

To understand why Heath gains in the one series of items and loses in the other, we have only to consider the actual state of affairs as shown by the accounts in the light of our supposition. By the actual account Heath owes us \$500, which is due on Aug. 21. By our supposition we consider that the proper date for settling the account is Aug. 28th. When a man, who owes \$500 due on Aug. 21st, is given until Aug. 28th to pay his account, he is gaining use of his money for the intervening 7 days. The use of \$500 for 7 days is equal to the use of \$1 for 3,500 days. The same reasoning can be employed in all items on the debit side.

Coming to the credit side, we find that the account shows that Heath actually paid us \$300 on May 24th. Where a man makes any payment on May 24th, and we suppose that Aug. 28th is the correct date for settlement, it is clear that, in paying us \$300, Heath has deprived himself of the use of this money from May 24th to Aug. 28th. This means the use of \$300 for 96 days, which is equivalent to the use of \$1 for 28,800 days. The same line of reasoning may be employed with all items on the credit side.

### SERIES 81

1. On Feb. 10, 1908, I bought a bill of goods amounting to \$1,200 on the following terms: \$300 cash, \$500 on a credit of 30 days, and \$400 on a credit of 60 days. On what day might I pay the whole bill of \$1,200 without gain or loss?

2. Smith bought goods of Brown as follows: On May 30, \$200; June 5, \$400; June 15, \$300. No payments having been made by Smith, on what date should he start to pay interest on the whole amount?

3. Bought goods as follows:

Mar. 3, 1908, \$350 on 3 months' credit.

April 10, 1908, \$270 on 2 months' credit.

May 15, 1908, \$125 on 30 days' credit.

What is the average date of payment?

4. John Brown bought goods as follows:

On Jan. 15, 1907, \$500 worth at 30 days' credit.

On Feb. 25, 1907, \$300 worth at 40 days' credit.

On Mar. 20, 1907, \$800 worth at 15 days' credit.

Find the equated date of payment.

5. A bill of goods was purchased on the following terms:  $\frac{1}{4}$  cash,  $\frac{1}{3}$  payable in 10 days,  $\frac{1}{4}$  in 30 days, and the balance in 60 days. Find the average term of credit.

6. A man bought a farm on Mar. 10, and was to pay \$1,000 cash, \$800 in 3 months, \$1,200 in 6 months, and \$1,500 in 9 months. Find the average term of credit and the equated time.

7. Find the equated date of payment of the following account:

June 3, Mdse. @	3 mos. ....	\$1,275.00
" 15, " "	60 days ....	500.00
July 12, " "	3 mos. ....	450.50
Aug. 18, " "	90 days ....	320.87
Sept. 25, " "	3 mos. ....	145.63

8. A person owes a debt of \$1,920, due in 9 months. He pays  $\frac{1}{3}$  in 4 months,  $\frac{1}{4}$  in 5 months,  $\frac{1}{5}$  in 6 months, and  $\frac{1}{6}$  in 7 months. By how much should the term of credit on the balance be extended?

9. H. J. Boyle owes C. Crawford: \$75, due to-day; \$90, due in 30 days; \$80, due in 40 days; and \$120, due in 60 days. If Mr. Boyle pays Mr. Crawford \$60 to-day and \$120 in ten days, how long a term of credit should he have for the payment of the balance?

10. On a debt of \$3,000 due in 9 months from April 1, the following payments were made: July 1, \$600; Aug. 1, \$500; Oct. 1, \$700. What is the equitable date for the payment of the balance?

11. On Feb. 10, 1908, a wholesale fruit dealer made purchases as follows: \$600 on 10 days, \$750 on 20 days, \$625 on 30 days, \$725 on 60 days, and \$850 on 90 days. On what date must he give his note at 30 days to have it mature on the average date of payment?

12. When may the remainder of the following account be paid without loss to either debtor or creditor?

Dr.		THOMAS BROWN.		Cr.			
1907.				1907.			
Aug.	1	To Mdse. ....	375	Sept.	1	By Cash .....	200
"	25	" " .....	425	"	20	" " .....	410

13. Average the following account:

Dr.		J. B. DUNBAR.				Cr.	
1908.				1908.			
Mar.	9	To Mdse., 1 mo . . . .	250	Apr.	1	By Cash . . . . .	175
Apr.	11	" " 60 dys. . . . .	312 50	May	9	" " . . . . .	280
May	17	" " 3 mos. . . . .	275 40	June	21	" " . . . . .	202 90

14. What is the equated time for the payment of the balance of the following account?

Dr.		A. C. NEWCOMB.		Cr.			
1907.				1907.			
May	15	To Mdse., 30 dys. . . .	500	June	20	By Cash . . . . .	300
June	8	" " 60 " . . . . .	750	Aug.	15	" " . . . . .	600
July	12	" " 90 " . . . . .	650	Sept.	1	" Note, 30 dys. . . .	800



<i>Dr.</i>				JAMES MUNRO.				<i>Cr.</i>			
1907.								1907.			
Mar.	1	To Balance .....	325	Mar.	16	By Cash .....	250				
Apr.	16	" Mdse., 2 mos. ..	623 47	May	20	" " .....	300				
May	11	" " 2 " ..	1722 30	July	30	" Note, 30 dys. ..	1000				
June	16	" " 30 dys. ..	975 12	Aug.	31	" Cash .....	500				
July	30	" " 60 " ..	150								
Aug.	17	" " 30 " ..	1600								

## AVERAGING ACCOUNT SALES

Accounts between principal and agent are kept as personal accounts are usually handled. For instance, the agent, in his books, credits the principal's account with all amounts received from sales, collections, or whatever work he is doing for the principal. He debits the account with all amounts paid on account of charges incurred in handling the principal's business, and also with all amounts representing the value of his services, such as commissions, and amounts representing the value of conveniences placed at principal's disposal, such as storage or cartage. The difference between the two sides of the account represents a balance due to principal.

Where it is a case of selling goods for the principal, the statement that is rendered to the principal is spoken of as an *Account Sales*.

Rendering an account sales, like rendering a statement of account, necessitates averaging the account in order that the equated date for the payment of the balance may be found.

Averaging an account sales presents no difficulties that have not been handled in averaging accounts.

There is only one feature to be noticed. Sometimes the commission is put in at the date of rendering the account sales, sometimes at the average date of the sales as made, and sometimes at the average due date of the sales. It is all a matter of the custom of the commission house or of agreement between the parties.

An inquiry among a number of commission houses reveals the fact that, in many of them averaging is a thing unknown. When the last sale is made a cheque is made out for the proceeds, and, along with the account sales, is mailed to the consignor. The transaction is immediately closed.

## ILLUSTRATION

Toronto, March 4, 1907.

Account sales of 1,200 barrels of apples received from J. LLOYD, Stratford Ont., to be sold on their account and risk by

RUTHERFORD &amp; MARSHALL.

1907.		SALES.			
Mar.	5	To A. Brown, at 30 days, 200 bbls..	\$1.50	\$300.00	
"	12	To C. Dunn, note at 30 days, 600 bbls.	1.25	750.00	
"	27	To E. Fall, note at 30 days, 400 bbls.	1.25	500.00	\$1,550.00
		CHARGES.			
Mar.	3	Freight in cash .....		\$110.00	
"	4	Drayage, note 30 days .....		25.00	
		Storage .....		31.75	
		Commission, 3% of sales .....		46.50	213.25
		Net proceeds .....			\$1,336.75

Find the average date for the payment of the proceeds of the above account sales. The date of the storage and of the commission to be the same as the average due date of the sales.

## SOLUTION

1st Step: Set down the sales at their due dates, and find average date as before directed.

April	4	.....	\$300
"	14	.....	750
"	30	.....	500
			<hr/>
			\$1,550

Average date, April 17

2nd Step: Arrange an account showing sales on Cr. side and charges on Dr. side, putting storage and commission in at date just found.

Dr.		LLOYD'S CONSIGNMENT.		Cr.	
1907.		CHARGES.		1907.	
Mar.	3.....	\$110.00		Apr.	17.....
Apr.	6.....	25.00			
"	17.....	31.75			
"	17.....	46.50			
					\$1,550.00

3rd Step : Average this account as before directed.

Average date for balance of account, April 21st.

Therefore net proceeds, \$1,336.75, are due, at per average, April 21st.

NOTE.—In performing the second average, it will be found much shorter to take as the focal date the average due date of the sales.

### SERIES 82

Find the net proceeds of the following account sales, and when due by equation. Consider the commission due as per average sales.

I. Montreal, April 3, 1908.  
Account Sales of Flour.

Sold for acct. of THE LAKE OF THE WOODS MILLING CO.

By C. PINEO & SON.

		SALES.					
Mar.	3 124 bbls.	@	\$6.10	Cash	.....		
"	8 48 "	@	6.20	"	.....		
"	28 120 "	@	5.95	"	.....		
Apr.	2 60 "	@	6.20	"	.....		
		CHARGES.					
Mar.	4	Freight	.....		124	50	
"	20	Advertising	.....		18	25	
Apr.	2	Storage	.....		25	80	
		Commission, 3%	.....				

2. Winnipeg, Dec. 12, 1907.  
Account Sales of Apples. For acct. of V. WEISBROD,  
By F. RUTHERFORD. Aylmer, Ont.

		SALES.					
Nov.	10 85 bbls.	@	\$2.75, 30 days	.....			
"	14 63 "	@	3.10, 20 "	.....			
"	19 110 "	@	2.90, 2 months	.....			
"	23 47 "	@	3.20, 10 days	.....			
Dec.	12 115 "	@	2.95, cash	.....			
		CHARGES.					
Nov.	8	Freight	.....		65	20	
"	15	Cooperage	.....		13		
"	20	Cash, advanced	.....		300		
Dec.	12	Storage	.....		23	60	
		Commission, 4%	.....				

3.

Halifax, N.S., April 3, 1908.

Account Sales of Bacon.

Sold for acct. of THE WM. DAVIES Co.

By L. LANG &amp; SONS.

SALES.							
Feb.	28	800 lbs. @	7½c, cash	.....			
Mar.	20	900 " @	8¼c, 30 days	.....			
"	27	700 " @	7¾c, 10 "	.....			
Apr.	3	600 " @	8½c, 20 "	.....			
CHARGES.							
Feb.	28	Freight and cartage		.....	23	90	
"	28	Insurance		.....	3	50	
Mar.	9	Advertising		.....	8	25	
Apr.	3	Storage		.....	4	70	
		Commission, 3½%		.....			

4.

Toronto, Feb. 10, 1908.

Account Sales of Coffee.

Sold for acct. of JOSE MARQUEZ.

By A. REGAN.

SALES.							
Jan.	3	47 bgs., 5,380 lbs. @	11½c, 30 days	.....			
"	10	56 " 6,720 " @	11¾c, 20 "	.....			
Feb.	10	38 " 4,320 " @	11⅞c, 1 month	.....			
CHARGES.							
Jan.	2	Freight and cartage		.....	78	35	
"	3	Duty		.....	89	25	
Feb.	10	Storage		.....	16	70	
		Commission, 4%		.....			

5.

Montreal, July 3, 1908.

Account Sales of Tea.

Sold for acct. of T. LIPTON.

By TODD &amp; HEPBURN.

SALES.							
June	20	20 half-chests, 1,210 lbs., @	65c, 30 days	.....			
"	30	44 " " 2,640 " @	72c, cash	.....			
July	3	130 " " 7,635 " @	68c, 20 days	.....			
CHARGES.							
June	18	Freight		.....	25		
"	19	Cartage		.....	2	25	
June	25	Cash, advanced		.....	250		
		Commission, 3%		.....			



6.

Toronto, June 17, 1908.

Account Sales of Pork.

Sold for acct. of BROWN &amp; SMITH.

By W. J. LONG &amp; SONS.

		SALES.					
May	28	100 bbls. @ \$17.60,	30 days .....				
June	1	130 " @ 16.35,	1 month .....				
"	10	240 " @ 17.50,	2 months .....				
"	17	30 " @ 15.25,	cash .....				
CHARGES.							
May	25	Freight .....		375			
"	25	Cartage .....		43	50		
June	17	Storage .....		12	50		
"	17	Cash, advanced .....		2500			
		Commission, 3% .....					

## INTEREST ON PARTNERS' ACCOUNTS

Where partners have not the same amounts invested in a business, it is customary for the articles of agreement to contain an interest clause.

The interest clause provides for interest to be allowed to each partner on all investments. If withdrawals are made, a clause may also be inserted providing that each partner is to be charged interest on all sums withdrawn.

ILLUSTRATION.—J. R. Coulan and N. Miller become partners under agreement to share gains and losses equally. It is also agreed that interest at the rate of 10% is to be allowed on all investments and charged on all withdrawals. From the following statement of the partners' accounts find the net credit interest due to each at the end of the year :

Dr.		J. R. COULAN.	Cr.	
1907.	WITHDRAWALS.	1907.	INVESTMENTS.	
Apr. 4 .....	\$3,000.00	Jan. 1 .....	\$6,000.00	
		Sept. 19 .....	5,000.00	
Dr.		N. MILLER.	Cr.	
1907.	WITHDRAWALS.	1907.	INVESTMENTS.	
Mar. 7 .....	\$1,000.00	Jan. 1 .....	\$4,000.00	
Oct. 1 .....	1,200.00	Oct. 6 .....	1,500.00	

### SOLUTION

<i>Dr.</i>				J. B. COULAN.				<i>Cr.</i>			
DATE.		SUMS.	DAYS.	INT.	DATE.		SUMS.	DAYS.	INT.		
1907.					1907.						
Apr. 4....	\$3,000	272	\$223.56		Jan. 1....	\$6,000	365	\$600.00			
					Sept. 19....	5,000	104	124.47			
									\$742.47		
Total credit interest.....							\$742.47				
Total debit interest.....							223.56				
Net credit interest.....							\$518.91				

<i>Dr.</i>					N. MILLER.				<i>Cr.</i>			
DATE.		SUMS.	DAYS.	INT.	DATE.		SUMS.	DAYS.	INT.			
1907.					1907.							
Mar.	7....	\$1,000	298	\$ 81.64	Jan.	1....	\$4,000	365	\$400.00			
Oct.	1....	1,200	92	30.25	Oct.	6....	1,500	87	35.75			
				<u>\$111.89</u>					<u>\$435.75</u>			
Total credit interest.....							\$435.75					
Total debit interest.....							111.89					
Net credit interest.....							<u>\$323.86</u>					

NOTE.—Under the agreement J. B. Coulan is entitled to receive from the firm \$518.91, and N. Miller to receive \$323.86. That is, the firm of Coulan & Miller pays Coulan \$518.91 and the same firm pays Miller \$323.86.

Each partner is, therefore, in the position of receiving money from a firm in which he is a partner. In other words, the payment of these two amounts (\$518.91 and \$323.86, or \$842.77) is a loss to the business, of which loss each partner is under agreement to share one-half. Dividing \$842.77 into two parts, we have \$421.38 to be charged to Coulan and \$421.39 to be charged to Miller.

If Coulan is entitled to draw out \$518.91, he must understand that he is really paying half of the amount required for this interest settlement, or \$421.38. He nets or clears only \$518.91 - \$421.38, or \$97.53. Miller, while he receives from the business \$323.86, has a portion of loss to sustain of \$421.39. In other words, he owes the business \$421.39 - \$323.86, or \$97.53.

Whenever any allowance for interest, salary, or the like is made to a partner, this interesting fact of dual-relationship must not be overlooked.

## SERIES 83

1. On January 1st, 1907, Secord & Caldwell became partners under agreement to share gains and losses equally. Secord invests \$3,000 and Caldwell \$2,500. On April 1 Secord invests \$1,000 more and Caldwell \$2,000 more. On July 1 Secord withdraws \$1,500, and on Sept. 1 Caldwell withdraws \$1,000. It is agreed that interest at the rate of 6% is to be allowed on all investments and charged on all withdrawals. Find the net credit interest due each at the end of the year. If they decide to adjust the interest privately, how much should the one pay the other to make an equitable settlement ?

2. On Mar. 1, 1907, Leeson, Martyn & Baker engaged in business. Leeson invested \$8,000, Martyn \$7,000, and Baker \$6,000. On Aug. 1 Leeson invested \$2,000 additional, but on Dec. 1 withdrew \$1,500. On July 1 Martyn withdrew \$2,000, but on Sept. 1 invested \$1,500 additional. On June 1 Baker invested \$2,000 additional, but on Oct. 1 withdrew \$1,000. If it was agreed to pay interest at the rate of 8% on net investments, find the net credit interest due each on March 1, 1908. If they agreed to adjust the interest without the use of the books, how would it be managed ?

3. On July 1, 1907, White & Johnson became partners under agreement to share gains and losses in proportion of  $\frac{2}{3}$  and  $\frac{1}{3}$ . White invested \$7,000 and Johnson \$4,000. On Jan. 1 White invested \$2,000 additional, but on Feb. 1 withdrew \$2,500, and on April 1 withdrew \$500. On Nov. 1 Johnson invested \$3,000 additional, but on Jan. 1 withdrew \$2,000. If they agreed to pay interest at 7% on net investments, find the net credit interest due each on July 1, 1908. How much should one pay to the other to adjust the interest privately ?

4. On Jan. 1, 1907, Brown & James formed a partnership, agreeing to share gains and losses in the proportion of  $\frac{3}{5}$  and  $\frac{2}{5}$ . Brown's investments were : On Jan. 1, \$9,000 ; on Mar. 10, \$2,000 ; on Oct. 23, \$2,000. Brown's withdrawals were : On May 15, \$1,200 ; on Sept. 10, \$1,500 ; and on Nov. 3, \$1,000. James' investments were : On Jan. 1, \$6,000 ; on May 17, \$2,000 ; and on Sept. 4, \$3,000. James' withdrawals were : On Feb. 7, \$1,000 ; on Oct. 1, \$3,000. If they agreed to pay interest at the rate of 5% on all investments and be charged 5% on all sums withdrawn, find the net credit interest due each on Jan. 1, 1908. How might they adjust the interest without the use of the books ?

5. On May 1, 1907, Kent, Todd & Tufford became partners, agreeing to share gains and losses in proportion of  $\frac{4}{9}$ ,  $\frac{3}{9}$ , and  $\frac{2}{9}$ . Kent invested on May 1 \$8,000, and on Nov. 10 \$3,000, but on Feb. 17 withdrew \$4,000. Todd invested on May 1 \$6,000, but on Nov. 3 withdrew \$500, and on Feb. 20 \$1,000. Tufford invested on May 1 \$4,000, and on Oct. 25 \$2,000, but on Dec. 1 withdrew \$1,400, and on Mar. 12 \$1,200. Find the net credit interest due each on May 1, 1908, if they were to receive interest at the rate of 9% on net investments. With what single amount might each partner's account be debited or credited to make the adjustment?

### ACCOUNTS WITH BANKS

Depositors in banks are of three classes :

1. Depositors on current account, who have the privilege of issuing cheques, but do not, as a rule, receive interest.

2. Depositors in savings banks, who receive interest, but, as a rule, do not issue cheques, except in their own favor.

3. Depositors on deposit receipt, who lend a bank one definite sum, and receive a written promise for the return of the same, and receive interest.

The methods adopted by banks for reckoning interest show considerable variety. They may reckon it on the minimum balance the depositor has to his credit each day, each month, each quarter, or each half-year. And the interest may be added to the principal each quarter or each half-year. In Canada since January, 1909, interest is added half yearly only.

However, if the student familiarizes himself with the two illustrations given, he should have no difficulty in making any calculations desired along these lines.

NOTE 1.—Banks do not pay interest on a fraction of a dollar.

NOTE 2.—In reckoning interest on monthly balances, banks allow  $\frac{1}{12}$  of the interest for one year.

ILLUSTRATION 1.—Verify the amount of \$229.64 appearing in John Smith's bank pass book as the amount which he has to his credit on Dec. 31, 1907, taking his deposits and withdrawals as they appear in the illustration shown. The bank pays interest

at the rate of 3% per annum on minimum monthly balances for each calendar month, and credits same to the accounts on June 30th and Dec. 31st of each year.

DATE.		DEPOSITS.		WITHDRAWALS.		BALANCES.	
1907.							
Jan. ....	10	75				75	
Feb. ....	3	30				105	
" .....	16			20		85	
Mar. ....	12	50				135	
" .....	20	15				150	
May .....	5	80				230	
June .....	7			30		200	
" .....	13	75				275	
" .....	30	<i>Cr. Int.</i> 1	65			276	65
July .....	12	28				304	65
Sept. ....	20			20		284	65
Oct. ....	11			30		254	65
Nov, .....	15	46				300	65
Dec. ....	20			75		225	65
" .....	31	<i>Cr. Int.</i> 3	99			229	64

## SOLUTION

First verify the amounts in the balance column, adding deposits and subtracting cheques.

Next calculate the minimum balance he had to his credit for each calendar month, by considering the amount he had to his credit during each part of the month. Thus, up to Jan. 10 he had nothing to his credit; from Jan. 10 to end of month he had a balance of \$75. So his minimum balance for January was nil. From Feb. 1 to Feb. 3 his balance was \$75; from Feb. 3 to Feb. 16, \$105; from Feb. 16 to end of month, \$85. So his minimum balance for February was \$75, and so on.

The interest to be credited to his account on June 30 was one month's interest on  $\$75 + \$85 + \$150 + \$150 + \$200 = \$660$ .

The interest on \$660 for 1 month at 3% =  $\$660 \times \frac{3}{100} \times \frac{1}{12} = \$1.65$ .

The minimum monthly balances for the last six months of the year were \$276, \$304, \$284, \$254, \$254, and \$225.

Sum of the minimum monthly balances is \$1,597.

The interest on \$1,597 for 1 month at 3% =  $\$1,597 \times \frac{3}{100} \times \frac{1}{12} = \$3.99$ .

The balance to his credit therefore on Dec. 31, 1907, is  $\$225.65 + \$3.99 = \$229.64$ .



ILLUSTRATION 2.—In the previous case what would John Smith have had to his credit on Dec. 31st had the bank allowed interest at 3% per annum on daily balances and credited the interest on June 30th and Dec. 31st ?

DATE.		DEPOSITS.		WITH- DRAWALS	BALANCES.		DYS	PRODUCTS.
1907.								
Jan. ....	10	75			75		24	1800
Feb. ....	3	30			105		13	1365
" .....	16			20	85		24	2040
Mar. ....	12	50			135		8	1080
" .....	20	15			150		46	6900
May .....	5	80			230		33	7590
June .....	7			30	200		6	1200
" .....	13	75			275		17	4675
								26650
" .....	30	<i>Cr. Int.</i>	2	19	277	19	12	3324
July .....	12		28		305	19	70	21350
Sept. ....	20			20	285	19	21	5985
Oct. ....	11			30	255	19	35	8925
Nov. ....	15	46			301	19	35	10535
Dec. ....	20			75	226	19	11	2486
								52605
" .....	31	<i>Cr. Int.</i>	4	32	230	51		

#### SOLUTION

Fill in the "Days" column in the diagram with the number of days each balance was the amount he had to his credit. Thus, \$75, was the amount he had to his credit from Jan. 10 to Feb. 3, or 24 days ; \$105 was his balance from Feb. 3 to Feb. 16, or 13 days, and so on.

He is entitled to interest on \$75 for 24 days, which is the same as the interest on \$1,800 for 1 day. Enter 1800 in the "Products" column. Similarly, the interest on \$105 for 13 days is the same as the interest on \$1,365 for 1 day. Enter 1365 in the "Products" column. Complete the "Products" column by multiplying each balance by the number of days it was the balance.

The total of the "Products" column at the end of June will be the number of dollars upon which 1 day's interest is to be calculated at 3%, and added to his balance.

The interest on \$26,650 for 1 day at 3% =  $\$26,650 \times \frac{3}{100} \times \frac{1}{365} = \$2.19$ .

Similarly the interest to be credited at the end of the second half-year may be found to be \$4.32. So his balance on Dec. 31, 1907, is  $\$226.19 + \$4.32 = \$230.51$ .

#### SERIES 84

1. On January 1, 1907, my bank balance was \$450. During the year I made the following deposits: Jan. 20, \$125; Feb. 4, \$75, April 3, \$37.50; April 25, \$48; July 11, \$140; Aug. 16, \$80; Oct. 28, \$50; Nov. 20, \$76; Dec. 5, \$85. The bank allows interest at 3% on minimum monthly balances, and credits the interest due to depositor's accounts on June 30 and Dec. 31. Rule up form of bank pass book as shown in Illustration 1, and show the balance to my credit on Dec. 31, 1907.

2. On January 1, 1906, John Wilson's bank balance was \$500. During the year he made the following deposits and withdrawals: On Jan. 27, deposited \$250; on Feb. 12, deposited \$100; on March 20, withdrew \$150; on March 28, withdrew \$50; on May 22, deposited \$200; on July 18, deposited \$375; on Aug. 10, withdrew \$90; on Nov. 8, deposited \$200; on Dec. 20, withdrew \$75. The bank allows interest at 3% on minimum monthly balances, and credits interest due to depositors' accounts on Mar. 31, June 30, Sept. 30, and Dec. 31. Rule up bank book showing Mr. Wilson's balance on Dec. 31, 1906.

3. On January 1, 1907, a merchant had a balance in bank of \$600, and during the first half of the year he made the following deposits and withdrawals. Deposits: Jan. 4, \$200; Jan. 24, \$250; Mar. 3, \$275; April 16, \$225; May 5, \$187; June 5, \$300. Withdrawals: Jan. 20, \$125; Feb. 11, \$130; Mar. 17, \$75; April 3, \$150; June 2, \$50; June 20, \$89. If the bank allows interest at 4% on daily balances and credits interest due to depositors' accounts on Mar. 31, June 30, Sept. 30, and Dec. 31, find the merchant's balance on June 30, 1907.

4. On Jan. 1, 1906, Emerson McTaggart's bank balance was \$720. His subsequent deposits were: Jan. 20, \$85; Feb. 27, \$100; April 20, \$80; May 10, \$110; July 13, \$215; Sept. 28, \$150; Dec. 3, \$90. His withdrawals were: Feb. 12, \$175; May 10, \$300; June 16, \$35; Aug. 25, \$140; Nov. 12, \$100. If the bank allows interest at  $3\frac{1}{2}\%$  per annum on minimum quarterly balances and credits interest due to depositors' accounts on June 30 and Dec. 31, find Mr. McTaggart's balance on Dec. 31, 1906.

5. A person made the following deposits in a bank : Jan. 25, 1907, \$200 ; Mar. 13, \$152.75 ; April 10, \$175 ; June 7, \$225 ; Sept. 17, \$160 ; Oct. 7, \$436.28. He withdrew as follows : April 21, \$165 ; June 11, \$215 ; Aug. 9, \$135 ; Dec. 10, \$240. If the bank allows interest at  $4\frac{1}{2}\%$  on daily balances and credits interest due to depositor's accounts on June 30 and Dec. 31, find his balance on Dec. 31, 1907.

## ACCOUNTS WITH STOCK BROKERS

The adjustment of accounts with stock brokers gives us a familiar example of the application of the principles of cash balance.

Stock is either bought outright or on margin.

Where stock is bought outright, through a broker, it is expected that the buyer shall furnish the broker with sufficient money to buy, at the market price, the stock required. Thus, if it is desired to buy outright 50 shares of C.P.R., selling at 155, the buyer must put into the hands of his broker 50 times \$155, or \$7,750, plus the broker's charge for his services.

Buying on margin is simply a form of speculation. The buyer orders his broker to buy, but does not intend to put up the purchase price. Instead, he puts up a margin or small percentage of the par value of the stock bought. This is usually 10% of the par value, but varies with different brokers. The broker then buys the stock with his own funds.

If the stock, after purchase, starts to decline in value, the broker has the margin amount to protect himself. If the speculator does not keep his margin good, the broker will sell the stock to protect himself. This he will always do before the decline has become sufficient to wipe out the margin.

If the stock starts to go up in value, the broker awaits the speculator's order in the matter of selling, and when the sale is ordered, the speculator expects to get back his margin plus his profits.

In theory, this is the idea of buying on margin. In practice, it often degenerates into a pure gamble, as the broker never puts up the purchase price, and it becomes simply a case of wager between the broker and the speculator as to whether the stock will go up or down.

ILLUSTRATION 1.—On January 2nd J. Walters deposited with his broker \$500 as margin for the purchase of 50 shares of Halifax R.R. stock at  $92\frac{1}{4}$ . The stock was sold January 26th at  $96\frac{3}{4}$ . Allowing 6% on deposit, and charging 6% on purchase price, and  $\frac{1}{8}\%$  brokerage each way, what does the broker owe Walters ?

## SOLUTION

## Broker's Account with

<i>Dr.</i>	J. WALTERS.	<i>Cr.</i>	
Jan. 2—To 50 shares Halifax R.R., at 92 $\frac{1}{4}$ .....	\$4,612.50	Jan. 2—By Cash ..... \$ 500.00	
“ 2—To Brokerage, $\frac{1}{8}\%$ .....	6.25	“ 26—By 50 shares Halifax R.R., at 96 $\frac{3}{4}$ .....	4,837.50
“ 26—To Interest on \$4,618.75 for 24 days at 6%	18.22	“ 26—By Interest on \$500 for 24 days at 6% ..	1.97
“ 26—To Brokerage, $\frac{1}{8}\%$ .....	6.25		
“ 26—Balance due J. Walters .....	696.25		
	<hr/> \$5,339.47		<hr/> \$5,339.47

## EXPLANATION

In this account J. Walters is charged with purchase price of the stock, the interest on this money plus the brokerage for buying for the 24 days from date of purchase to date of sale,  $\frac{1}{8}\%$  brokerage for buying, and  $\frac{1}{8}\%$  for selling. Brokerage is always reckoned on par value.

Walters is credited with \$500 margin, the interest on it for 24 days, and the selling price of the stock.

The difference of \$696.25 shows the cash balance due Walters. If he wants to figure his profit, he compares the \$500 that he invested with the \$696.25 he received from the broker. The profit is \$196.25.

NOTE.—In charging interest on purchase price plus brokerage, we are following the custom of brokers.

ILLUSTRATION 2.—Suppose in the previous question that on Jan. 26th the stock had been quoted at 88, how much would Walters need to deposit to make good his margin ?

SOLUTION	
<i>Dr.</i>	<i>Cr.</i>
J. WALTERS.	
Jan. 2—To 50 shares Halifax R.R., at $92\frac{1}{4}$ ..... \$4,612.50	Jan. 2—By Cash ..... \$ 500.00
" 2—To Brokerage, $\frac{1}{8}\%$ ..... 6.25	" 26—By 50 shares Halifax R.R., at 88..... 4,400.00
" 6—To Interest on \$4,618.75 for 24 days at 6% 18.22	" 26—By Interest on \$500 for 24 days at 6% .. 1.97
" 26—Balance ..... 265.00	
<u>\$4,901.97</u>	<u>\$4,901.97</u>

Walters would need to deposit \$500 - \$265, or \$235, to make his margin good.

ILLUSTRATION 3.—On July 1 a broker bought for W. C. McCarter 200 shares Dominion Coal at  $43\frac{5}{8}$ . 30 days later he sold 100 shares at  $41\frac{1}{8}$ , and on Aug. 18th he sold 100 shares at  $39\frac{3}{4}$ . Allowing  $\frac{1}{8}\%$  brokerage each way and 6% interest, how does the account stand ?

SOLUTION	
<i>Dr.</i>	<i>Cr.</i>
W. C. McCARTER.	
July 1—200 shares Dom. Coal at $43\frac{5}{8}$ .. \$8,725.00	July 1—Margin ..... \$2,000.00
" 1—Brokerage, $\frac{1}{8}\%$ . 25.00	" 31—100 shares Dom. Coal at $41\frac{1}{8}$ ... 4,112.50
" 31—Brokerage, $\frac{1}{8}\%$ on 100 shares. 12.50	Aug. 18—Int., 48 days, on \$2,000 ..... 15.78
Aug. 18—Brokerage $\frac{1}{8}\%$ on 100 shares 12.50	" 18—Int., 18 days, on \$4,100 ..... 12.13
" 18—Int., 48 days, on \$8,750 ..... 69.04	" 18—100 shares Dom. Coal at $39\frac{3}{4}$ ... 3,975.00
" 18—Balance due W. C. McCarter .. 1,271.37	
<u>\$10,115.41</u>	<u>\$10,115.41</u>

McCarter put up \$2,000 margin. He receives from the broker only \$1,271.37. He is therefore a loser by \$2,000 - \$1,271.37, or \$728.63.



## SERIES 85

1. On May 10th W. G. Harwood deposited with his broker \$1,000 for the purchase of 100 shares of Bell Telephone stock at  $132\frac{3}{4}$ . The stock was sold June 12th at  $136\frac{1}{2}$ . What were the profits of the sale, if money was worth 5% and there was brokerage each way of  $\frac{1}{8}\%$ ?

2. On Aug. 3rd I purchased through a broker 25 shares of Sao Paulo at  $126\frac{1}{2}$ , and deposited \$250 as margin. On Aug. 30th the stock was sold at  $123\frac{3}{4}$ . What was my loss, allowing  $\frac{1}{8}\%$  brokerage each way and 6% interest on purchase and margin?

3. On June 6th a speculator deposited with his broker \$300 as margin, with instructions to buy Richelieu Navigation stock at  $75\frac{1}{2}$ . On June 20th the stock was quoted at 70. How much should he have deposited with his broker to make his margin of 10% good?

4. On Sept. 20th I bought through my broker 200 shares of Canadian General Electric at  $87\frac{1}{4}$ , and deposited with him \$2,000 as margin. On Oct. 15th he sold 125 shares at  $85\frac{1}{2}$ , and on Oct. 23rd the balance at  $83\frac{3}{4}$ . Find my loss, if the broker's charges were  $\frac{1}{8}\%$  for buying and the same for selling, and money was worth 8%.

5. On May 2nd I deposited with my broker \$800, with instructions to buy 10 shares C.P.R. at  $156\frac{3}{4}$ , 30 shares Consumers' Gas at  $192\frac{1}{4}$ , and 40 shares Winnipeg Ry. at  $157\frac{1}{2}$ . On May 29th the C.P.R. was sold at  $160\frac{1}{2}$ , the Consumers' Gas at  $194\frac{1}{2}$ , and the Winnipeg Ry. at  $158\frac{1}{4}$ . What was my profit if my broker charged me  $\frac{1}{8}\%$  for buying and same for selling, and money was worth 5%?

6. On Mar. 3rd I deposited with my broker \$8,500 as margin, and he purchased for me 150 shares Niagara Nav. at  $123\frac{1}{4}$ , 200 shares Twin City at  $93\frac{3}{4}$ , and 500 shares Toronto Ry. at 108. The stocks on Mar. 28th were quoted as follows: Niagara Nav.,  $120\frac{3}{4}$ ; Twin City,  $88\frac{1}{2}$ ; and Toronto Ry.,  $102\frac{3}{4}$ . How much should I have deposited with my broker to make my margin of 10% good, and to cover commission of  $\frac{1}{8}\%$  for buying and interest at 6%? If I had been unable to have made an additional deposit, and the broker had "sold me out," what would have been my loss?

# COMPOUND INTEREST

In computing simple interest the principal is the same throughout the whole time. In reckoning compound interest, it increases with each interest period—the amount at the end of the first period becoming the principal for the second, that at the end of the second becoming the principal for the third, and so on.

Unless otherwise stated, the interest is compounded annually, thus making the interest period one year. But it may be compounded half-yearly, quarterly, or at any other time agreed upon.

Suppose A borrows from B \$100 for 3 years, and agrees to pay him compound interest at the rate of 5%. At the end of the first year A would owe B \$105. For the second year he would pay interest on \$105. At the end of the second year he would owe \$110.25. For the third year he would pay interest on \$110.25, and at the end of that year would owe \$115.76. The compound interest, therefore, on \$100 for 3 years at 5% is  $\$115.76 - \$100 = \$15.76$ .

## QUESTIONS OF THE FIRST ASPECT

(Do not use interest tables.)

ILLUSTRATION 1.—Find the compound interest on \$640 for 3 years at 4% per annum, compounded yearly.

### SOLUTION 1

Principal .....	\$640.
Interest for 1st year ( $\$640 \times .04$ ) .....	25.60
<hr/>	
Amount for 1st year or principal for 2nd year.....	\$665.60
Interest for 2nd year ( $665.60 \times .04$ ) .....	26.624
<hr/>	
Amount for 2nd year or principal <sup>1</sup> for 3rd year.....	\$692.224
Interest for 3rd year ( $692.224 \times .04$ ).....	27.68896
<hr/>	
Amount for 3rd year .....	\$719.91296
Original principal to be subtracted .....	640.
<hr/>	
Compound interest for 3 years to nearest cent.....	\$ 79.91

NOTE 1.—If interest is compounded semi-annually, work in exactly the same manner as above, but take  $\frac{1}{2}$  the rate for twice the number of years. If quarterly, take  $\frac{1}{4}$  the rate for 4 times the number of years, etc.

NOTE 2.—If the time is not an exact number of interest periods, calculate the interest for the integral number of periods; then, for the remaining time, take the interest for the full period, multiply it by the fraction which this remaining time is of a full period, and add to the amount for the exact periods.

SOLUTION 2

$$\text{The interest for 1 year} = \frac{4}{100} \text{ of principal.}$$

$$\text{The amount at end of 1 year} = \frac{104}{100} \text{ of principal.}$$

$$\text{The interest for 2nd year} = \frac{4}{100} \text{ of this amount.}$$

$$\text{The amount at end of 2nd year} = \frac{104}{100} \text{ of this amount.}$$

$$= \frac{104}{100} \text{ of } \frac{104}{100} \text{ of principal.}$$

$$= \left(\frac{104}{100}\right)^2 \text{ of principal.}$$

$$\text{Similarly, the amount at end of 3rd year} = \left(\frac{104}{100}\right)^3 \text{ of principal.}$$

$$= (1.04)^3 \text{ of principal.}$$

$$= \$640 \times (1.04)^3$$

$$\text{Hence the interest for the 3 years} = 640 \times (1.04)^3 - 640$$

$$= 640 \{(1.04)^3 - 1\}$$

The above solution will be found very valuable in finding the compound interest or the amount when the number of years is large and no interest table is available. This solution really depends on finding  $1.04^3$ , which in this case is easily found. If the time had been 16 years, however, we would have to find  $1.04^{16}$ , which by multiplying by individual factors would be a laborious process, but may be shortened by squaring 1.04, then squaring the result, which would give  $1.04^4$ , then squaring this result, which would give  $1.04^8$ , and again squaring this result, which would give  $1.04^{16}$ , in each case using the contracted method of multiplication of decimals.

A little thought will show how this plan may be adapted to find any power for any rate. Thus,  $1.05^{20}$  would be got by finding  $1.05^{16}$  as above, and multiplying this result by  $1.05^4$ , and so on.

## Compound Interest Table

Showing the amount of \$1 at compound interest for any number of years from 1 year to 55 years inclusive.

Yrs.	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	Yrs.
1	1.0100 000	1.0150 000	1.0200 000	1.0250 000	1.0300 000	1.0350 000	1.0400 000	1
2	1.0201 000	1.0302 250	1.0404 000	1.0506 250	1.0609 000	1.0712 250	1.0816 000	2
3	1.0303 010	1.0456 784	1.0612 080	1.0768 962	1.0927 270	1.1087 178	1.1248 640	3
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	4
5	1.0510 101	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	5
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	6
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	7
8	1.0828 567	1.1264 926	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905	8
9	1.0936 853	1.1433 900	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	9
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1658	1.4105 9876	1.4802 4428	10
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406	11
12	1.1268 250	1.1956 182	1.2682 4179	1.3448 8892	1.4257 6089	1.5110 6866	1.6010 3222	12
13	1.1380 933	1.2155 524	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351	13
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	14
15	1.1609 690	1.2502 321	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4551	15
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125	16
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050	17
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	18
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918	19
20	1.2201 900	1.3463 550	1.4859 4740	1.6366 1644	1.8061 1123	1.9897 8886	2.1911 2314	20
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 8147	2.2787 6807	21
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879	22
23	1.2571 630	1.4083 772	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	23
24	1.2697 346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417	24
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	25
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979	26
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858	27
28	1.3212 910	1.5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332	28
29	1.3345 039	1.5399 805	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145	29
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9570	3.2433 9751	30
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341	31
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875	32
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110	33
34	1.4025 770	1.6589 964	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634	34
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899	35
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3523	2.8982 7833	3.4502 6611	4.1039 3255	36
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 8986	37
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4.4388 1345	38
39	1.4741 225	1.7872 103	2.1647 4177	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 6599	39
40	1.4888 637	1.8140 184	2.2080 3960	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063	40
41	1.5037 524	1.8412 287	2.2522 0046	2.7521 9043	3.3598 9893	4.0978 3381	4.9930 6145	41
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391	42
43	1.5339 778	1.8968 798	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9527	43
44	1.5493 176	1.9253 330	2.3900 5314	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508	44
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568	45
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5086	3.8950 4372	4.8669 4110	6.0748 2271	46
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562	47
48	1.6122 261	2.0434 783	2.5870 7039	3.2714 8956	4.1322 5188	5.2135 8898	6.5705 2824	48
49	1.6283 483	2.0741 305	2.6388 1179	3.3532 7680	4.2562 1944	5.3960 6459	6.8333 4937	49
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335	50
51	1.6610 781	2.1368 211	2.7454 1979	3.5230 3644	4.5154 2320	5.7803 9930	7.3909 5068	51
52	1.6776 889	2.1688 734	2.8003 2819	3.6111 1235	4.6508 8590	5.9827 1327	7.6865 8871	52
53	1.6944 658	2.2014 065	2.8563 3475	3.7013 9016	4.7904 1247	6.1921 0824	7.9940 5226	53
54	1.7114 105	2.2344 276	2.9134 6144	3.7939 2491	4.9341 2485	6.4088 3202	8.3133 1435	54
55	1.7285 246	2.2679 439	2.9717 3067	3.8887 7303	5.0821 4859	6.6331 4114	8.6463 6692	55

NOTE 1.—To find the amount to which any given principal, at compound interest, will increase at any rate per annum, and for any number of years given in the preceding tables:

Multiply the given principal by the amount of \$1 of principal for the given time and rate per annum, as shown in the table.

NOTE 2.—To find the compound interest which any given principal will produce at any rate per annum, and for any number of years given in the preceding tables:

Subtract \$1 from the amount of \$1 for the given time and rate per annum, as shown in the table; the result will be the compound interest of \$1 of principal. Multiply the compound interest of \$1 of principal by the given principal.



# Compound Interest Table

Showing the amount of \$1 at compound interest for any number of years from 1 year to 55 years inclusive.

Yrs.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.	Yrs.
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000	1
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2109 000	2
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000	3
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000	4
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 249	1.6106 100	5
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5868 743	1.6771 001	1.7715 610	6
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171	7
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888	8
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477	9
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425	10
11	1.6228 5505	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167	11
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1334 284	12
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712	13
14	1.8519 4492	1.9799 516	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 963	14
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482	15
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730	16
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703	17
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173	18
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390	19
20	2.4117 1402	2.6532 977	3.2071 305	3.8696 845	4.6609 571	5.6044 108	6.7275 000	20
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0358 337	6.1088 077	7.4002 499	21
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749	22
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024	23
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327	24
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059	25
26	3.1406 7901	3.5556 727	4.5493 850	5.8073 529	7.3963 532	9.3991 579	11.9181 765	26
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942	27
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 614	11.1671 395	14.4209 936	28
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930	29
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023	30
31	3.9138 5745	4.5390 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425	31
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768	32
33	4.2740 3018	5.0031 885	6.8405 899	9.3253 598	12.6760 496	17.1820 284	23.2251 544	33
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699	34
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7783 443	20.4139 679	28.1024 369	35
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805	36
37	5.0968 6049	6.0814 069	8.6560 871	12.2236 181	17.2456 256	24.2538 353	34.0038 486	37
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434	38
39	5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778	39
40	5.8163 6454	7.0539 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556	40
41	6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2562 679	49.7851 811	41
42	6.3516 1548	7.7615 876	11.5570 327	17.1442 568	25.3394 810	37.3175 320	54.7636 992	42
43	6.6374 5818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	40.6761 098	60.2400 692	43
44	6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761	44
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837	45
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321	46
47	7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853	47
48	8.2714 5557	10.4102 697	16.3938 717	25.7289 065	40.2105 731	62.5852 370	97.0172 338	48
49	8.6436 7107	10.9213 351	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106.7189 572	49
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.3908 529	50
51	9.4391 0490	12.0407 698	19.5253 635	31.5190 168	50.6537 415	81.0496 969	129.1299 382	51
52	9.8638 6463	12.6428 083	20.6968 853	33.7253 480	54.7060 408	88.3441 696	142.0429 320	52
53	10.3077 5853	13.2749 487	21.9386 985	36.0861 224	59.0825 241	96.2951 449	156.2472 252	53
54	10.7715 8677	13.9386 961	23.2550 204	38.6121 509	63.8091 260	104.9617 079	171.8719 477	54
55	11.2563 6817	14.6356 309	24.6503 216	41.3150 015	68.9138 561	114.4082 616	189.0591 425	55

NOTE 3.—The amount of \$1 for more than 55 years is equal to the product of the amount of \$1 for any two or more numbers of years in the table whose sum is equal to the given time. Thus, the amount of \$1 at compound interest for 98 years at 8 per cent. per annum, is equal to the product of the amount of \$1, at 8 per cent., for fifty years (\$46.9016125) multiplied by the amount of \$1, at 8 per cent., for 48 years (40.2105731), that is, \$46.9016125 x 40.2105731, or \$1885.9407179+.

NOTE 4.—When the interest is to be compounded semi-annually, find, from the table, the amount or interest at one-half the rate per annum for twice the number of years. Thus, the compound interest of \$1 for 10 years at 5 per cent. per annum, payable semi-annually, is the same as the compound interest of \$1 for 20 years at 2½ per cent. per annum, payable annually; and the compound interest of any sum for 8 years at 4 per cent. per annum, compounded quarterly is the same as the compound interest of the same sum for 32 years, at 1 per cent. per annum, compounded annually.



## SERIES 86

1. Find the compound interest on \$7,500 for 3 years at 5%, compounded annually.
2. Find the compound interest on \$3,700 for 4 years at 7%, compounded annually.
3. Find the compound interest on \$3,250 for 3 years at 6%, compounded semi-annually.
4. Find the amount of \$1,850 for 1 year 6 months at 8%, compounded quarterly.
5. What is the compound amount of \$1,520 for 5 years at 5%?
6. Find the compound interest on \$1,415 from July 16, 1902, to Dec. 27, 1907, at 3%.
7. What is the amount due Oct. 17, 1907, upon a note of \$468.75, dated Nov. 12, 1903, and bearing interest at 5% per annum, compounded annually, no payment having previously been made?
8. Find the compound interest on \$250 for 2 years at 5% per annum, compounded semi-annually.
9. By what decimal must a sum be multiplied to give the amount at compound interest for 4 years at 4%, compounded yearly?
10. Find  $1.06^{12}$  correct to 6 places of decimals.
11. Find  $1.04^{24}$  correct to 6 decimal places.
12. By the process suggested in the previous example find the compound interest on \$362.75 for 15 years at 7%, compounded yearly.
13. In the same way find the compound interest on \$745.80 for 11 years at 4% per annum, compounded semi-annually.

## QUESTIONS OF THE SECOND ASPECT

(Interest table not to be used.)

ILLUSTRATION.—What sum will amount to \$4,024.55 in 3 years at 7% compound interest?

## SOLUTION

\$1 will amount to \$1.225043 in 3 years at 7%.

\$1.225043 is amount of \$1.

\$4,024.55 is amount of  $\$4,024.55 \div \$1.225043 = \$3,285.23$ .

SERIES 87

1. What sum will amount to \$182.33 in 4 years at 5% per annum, calculated yearly?
2. What is the present worth of \$984.54 for 3 years at 4%, compounded yearly?
3. What sum of money will give \$150 interest in 4 years at  $3\frac{1}{2}\%$  per annum, compounded yearly?
4. The difference between the simple and compound interest on a certain sum for 4 years at 6%, compounded yearly, is \$100. Find the sum.
5. Find the present worth of \$367.25 for 3 years at 5% per annum, compounded half-yearly.
6. What sum put out at interest now at  $4\frac{1}{2}\%$  will in 3 years at compound interest, compounded yearly, amount to \$1,200?
7. Find the true discount on \$1,450 for 11 years at 3%, compounded yearly.
8. The compound interest, compounded yearly at 5%, on a certain sum from Jan. 10, 1901, to Sept. 17, 1907, was \$216.15. Find the sum.

QUESTIONS OF THE THIRD ASPECT

(Interest table to be used.)

ILLUSTRATION.—At what rate per annum will \$750 amount to \$2,549.6747 in 21 years?

SOLUTION

\$750 amounts to \$2,549.6747.

\$1 amounts to \$3.3995636.

Then run the eye down the column of years until the given number of years (21) is reached, and thence to the right until the amount is reached, finding it in the 6% column. Hence the required rate is 6%.

SERIES 88

1. At what rate will the compound interest on \$200 for 3 years be \$38.81, if the interest is compounded half-yearly?
2. At what rate, compounded yearly, will \$9,600 in 17 years amount to \$30,324.63?
3. The compound interest on \$1,250 for 16 years, compounded half-yearly, is \$1,105.67 $\frac{1}{2}$ . What is the rate?
4. The amount of \$850 for 23 years, interest compounded yearly, is \$1,677.5515. What is the rate?

## QUESTIONS OF THE FOURTH ASPECT

(Interest tables to be used.)

ILLUSTRATION.—In what time will \$5,000 amount to \$6,100.95 if compounded quarterly at 4% per annum.

SOLUTION

\$5,000 amounts to \$6,100.95.

\$1 amounts to \$1.22019.

Since the given rate is 4% per annum, or 1% per quarter, run the eye down the 1% column until the amount \$1.22019 is reached, which will be found to be the amount for 20 years, or, in this instance, 20 quarters. Hence, the required time is 20 quarters, or 5 years.

### SERIES 89

1. In what time will \$1,500 amount to \$2,984.68 $\frac{1}{2}$  at 3 $\frac{1}{2}$ % compounded yearly?
2. In what time will the compound interest on \$800 at 5% per annum, compounded half-yearly, be \$720.232?
3. In what time will \$750 amount to \$3,322.80 at 7% per annum, compounded yearly?
4. In what time will the compound interest on \$625 be \$463.13 $\frac{3}{4}$ , if compounded quarterly at 8%?

### SERIES 90

Work the following questions with the use of the compound interest tables:

1. Find the interest on \$3,600 for 18 years at 7%, compounded annually.
2. Find the amount of \$787.30 for 36 years if compounded annually at 5%.
3. Find the interest on \$1,960 for 20 years at 5%, compounded half-yearly.
4. Find the amount of \$4,500 for 8 years at 6%, compounded quarterly.
5. Find the compound interest on \$1,600 for 28 years at 7%, compounded yearly.
6. If \$300 be deposited in a savings bank for 16 years, and the interest be compounded semi-annually at 7%, how much would the amount be at the end of the time?

7. What sum will amount to \$2,912.71 in 15 years at 6%, compounded semi-annually ?

8. On what sum will the compound interest for 24 years at 5% per annum, compounded yearly, be \$4,105.31 ?

9. At what rate will \$3,500 amount to \$8,007.74688 if compounded semi-annually for 14 years ?

10. At what rate will \$750 produce \$1,355.09528 interest if compounded semi-annually for 15 years ?

11. In what time will \$750 amount to \$1,578.639 if compounded annually at 7% ?

12. In what time will \$100 yield \$75.350605 interest if compounded half-yearly at 6% ?

13. In how many years will the compound amount of a sum at 6%, compounded yearly, be 3.60354 times the sum ?

14. In how many years at 8%, compounded half-yearly, will the interest be 2.50806 times the sum ?

15. In how many years will a sum double itself at  $3\frac{1}{2}\%$  per annum, compounded yearly ?

16. What rate per cent. per annum, compounded yearly, is equivalent to 6% per annum compounded half-yearly ?

17. What rate per cent. per annum, compounded half-yearly, is equivalent to 6% per annum compounded yearly ?

18. The difference between the simple and compound interest on a certain sum for 4 years at 6%, compounded yearly, is \$100. Find the sum.

19. The difference between the interest at 10% per annum, added yearly, and that added half-yearly, for two years, is \$55.06 $\frac{1}{4}$ . Find the sum.

20. The difference between the interest at 8% per annum, added yearly, and that added half-yearly, for 6 years, is \$17.70. Find the sum.

## ANNUITIES

An **Annuity** is a specified sum of money paid annually or at equal periods, half-yearly, quarterly, and so on.

A **Certain Annuity** is one which begins and ends at a fixed time.

For instance, A is told on Jan. 1, 1902, that, starting with Jan. 1, 1903, he will be entitled to receive \$100 each year for three years. This might be called a three-year annuity of \$100, and might be presented to the mind thus :

Jan. 1, 1902....	\$100	\$100	\$100
	Jan. 1, 1903.	Jan. 1, 1904.	Jan. 1, 1905.

If this annuity is to be derived from an investment of money, loaned at 5%, it would require the use of \$2,000 for three years to secure the income. The principal amount of \$2,000 would need to be put out at interest on Jan. 1, 1902, to have the first \$100 ready on Jan. 1, 1903.

This explains the law that an annuity begins, not at date of first payment, but one annuity interval before.

A **Perpetual Annuity**, or **Perpetuity**, is one which continues for ever. Thus, \$100, if paid each year for ever, would require the perpetual use of \$2,000 capital at 5%.

An **Annuity in Possession**, or an **Immediate Annuity**, is one that begins immediately.

A **Deferred Annuity**, or an **Annuity in Reversion**, is one that begins at some future time.

An **Annuity in Arrears**, or **Forborne**, is one on which the payments are not made when due.

The **Amount**, or **Final Value**, of an annuity is the sum to which all its payments, with interest on each, will amount at its termination.



## QUESTIONS OF THE FIRST ASPECT

Given the annuity, the time, and the rate, to find its final value

Suppose in the case mentioned above that A did not get his \$100 each year as intended. The amount due him at the end of the three years might be expressed as follows :

\$100 plus interest on it for 2 years  
 \$100 plus interest on it for 1 year  
 \$100

In other words, he would be entitled to his \$300 and also to the interest for the time for which he is kept out of the use of his money. Note that the first interest period is one year less than the number of years marking the duration of the annuity.

Annuities in actual practice are worked out on the lines of compound interest. It is absolutely necessary, therefore, that a good knowledge of compound interest should be in hand. Granting this, we have acquired, it may be insensibly, the introductory steps in calculating the final value of an annuity.

Let us explain our position. Suppose we have to find the amount of \$1 for 3 years at 3%, compound interest.

\$1	\$1
.03	.03
<hr style="width: 100px; margin: 0;"/> \$1.03 end of 1st year	<hr style="width: 100px; margin: 0;"/> \$ .03
.0309	\$1.03
<hr style="width: 100px; margin: 0;"/> \$1.0609 end of 2nd year	<hr style="width: 100px; margin: 0;"/> .03
.031827	<hr style="width: 100px; margin: 0;"/> \$ .0309
<hr style="width: 100px; margin: 0;"/> \$1.092727 end of 3rd year	<hr style="width: 100px; margin: 0;"/> \$1.0609
	<hr style="width: 100px; margin: 0;"/> .03
	<hr style="width: 100px; margin: 0;"/> \$ .031827

The amount of \$1 is \$1.092727, or the interest alone is \$.092727. Now, of what does this interest of \$.092727 consist? Simply the interest on \$1 each year—3c—deposited at the commencement of each year, and also allowed to gather interest. Suppose, for instance, we kept our \$1 in one bank, and took the interest every year and placed it in another bank. The amount we would have in the second bank at the end of the time would be the amount of an annual deposit of 3c with the interest at the rate of 3%, compound, added to it.

Now, an annual deposit is called an annuity, and we see that in finding the amount of \$1 for 3 years at 3%, compound interest, we cannot help finding the amount of an annuity of 3c for 3 years at 3%, compound interest.

Knowing, then, the amount of an annuity of 3c to be \$.092727, we can easily find the amount of an annuity of \$1. Thus :

If annuity of 3c amounts to \$.092727

∴ annuity of 1c would amount to  $\frac{$.092727}{3}$

And annuity of 100c would amount to  $100 \times \frac{$.092727}{3}$

That is, we multiply our figure by 100 and divide it by 3, or, which is the same thing, we divide our figure by .03.

From all of which we deduce this useful rule :

### RULE

*To find the value at maturity of an annuity of \$1, we find the compound interest of \$1 for the given time at the given rate, and divide by the given rate expressed decimally.*

When we know what an annuity of \$1 amounts to, we can easily find by multiplication the amount of any annuity.

Now, even in finding the compound interest, we can take a shorter method than that already expressed. The amount of \$1 for 3 years at 3% can be expressed thus :

$$(1.03)^3$$

To get the interest alone we would subtract our principal of \$1 from this, giving us

$$(1.03)^3 - 1$$

and when we divide this interest by the rate we have

$$\frac{(1.03)^3 - 1}{.03}$$

or, putting it generally for all cases, the amount of \$1 annuity

$$= \frac{(1 + \text{rate})^{\text{time}} - 1}{\text{rate}}$$

ILLUSTRATION 1.—What is the amount of an annuity of \$400 for 23 years at 5% compound interest ?

SOLUTION BY RULE ESTABLISHED

$$\begin{aligned}
 & \$400 \times \left\{ \frac{(1.05)^{23} - 1}{.05} \right\} \\
 &= \$400 \times \left\{ \frac{(3.071524) - 1}{.05} \right\} \\
 &= \$400 \frac{(2.071524)}{.05} \\
 &= \frac{\$828.6096}{.05} \\
 &= \$16572.19
 \end{aligned}$$

ILLUSTRATION 2.—Find the amount accumulated at the end of 15 years by a person who deposits in a bank at the beginning of each year the sum of \$200, the bank paying 4% interest, compounded half-yearly.

SOLUTION

Since each deposit is made at the beginning of the year, the first deposit bears interest for 15 years, not for 14 years, as in the case of a 15-year annuity ; and there will be no deposit at the end of the 15th year. We can, therefore, get the result by finding the amount of a 15-year annuity for the same time and rate, and adding to it the amount of one payment for 15 years and subtracting one payment. Or, which is the same thing, to the amount of the 15-year annuity we can add the compound interest on one payment for 15 years.

The rate of 4%, when compounded semi-annually, is equal to an effective rate of 4.04% per annum.

$$(1.0404)^{15} = (1.02)^{30} = 1.81136.$$

$$\text{Result} = \$200 \times \left( \frac{1.0404^{15} - 1}{.0404} + 1.0404^{15} - 1 \right) = \$4178.91$$

### SERIES 91

1. What is the final value of an annuity of \$350 for 10 years at 6% per annum, compounded yearly ?

2. What will an annuity of \$74 amount to in 30 years at 4%, compound interest ?

3. What is the final value of an annual pension of \$210 for 12 years at 4%, compound interest ?

4. A man pays \$150 yearly for 15 years for an endowment policy of \$2,500. Find the accumulated value of payments, reckoning money at 6% per annum.

5. Find the amount of an annuity of \$500 for 3 years, reckoning money at 10% per annum, compounded semi-annually.

6. If \$500 is deposited at the beginning of each year for 8 years, what amount is due at the end of the time, money being worth 5% per annum, compound interest ?

7. What amount will a man have to his credit who deposits in a savings bank \$450 at the beginning of each year for 10 years, if the bank allows interest at the rate of 5% per annum, compounded half-yearly ?

8. If a man deposits in a savings bank \$100 at the beginning of each year for 3 years, what amount will there be to his credit at the end of the third year if the bank allows interest in the meanwhile at 3%, compounded half-yearly ?

### QUESTIONS OF THE SECOND ASPECT

Given the amount of the annuity, the time, and the rate, to find the annuity.

ILLUSTRATION 1.—What sum of money deposited at the end of each year for the next ten years will amount to \$800, money being worth 5% ?

#### SOLUTION

A deposit of \$1 at the end of each year for ten years will amount to

$$\frac{\$1 \times (1.05^{10} - 1)}{.05}$$

\$800 will, then, be the amount of an annual deposit of

$$\$800 \div \frac{(1.05^{10} - 1)}{.05} = \frac{\$800 \times .05}{1.05^{10} - 1} = \frac{\$40}{.62889} = \$63.60.$$

ILLUSTRATION 2.—The town of Woodstock borrowed \$20,000, and agreed to pay 5% compound interest. What sum must be set apart annually as a sinking fund to pay the debt in 12 years ?

**SOLUTION.**—This time the question within the main question is to find what is due at the end of the time, viz., the amount of \$20,000 for 12 years at 5% compound interest.

$$\begin{aligned}\text{Amount} &= \$20,000 \times (1.05)^{12} \\ &= \$35,917.12 \\ \text{Then} \quad & \$35,917.12 \div \frac{(1.05)^{12} - 1}{.05} \\ &= \$2,256.58 \text{ Answer.}\end{aligned}$$

**ILLUSTRATION 3.**—A farmer has a mortgage on his farm of \$5,340, principal payable in 5 years from Jan. 1, 1898, interest payable half-yearly at 5%. He has provided for the interest, but desires to place in a savings bank an annual deposit which, compounded at 4% every 6 months, will give him enough to pay off the mortgage at the end of 5 years. What is the annual deposit to be made on Jan. 1, 1898, and the four following years, to have \$5,340 at end of 5 years ?

**SOLUTION.**—A reference to Illustration 2, Questions of First Aspect, will show the effect of the deposit being made at the beginning of the year, instead of at the end—the value of \$1 annuity must be increased by the compound interest on \$1 for the full time. The solution then appears as follows :

$$\begin{aligned}\$5,340 \div \left\{ \frac{(1.0404)^5 - 1}{.0404} + (1.0404)^5 - 1 \right\} \\ = \$946.87 \text{ Answer.}\end{aligned}$$

#### SERIES 92

1. What annual deposit for 15 years at 5% per annum will amount to \$5,000 ?

2. A railroad company issued sinking fund bonds at 6% for \$200,000, payable in 10 years. If at compound interest, what sum must be set apart annually to meet interest and principal when due ?

3. What sum must be set apart annually to rebuild a bridge costing \$30,000, estimated to last 17 years, allowing 5% compound interest ?

4. A merchant is mortgagor on a mortgage of \$5,000 due in 1902. He has provided for interest, but intends to pay principal out of proceeds of four mortgages of \$1,250, each of which he holds, and which expire one in 1898, 1899, 1900, 1901. He



proposes to invest sufficient of the principal of these at compound interest, 4% half-yearly, to provide for the \$5,000 he owes in 1902. How much of each does he invest ?

5. A man pays \$240 yearly for 15 years for an endowment policy of \$4,000. Reckoning money worth 6% per annum, payable yearly, how much is he paying each year for the life risk ?

6. A corporation obtains a loan of \$100,000, to be paid during or at the end of 15 years. Show, by finding in each case the annual tax to be collected, which of the following ways is the best, and by how much :

(a) Pay the interest annually at 5% per annum, and create a sinking fund to meet the debt, to be invested at 4% per annum, convertible half-yearly.

(b) Repay loan and interest at  $5\frac{1}{2}\%$  per annum in 15 equal annual instalments.

(c) Invest annually an amount bearing interest at  $6\frac{1}{2}\%$  per annum which, at the end of 15 years, will repay the loan and interest at 6% per annum. ( $1.055^{15} = 2.232479$ ;  $1.065^{15} = 2.57184$ .)

### QUESTIONS OF THE THIRD ASPECT

Given the necessary data, to find the present worth of an annuity.

ILLUSTRATION 1.—What is the present value of an annuity of \$154 for 19 years at 5% compound interest ?

#### SOLUTION

$$\text{Find value of annuity} = \frac{\$154 (1.05^{19} - 1)}{.05}$$

Amount of \$1 for 19 years at 5% compound interest =  $\$(1.05)^{19}$   
 $\$(1.05)^{19}$  has for its present value \$1.

$$\begin{aligned} \frac{\$154 (1.05^{19} - 1)}{.05} \text{ has for its present value } & \frac{\$154 (1.05^{19} - 1)}{.05} \div 1.05^{19} \\ & = \frac{\$154 (1.05^{19} - 1)}{.05 (1.05)^{19}} = \frac{\$3,080 (1.52695)}{2.52695} = \$1,861.14. \end{aligned}$$

ILLUSTRATION 2.—What is the present value of an annuity of \$200, deferred 5 years and to run 10 years, if money is worth 5% per annum, payable yearly ?

SOLUTION

$$\text{Final value of annuity} = \frac{\$200 (1.05^{10} - 1)}{.05}$$

Present worth for 15 years (5 years + 10 years)

$$= \frac{\$200 (1.05^{10} - 1)}{.05} \times \frac{1}{1.05^{15}} = \$1,210.04.$$

ILLUSTRATION 3.—A town issues debentures for \$12,000, bearing interest at 6%, payable yearly, and to run 5 years. For what sum should they sell, money being worth 5% yearly?

SOLUTION.—In this case there are really two questions to solve, viz., to find the present worth of the debenture itself and the present worth of the interest on the debenture. The interest, 6% of \$12,000, gives us a figure of \$720 to be paid each year. The two results added give us the complete P.W.

$$\frac{\$12,000}{1.05^5} + \frac{\$720 (1.05^5 - 1)}{.05 (1.05)^5} = \$12,519.54.$$

ILLUSTRATION 4.—If money is worth 5%, find the present value of a perpetuity of \$450.

SOLUTION.—To produce \$450 yearly for ever would require the perpetual use of  $\$450 \times \frac{100}{5} = \$9,000$ .

The present value of the perpetuity is, therefore, \$9,000.

SERIES 93

1. If money is worth 5% per annum, what should be paid for an annuity of \$400 to run 4 years?

2. Find the present worth of an annuity of \$1,000 to run 15 years, the first payment to be made at the end of 1 year, calculated at 4% per annum.

3. A man agrees to pay for a farm \$1,600 a year for 5 years. What sum paid now would be equivalent to this price, money being worth 4% per annum, payable yearly?

4. Compare the present values of two scholarships payable half-yearly, one of \$50 to continue  $2\frac{1}{2}$  years, and the other for \$35 to continue  $4\frac{1}{2}$  years, reckoning 5% compound interest, compounded half-yearly.

5. What is the present value of a perpetual annuity of \$250, money being worth 4% per annum, payable yearly?

6. A farm yields a rental of \$400 yearly. What is the value of this farm, money being worth 6% per annum?

7. Find how much should be paid for a perpetual annuity of \$100, deferred 7 years, calculated at 5% yearly.

NOTE.—The expression, “deferred 7 years,” means that the annuity begins in 7 years; this means that the first payment will be made at the end of the eighth year. The present worth in this case will be for 7 years.

8. What is the present value of a perpetual annuity of \$250, the first payment to be made at the end of 6 years, calculated at 4% per annum?

9. Find the present value of an annuity deferred 1 year and to run 4 years, calculated at 5% yearly.

10. A mortgage of \$2,500, bearing interest at 8% per annum, payable half-yearly, has four years to run. Find its present value, calculated at 6% per annum, payable half-yearly.

11. A mortgage of \$5,000, bearing interest at 6% per annum, payable yearly, has 10 years to run. Find its present value, money being worth 5% per annum, payable half-yearly?

12. Find the present value of a mortgage for \$10,000, bearing interest at 6% per annum, payable half-yearly, and having three years to run, money being worth 4% per annum, payable half-yearly.

13. Bonds bearing 6%, payable annually, have 3 years to run. What can I afford to pay for them so that I shall make 5% per annum on my money as long as it is outstanding?

14. What should be paid for a \$1,000 coupon bond maturing five years hence, and bearing 4% interest, payable annually, so that the investor will receive 5% per annum compound interest on his money?

15. What can I afford to pay for a \$2,000 bond maturing 5 years hence, and bearing interest at 7%, payable annually, so that I shall be making 6% compound interest on my money?

16. What can I afford to pay for a debenture for \$100, having 3 years to run and bearing interest at 5%, payable yearly, so that I may realize  $4\frac{1}{2}\%$  per annum on the investment?

17. What amount will be required to be raised annually at the beginning of each year for 5 years so that there may be sufficient to provide a sinking fund of \$25,000 at the end of the 5th year, and to pay interest on \$25,000 in the meanwhile at 5% per annum, money being worth 4%?

18. Find the present value of an annuity of \$200, payable for 12 years, the first payment to be made at the end of 2 years, money at 3% per annum yearly.

19. What sum of money, deposited at the end of each year for the next five years, will then be sufficient to purchase a perpetual annuity of \$50, deferred 2 years, money being worth 6% yearly?

20. What sum of money, deposited at the end of each year for the next six years, will then be sufficient to purchase an annuity of \$500, deferred 2 years, to run 5 years, money being worth 4% per annum, payable yearly ?

21. What sum of money deposited at the end of each year for the next five years, will then be sufficient to purchase a perpetual annuity of \$100, deferred 2 years, money being worth 5% ?

22. Money being worth 6% per annum, compound interest,

(a) What would be the cost of a 5-year annuity of \$400 ?

(b) What annuity to run 5 years could be bought for \$2,000 ?

23. What is the present value of a mortgage of \$5,000, bearing interest at 6%, payable yearly, having 5 years and 6 months to run, the next payment of interest being due in 6 months, money being worth 5% per annum, payable yearly ?

24. A farm bears a mortgage of \$3,000 at 8% interest, payable half-yearly ; the mortgage has 5 years to run. What sum paid now would be equivalent to reducing the interest on the mortgage to 5%, money being worth 4% per annum, payable half-yearly ?

25. Find the present value of an annuity of \$100, deferred 3 years and to run 4 years, calculated at 5% yearly.

# FOREIGN TRADE

## BILLS OF EXCHANGE

Importing goods into Canada makes it necessary that remittances of money should be made in payment of these goods to the persons from whom the goods are bought.

As in the case of cancelling domestic debts, where the creditor lives in a different city in Canada, we find that the usual custom in making a foreign remittance is to purchase a bill of exchange, which may in turn be remitted in cancellation of the debt.

Foreign bills of exchange are usually drawn in sets of three to provide against loss in transmitting. This custom is a long-established one and still obtains, although the reason for the custom in these days of safe and rapid ocean transit is not so apparent. The wording of each of the bills in a set of exchange will make it clear that the honoring of one practically cancels the other two. The following forms will illustrate a set of foreign exchange :

### Set of Exchange

1. Exchange for £500.	Toronto, Ont., April 22, 1908.
Sixty days after sight of this First of Exchange.....	
(Second and Third of the same tenor and date unpaid),	
Pay to the order of Messrs. S. E. Hill & Co. ....	
Five Hundred Pounds Sterling .....	
Value received, and charge the same to account of	
To J. S. Morgan & Co.,	A. E. AMES & Co.
London.	
No. 149.	



2. *Exchange for £500.* *Toronto, Ont., April 22, 1908.*  
*Sixty days after sight of this Second of Exchange.....*  
*(First and Third of the same tenor and date unpaid),.....*  
*Pay to the order of Messrs. S. E. Hill & Co. ....*  
*Five Hundred Pounds Sterling .....*  
*Value received, and charge the same to account of*  
*To J. S. Morgan & Co.,*  
*London.* *A. E. AMES & Co.*  
 No. 149.

3. *Exchange for £500.* *Toronto, Ont., April 22, 1908.*  
*Sixty days after sight of this Third of Exchange.....*  
*(First and Second of the same tenor and date unpaid),*  
*Pay to the order of Messrs. S. E. Hill & Co. ....*  
*Five Hundred Pounds Sterling .....*  
*Value received, and charge the same to account of*  
*To J. S. Morgan & Co.,*  
*London.* *A. E. AMES & Co.*  
 No. 149

The **Par of Exchange** is the established value of the monetary unit of one country expressed in the currency of the other.

Thus, we read in our Canadian Currency Act that "The currency of Canada shall be such that the British sovereign of the weight and fineness now prescribed by the laws of the United Kingdom shall be equal to and pass current for  $\$4.86\frac{2}{3}$  of the currency of Canada." This means that the par of exchange between Canada and Great Britain is  $\$4.86\frac{2}{3}$  for a sovereign or one pound sterling.

In the same way the value of the monetary units of all foreign countries has been established for us, and the table we append herewith will give these values.

The **Rate of Exchange** is the market value of the monetary unit of one country estimated in the currency of another.

Thus, we find that, while one pound is equal to  $\$4.86\frac{2}{3}$  of our money, it does not always follow that in purchasing a sterling bill of exchange that we can get it at the par value. The bank is entitled to a commission for issuing the bill of exchange. This always has a tendency to increase the cost of the exchange above the par value. Then, again, foreign bills may be drawn at any length of time. The customary bills, however, are drawn either at sight or at sixty days' sight. It is natural that a quotation on a sight bill is going to be higher than a quotation on a sixty-day bill.

## Values of Foreign Money in Canadian Currency

COUNTRY.	MONEY UNIT.	STANDARD.	VALUE IN CANADIAN MONEY.	STANDARD COIN.
Argentine Republic.....	Peso-fuerte.....	Gold and Silver.....	\$ .96.5	(See Great Britain.)
Australia.....	Florin.....	Silver.....	.50.7	
Austria.....	Franc.....	Gold and Silver.....	.19.3	5, 10 and 20 francs.
Belgium.....	Franc.....	Silver.....	.82.3	Boliviano.
Bolivia.....	Boliviano.....	Gold.....	.54.6	
Brazil.....	Milreis of 1,000 reis.....	Silver.....	.96.5	
Bogotá.....	Peso.....	Gold and Silver.....	.91.2	Condor, doubloon and escudo.
Central America.....	Dollar.....	Silver.....	.93.5	
Chili.....	Peso.....	Gold.....	.91.2	
China.....	Tael.....	Gold and Silver.....	1.38.0	
Cuba.....	Peso.....	Gold and Silver.....	.93.2	1-16, ¼, ½ and 1 doubloon.
Denmark.....	Crown.....	Gold.....	.26.8	10 and 20 crowns.
Ecuador.....	Peso.....	Silver.....	.82.3	Peso.
Egypt.....	Piaster.....	Gold.....	.04.9	5, 10, 25, 50 and 100 piasters.
France.....	Franc.....	Gold and Silver.....	.19.3	5, 10 and 20 francs.
Great Britain.....	Pound Sterling.....	Gold.....	4.86.66%	½ sovereign and sovereign.
Greece.....	Drachma.....	Gold and Silver.....	.19.3	5, 10, 20, 50 and 100 drachmas.
German Empire.....	Mark.....	Gold.....	.23.8	5, 10 and 20 marks.
India.....	Rupee of 16 annas.....	Silver.....	.59.0	
Italy.....	Lira.....	Gold and Silver.....	.19.3	5, 10, 20, 60 and 100 lire.
Japan.....	Yen.....	Silver.....	.88.8	1.2, 5, 10 and 20 yen, gold and silver yen.
Libertia.....	Dollar.....	Gold.....	1.00.0	
Mexico.....	Dollar.....	Silver.....	.89.4	Peso or dollar 5, 10, 25 and 50 centavo.
Netherlands.....	Florin.....	Gold and Silver.....	.40.2	10 and 20 crowns.
Norway.....	Crown.....	Gold.....	.26.8	
Peru.....	Sol.....	Silver.....	.82.3	Sol.
Portugal.....	Milreis of 1,000 reis.....	Gold.....	1.08.0	2, 5 and 10 milrees.
Russia.....	Roubles of 100 copecks.....	Silver.....	.65.8	¼, ½ and one rouble.
Sandwich Islands.....	Dollar.....	Gold.....	1.00.0	
Spain.....	Peseta of 100 centimes.....	Gold and Silver.....	.19.3	5, 10, 20, 50 and 100 pesetas.
Sweden.....	Crown.....	Gold.....	.26.8	10 and 20 crowns.
Switzerland.....	Franc.....	Gold and Silver.....	.19.3	5, 10 and 20 francs.
Tripoli.....	Mahbub of 20 piasters.....	Silver.....	.94.3	
Turkey.....	Piaster.....	Gold.....	.04.1	25, 50, 100, 250 and 500 piasters.
United States.....	Dollar.....	Gold and Silver.....	1.00.0	
U. S. of Columbia.....	Peso.....	Silver.....	.82.3	Peso.
Uruguay.....	Patacon.....	Silver.....	.94.9	
Venezuela.....	Bolivar.....	Gold and Silver.....	.19.2	5, 10, 20, 50 and 100 Bolivars.

## Peculiar Methods of Quoting the Rate of Exchange

Ordinarily we expect that the rate of exchange would be quoted by giving the value of the monetary unit of any country in terms of our own currency.

Thus, the obvious way of quoting the rate for sterling exchange would be to give the value of one pound in our currency. Accordingly, we may find that sterling exchange is quoted at \$4.88, \$4.89, and so on.

There is another method, however, still in use in Canada, of quoting sterling exchange, which the student will do well to recognize. This method of quoting is based on a reference to the old par of exchange, which used to be  $\$4\frac{1}{2}$  for one pound sterling. As this was below the intrinsic value, the par was raised to its present figure of  $\$4.86\frac{2}{3}$ . The old par had been in use so long, however, that the custom grew up, even after the new par had been established, of quoting sterling exchange at a certain increase on the old par. Thus, if we find that sterling exchange is quoted at a premium of  $9\frac{1}{2}\%$ , or, which is the same thing, at  $109\frac{1}{2}$ , we are to understand this as being a premium of  $9\frac{1}{2}\%$  on the old par of  $\$4\frac{1}{2}$ . This increase of  $9\frac{1}{2}\%$ , by the way, just brings us to the present par of  $\$4.86\frac{2}{3}$ , and explains the somewhat queer expression that sterling exchange is at par when it is at a premium of  $9\frac{1}{2}\%$ . This means that it is at the present par when it is at  $9\frac{1}{2}\%$  premium over the old par. If sterling exchange is quoted at  $10\%$  premium, it means  $10\%$  premium on the old par of  $\$4\frac{1}{2}$ . In this way any percentage may be quoted on this old par.

Exchange on France, Belgium, and Switzerland, in which countries, as it will be noted in the table, the franc is the monetary unit, would naturally be expected to be quoted by giving the value of one franc in our money. Thus, a franc at par is worth  $19\frac{3}{10}$  cents. If, therefore, we found French exchange quoted at  $19\frac{3}{10}$ , or 20, or  $20\frac{1}{4}$ , the meaning would be clear. However, we find that this exchange is more frequently quoted by giving the number of francs that go to make up one dollar of our money. Thus, when exchange is at 5.27, it means that one dollar will buy 5.27 francs.

Exchange on Germany, in which country the monetary unit is the mark, we would naturally expect to be stated by giving the value of one mark in our money. One mark at par is worth  $23\frac{8}{10}$  cents. Therefore, if we find German exchange quoted at  $23\frac{8}{10}$ , or  $23\frac{1}{2}$ , or 24, the meaning would be clear. These figures would represent the value in cents of one mark under different conditions. The commoner way, however, is to quote the value of four marks in cents. Thus, when we find German exchange quoted at 96, it means that four marks are worth 96 cents of our money.

## DIRECT EXCHANGE

## Questions of the First Aspect

ILLUSTRATION 1.—Find the cost in Montreal of a bill of exchange on London, England, for £900 when exchange is quoted at \$4.87 $\frac{1}{2}$

## SOLUTION

Cost of bill for £1 = \$4.87 $\frac{1}{2}$ .

Cost of bill for £900 = \$4.87 $\frac{1}{2}$   $\times$  900 = \$4,386.

ILLUSTRATION 2.—How much must be paid in Toronto for a draft on Liverpool for £458 13s. 4d., exchange being quoted at 109 $\frac{3}{4}$ ?

## SOLUTION

£458 13s. 4d. = £458 $\frac{2}{3}$ .

Cost of draft for £1 when exchange is at 109 $\frac{3}{4}$  = \$4 $\frac{4}{9}$   $\times$  1.09 $\frac{3}{4}$ .

Cost of draft for £458 $\frac{2}{3}$  when exchange is at 109 $\frac{3}{4}$  = \$4 $\frac{4}{9}$   $\times$  1.09 $\frac{3}{4}$   $\times$  458 $\frac{2}{3}$   
= \$2,237.27 $\frac{11}{17}$  or \$2,237.27.

ILLUSTRATION 3.—Find the cost of a bill of exchange on Berlin for 1,600 marks, when exchange is quoted at 93 $\frac{3}{4}$ .

## SOLUTION

Cost of bill for 4 marks = \$.93 $\frac{3}{4}$

Cost of bill for 1,600 marks = \$.93 $\frac{3}{4}$   $\times$   $\frac{1600}{4}$  = \$375.

## SERIES 94

Find the cost of the following London drafts :

1. For £374 7s. 6d., exchange 108 $\frac{1}{2}$ .
2. For £193 6s. 8d., exchange 109 $\frac{1}{4}$ .
3. For £836 17s. 6d., exchange 110.
4. For £527 15s., exchange 108 $\frac{3}{4}$ .
5. For £1,638 14s. 7d., exchange 9 $\frac{3}{4}$ % premium.
6. For £947 11s. 9d., exchange 9 $\frac{1}{2}$ % premium.
7. For £385 19s. 5d., exchange 8 $\frac{3}{4}$ % premium.
8. For £593 16s. 4d., exchange \$4.86 $\frac{3}{4}$ .
9. For £267 5s. 9d., exchange par.

10. How much must be paid in Winnipeg for a bill of exchange on Paris for 3,048 francs when exchange is quoted at 5.19 $\frac{1}{2}$ ?

11. What is the cost in Toronto for a draft on New York for \$3,479.37 when exchange is at  $\frac{3}{8}\%$  premium?

12. Find the cost of a draft on Berlin for 7,329 marks when exchange is quoted at  $94\frac{1}{2}$ .

13. What will it cost to cancel a debt in Yokohama for 850 yen when exchange is at par?

14. Find the cost of a bill of exchange on Amsterdam for 2,367 guilders when exchange is quoted at  $40\frac{3}{8}$ .

15. Sold through a broker a bill of exchange on Geneva for 3,650 francs at 60 days' sight. What were the proceeds of the draft, exchange being  $5.20\frac{5}{8}$ , brokerage  $\frac{1}{8}\%$ ?

16. What will a sight draft on London for \$280 15s. cost in currency in Philadelphia when exchange is \$4.88 and gold is 105?

### Questions of the Second Aspect

ILLUSTRATION 1.—The cost of a draft on London for £383 12s. 6d. was \$1,854.18 $\frac{3}{4}$ . Find the rate of exchange.

#### SOLUTION

$$£383\ 12s.\ 6d. = £383\frac{5}{8}.$$

$$\text{Draft for } £383\frac{5}{8} \text{ cost } \$1,854.18\frac{3}{4}.$$

$$\text{Draft for } £1 \text{ cost } \$1,854.18\frac{3}{4} \div 383\frac{5}{8} = \$4.83\frac{1}{3}.$$

$$\$4.83\frac{1}{3} \div \$4\frac{2}{3} = 108\frac{3}{4}.$$

The rate of exchange may be given as \$4.83 $\frac{1}{3}$ , 108 $\frac{3}{4}$ , or 8 $\frac{3}{4}\%$  premium.

ILLUSTRATION 2.—The cost of a draft on Paris for 2,605 francs was \$500.62 $\frac{1}{2}$ . Find the rate of exchange, brokerage  $\frac{1}{8}\%$ .

#### SOLUTION

$$100\% + \frac{1}{8}\% = 100\frac{1}{8}\%.$$

If there had been no brokerage, the cost of the draft would have been

$$\frac{100}{100\frac{1}{8}} \text{ of } \$500.62\frac{1}{2}, \text{ or } \frac{800}{801} \text{ of } \$500.62\frac{1}{2} = \$500.$$

\$500 is cost of draft for 2,605 francs.

$$\$1 \text{ is cost of draft for } \frac{2605}{500} = 5.21 \text{ francs.}$$

$$\therefore \text{rate of exchange} = 5.21.$$



## SERIES 95

Find the rate of exchange

1. When a draft for £900 cost \$4,370.
2. When a draft for £306 13s. 6d. cost \$1,492.44 $\frac{4}{9}$ .
3. When a draft for £563 12s. 9d. cost \$2,724.25.
4. When a draft for 4,800 marks cost \$1,147.43 $\frac{1}{4}$ , brokerage  $\frac{1}{8}\%$ .
5. When a draft for 3,576 marks cost \$840.36.
6. When a draft for 3,575 guilders cost \$1,443.41
7. When a draft for 33,250 francs cost \$6,412.72.
8. When a draft for 366.20 francs cost \$72.09, brokerage  $\frac{1}{8}\%$ .
9. The cost of a bill on Bordeaux for 2,746 francs was \$538.20, including  $\frac{1}{8}\%$  brokerage. What was the rate of exchange?

## Questions of the Third Aspect

ILLUSTRATION 1.—A bill of exchange on London, England, cost \$2,104.22 $\frac{1}{2}$  when exchange was at 109 $\frac{1}{2}$ . What was the face of the bill?

## SOLUTION

Cost of bill for £1 when exchange is 109 $\frac{1}{2}$  =  $\$4.44\frac{4}{9} \times 1.09\frac{1}{2}$  =  $\$4.86\frac{2}{3}$ .

$\$4.86\frac{2}{3}$  will buy a bill for £1.

$\$2104.22\frac{1}{2}$  will buy a bill for  $\pounds 1 \times 2104.22\frac{1}{2} \div 4.86\frac{2}{3}$

= £432.375

= £432 7s. 6d.

ILLUSTRATION 2.—The cost of a bill of exchange on Paris was \$600, and exchange was at 5.19. Find the face of the bill.

## SOLUTION

\$1 will buy a bill for 5.19 francs

\$600 will buy a bill for 5.19 francs  $\times$  600

= 3,114 francs, face of bill.

## SERIES 96

1. A draft on London, England, cost \$1,194.94 when exchange was at \$4.88. Find the face of the draft.

2. Find the face of a bill of exchange on Manchester, England, which cost \$2,730 when exchange was at 9 $\frac{11}{16}\%$  premium.

3. An exporter sold through a broker a bill of exchange on Hamburg at 95 $\frac{3}{4}$ , and received \$5,953.49 as net proceeds. What was the face of the bill, brokerage  $\frac{1}{8}\%$ ?

4. I bought a bill of exchange on Paris and paid \$2,156 for it. What was the face of the bill, exchange being quoted at  $5.17\frac{2}{3}$  ?

5. What is the face of a bill of exchange on London which can be bought for \$5,807.25 if exchange is at \$4.85, brokerage  $\frac{1}{8}\%$  ?

6. An importer purchased a 60-day bill of exchange on Bremen at par for \$446.20. What was the face of the bill ?

7. The cost of a bill of exchange on Bordeaux, bought at 5.20, was \$941.17 $\frac{1}{2}$ , including  $\frac{1}{8}\%$  brokerage. What was the face of the bill ?

8. I bought a draft on Amsterdam when exchange was at 41 $\frac{3}{4}$ c to the guilder, and paid \$1,489.50 for it. What was the face of the draft ?

### INDIRECT OR CIRCUITOUS EXCHANGE

ILLUSTRATION 1.—A Toronto merchant paid a debt of 27,940 florins in Amsterdam by remitting to London when exchange was \$4.885; thence to Paris when exchange was 25.4 francs for £1; and thence to Amsterdam when exchange was 212 francs for 100 florins. What did he pay in Canadian money ?

#### SOLUTION

$$100 \text{ florins} = 212 \text{ francs}$$

$$\therefore 27940 \text{ florins} = \frac{27940 \times 212}{100} \text{ francs}$$

$$25.4 \text{ francs} = \text{£}1$$

$$\therefore \frac{27940 \times 212}{100} \text{ francs} = \text{£} \frac{27940 \times 212}{100 \times 25.4}$$

$$\text{£}1 = \$4.885$$

$$\text{£} \frac{27940 \times 212}{100 \times 25.4} = \$ \frac{4.885 \times 27940 \times 212}{100 \times 25.4} = \$11,391.82$$

#### SERIES 97

1. If 11.65 Dutch florins are given for 24.42 francs, 352 florins for 407 marks, and 58 $\frac{1}{4}$  marks for 32 silver roubles, how many francs should be given for 932 silver roubles ?

2. A Montreal merchant wishes to transmit 2,400 marks to Hamburg. He remits to London when exchange is \$4.83; thence to Paris when 26 francs are worth £1; and thence to Hamburg when 47 francs are worth 25 marks. What does he pay in Canadian money ?

3. If £1 is worth 12 florins or 25.56 francs, how many francs is one florin worth ?

4. A merchant in Montreal drew on Amsterdam for 10,000 guilders at \$.415. How much more would he have received if he had ordered remittance through London to Montreal, exchange at Amsterdam on London being  $11\frac{1}{4}$  guilders for £1, and at London on Montreal  $9\frac{1}{4}\%$  premium ?

5. If the exchange of London on Hamburg is 14 marks per pound sterling, that of Hamburg on Amsterdam is 20 marks for 18 florins, that of Amsterdam on Paris is 28 florins for 60 francs, and that of Paris on Toronto is 4 francs for 72 cents, what is the value of £1 sterling in Canadian money ?

6. Exchange at Paris upon London is at the rate of 25.7 francs for £1 sterling, and exchange at Vienna upon Paris is at the rate of  $40\frac{1}{2}$  Austrian florins for 20 francs. Find how many Austrian florins should be paid at Vienna for a £50 note.

### CUSTOM HOUSE BUSINESS

The **Customs Tariff** is a book issued by the Department of Customs at Ottawa, giving a list of dutiable goods, together with rates of duty thereon, and also giving the statutes which regulate the matter of levying duties.

The **General Tariff** is the rate of duties to which goods are subject, unless specially provided for under the British preferential tariff, the intermediate tariff, or a surtax.

The **British Preferential Tariff** applies to goods which are the produce or manufacture of British countries when imported direct from any British country. This duty is always less than the general tariff.

The **Intermediate Tariff** is, as the name implies, a tariff between the general tariff and the British preferential tariff. It may by order in council be extended, in consideration of benefits, to any country the produce or manufactures of which have previously been subject to the general tariff.

A **Surtax** is a rate duty above the general tariff, and is levied on the produce or manufactures of any foreign country which treats imports from Canada less favorably than those from any other country. For instance, up to March 1, 1910, German goods

imported into this country were subject to a surtax of  $\frac{1}{3}$  over and above the general tariff.

An **Ad Valorem Duty** is reckoned as a certain percentage of the "Value for duty in dollars."

A **Specific Duty** is reckoned according to quantity, as, for instance, a certain number of cents per pound, or per bushel, or per barrel, and so on.

Some articles imported are subject to both ad valorem and specific duties.

Dutiable goods arrive either by vessel or by rail or by express. For the purpose of handling these imports, the work is divided according to the amount of goods imported between the Long House and the Short House. All goods coming by rail or vessel are passed through the Long House, no matter what the amount of the invoice may be. Goods arriving by express are passed through the Short House if the amount is less than \$25.00. If the amount is \$25.00 or exceeds \$25.00, the goods must be passed through the Long House. (Toronto Custom House practice.)

**ILLUSTRATION.**—The following invoice represents an importation of goods from Germany into Canada in the days of the surtax. Compute the duty, filling in the usual Custom House Bill of Entry (Form B. 1).

**NOTE 1.**—The declaration, which is signed by the firm shipping the goods, is generally found printed on the back of the invoice. Three invoices are made out—one for the ordinary use of the house buying the goods, the other two for customs purposes. The two for customs purposes require to be certified by dating and signing the declaration on the back of the invoice. Two copies of the bill of entry are made out.

**NOTE 2.**—The invoice shown, while representing an importation from Germany, is, nevertheless, figured in sterling currency. This is very often the case where the firm selling the goods prefers to receive payment by exchange on London. It must not be taken for granted, however, that the invoice has to be figured in shillings, as it might just as well be figured in marks.

**NOTE 3.**—Each of the four items imported is subject to two duties—the specific duty, of so much per gallon; and the ad valorem duty, of a certain percentage of the "Value for duty in dollars." In addition, there is levied as a surtax  $33\frac{1}{3}$  per cent. of the ordinary duty (ad valorem and specific combined) on account of the goods coming from Germany.

**NOTE 4.**—The "Value for duty in dollars" is figured in the Long House to the nearest dollar. Thus, \$151.25 would be considered \$151, while \$151.50 or \$151.62 would be considered \$152. In the Short House the exact amount of the invoice is used.

Invoice of One Hundred cases Wine purchased by  
(class of goods)  
Jones & Brown of Toronto, Canada  
from C. F. Eccardt, L'td of Kreuznach, Rhineland  
to be shipped per Str. Lake Michigan—C.P. Ry. Co. Our Order No. 4718

•Marks and Numbers on Packages	Quantity.	DESCRIPTION OF GOODS	Fair Market Value as sold for home consumption	Selling price to Purchaser in Canada				Rate of Duty or Free	Leave Blank
				@	Amount				
			Shillings	Shillings	£	s.	d.		
J. & B. Toronto									
1/25	25	Cases, 12 bottles each, Laubenheimer	30	30	37	10	0		
26/50	25	" 24 ½ " " Rudesheimer	45	45	56	5	0		
51/70	20	" 12 bottles " Liebfraumilch	60	60	60	0	0		
71/100	30	" 12 " " Sparkling Moselle	110	110	165	0	0		
					£318	15	0		

\*Put a distin-  
guishing mark  
or number on  
packages and  
show same in  
this column.

Two invoices required for Customs. Fill in and sign declaration  
on back of each invoice.



## SOLUTION

**CUSTOMS, CANADA—ENTRY FOR HOME CONSUMPTION***Port of Toronto, April 30, 1908. Imported by Jones & Brown**Per C. P. R. Master from Germany to Toronto**Goods purchased in Germany and Imported via Montreal  
(State whether direct or through United States)**Report No.**Entry No.*

Marks and Numbers.	Number of Packages.	DESCRIPTION OF GOODS.	Quantity.	Invoice value in Currency of Invoice.	Value for Duty in Dollars.	Rate of Duty or Free.	Total Duty. \$ c.	Preferential or Net Duty. \$ c.
J. & B. Toronto #1/25	25	Cases, 12 bot. each, Laubenheimer Wine	50 galls.	37.10.0	\$183	50c.	25 00	
26/50	25	" 24 ½-bot. each, Rudesheimer "	50 "	56.5.0	274	30% 33 ½	54 90 26 63	
51/70	20	" 12 bot. each, Liebfraumilch	40 "	60.0.0	292	50c. 33 ½	82 20 35 73	
71/100	30	" 12 " " Sparkling Moselle	30 doz.	165.0.0	803	30% 33 ½	240 90 113 30	
	100		170	£318.15.0	\$1552		\$846 13	

## SERIES 98

1. Gowans, Kent & Co., Toronto, Ont., import from France 1 crate China dishes, containing 10 sets, at 10 francs a set, on which the duty is 30%. The crate is invoiced at 9 francs. Show invoice and form for passing goods at customs (B. 1) properly filled in.

NOTE.—Cases where invoiced are dutiable. The general tariff is 20%. The preferential is 15%.

2. Greenshield, Son & Co., of Montreal, import from Germany 1 case of Buttons, containing 42 gross, invoiced at 10 marks a gross. The general tariff on these buttons is 35%. The case is invoiced at 5 marks. Show invoice and customs form properly filled out.

3. George Hamilton, of Medicine Hat, imports from England : 1 Wool Suit at £2, 1 doz. Silk Ties at 1s. 8d. each, 1 Silk Hat at 15s., 1 pair of Shoes at 12s. The duty on the hat is 22½%, on the others 30%. The package is invoiced at 4s. Show invoice and customs forms.

4. Messrs. Johnston & Co., Quebec, import from Williams, Humbert & Co., Jerez De La Frontera, Spain, as follows : 1 butt Sherry Wine, 108 gallons, at £85 per butt, duty 30% and 43c per gallon ; 4 hhds. (2 butts) Sherry Wine, 216 gallons, at £96 per butt, duty 30% and 46c per gallon ; 10 quarters (2½ butts) Sherry Wine, 270 gallons, at £102 per butt, duty 30% and 40c per gallon. Show invoice and customs form.

5. Messrs. Foster & Smith, St. John, N.B., import from Warre & Co., Oporto, Spain, the following : 1 pipe Port Wine, 115 gallons, at £70 per pipe, duty 30% and 43c per gallon ; 1 hhd. Port Wine, 58 gallons (½ pipe) at £80 per pipe, duty 30% and 49c per gallon ; 2 quarters Port Wine, 58 gallons (½ pipe), at £90 per pipe, duty 30% and 46c per gallon ; 100 cases Port Wine, 200 gallons, at 50s. per case, and duty 30% and 49c per gallon. Show invoice and customs paper.

6. McLeod Bros., of Glasgow, sell to Brown, Morgan & Co., Montreal, a line of ready-made Skirts as follows :

8 @ 17/11, 8 @ 21/—, 6 @ 27/3, 6 @ 35/6. Case 13/5.  
 6 @ 19/6, 6 @ 25/3, 6 @ 27/—, 6 @ 34/6, 4 @ 40/—. Case 19/9.  
 6 @ 46/6, 4 @ 52/6, 6 @ 39/6, 4 @ 40/6, 4 @ 41/. Case 17/2.  
 14 @ 45/, 8 @ 56/—.

The duty on skirts is 30%. Show invoice and customs form.

NOTE.—It is not always necessary that the bill of entry should show the detail of the invoice. For instance, these goods may be entered as "3 cases wool clothing."

7. John McDonald & Co., Toronto, buy from Wm. Anderson & Co., Glasgow, as follows :

57 pcs. Madras Shirting at  $4\frac{3}{8}$ d. per yard.  $32\frac{1}{2}$ ,  $46\frac{1}{2}$ , 47,  $48\frac{1}{2}$ , 50, 54, 57,  $2/58\frac{1}{2}$ ,  $5/59$ ,  $2/60$ ,  $60\frac{1}{2}$ ,  $4/61$ ,  $4/61\frac{1}{2}$ ,  $6/62$ ,  $9/62\frac{1}{2}$ ,  $10/63$ ,  $3/63\frac{1}{2}$ , 64,  $64\frac{1}{2}$ , 71,  $76\frac{1}{2}$ . Case 13/-.

34 pcs. Oxford Shirting at  $5\frac{3}{4}$ d. per yard, 55,  $55\frac{1}{2}$ , 56,  $4/57$ ,  $3/57\frac{1}{2}$ ,  $2/58$ ,  $2/58\frac{1}{2}$ ,  $2/59$ ,  $4/59\frac{1}{2}$ ,  $3/60\frac{1}{2}$ ,  $3/61$ ,  $2/61\frac{1}{2}$ ,  $2/62$ ,  $3/63$ , 60. Case 11/4.

9 pcs. Madras Shirting at  $6\frac{3}{4}$ d. per yard,  $54\frac{1}{2}$ , 55,  $55\frac{1}{2}$ ,  $2/57$ ,  $2/58\frac{1}{2}$ , 60,  $61\frac{1}{2}$ . Case 12/9.

Duty on shirting is 25%. Show invoice and customs form.

8. Thomas Miller & Co., Winnipeg, Man., import from M. Myers & Son, of Birmingham, Eng., the following :

882	gross Pens	.....	@ 1/-
100 $\frac{1}{2}$	" "	.....	@ 1/3
25	" "	340	@ 1/-
30	" "	42203	@ 1/-
50	Binders M. 1	.....	@ 1/1
1	" M. 5	.....	@ 2/-
1	" M. 7	.....	@ 4/6
• Case			5/2

Duty on pens 15%, duty on binders 20%. Show invoice and customs paper.

9. Messrs. Bell & Stevens, Halifax, N.S., import from Spalding & Hodge, Ltd., London, England, the following :

10 reams C. Laid Duplicator Ledger Post at 4s. 4d. per ream ;  
10 reams C. Laid Manifold Ledger Post at 13s. 8d. per ream.

Deduct trade discount of 5% from total of both items. Add cost of packing, 2 cwt. 2 qrs. 26 lbs. at 30s. per ton. Duty 15%. Show invoice and customs paper.

10. Davies & Henderson, of Toronto, import from Andrew Whyte & Son, Ltd., Edinburgh, Scotland, the following :

96 reams Printing " Water Mill " Cream Wove, 17 by 28, 2,304 lbs., at  $2\frac{3}{4}$ d. per lb. ; 9 reams do. do., 2nds, 230 lbs., at  $2\frac{3}{4}$ d. per lb., less 10% ; 10 reams do. do., 3rds, 246 lbs., at  $2\frac{3}{4}$ d. per lb., less 20% ; 20 reams Printing " Water Mill " Cream Wove,  $16\frac{1}{2}$  by 21, 420 lbs., at  $2\frac{3}{4}$ d. per lb.

Deduct trade discount of 5% from whole bill. 7 cases cost £2 0s. 7d. ; duty 15%. Show invoice and customs paper.

II. F. Tiffin & Co., Vancouver, B.C., import from the American Lead Pencil Co., New York, the following :

6	gross	557 Pencils .....	@	\$3.50
3	"	450R " .....	@	3.00
6	"	295Bx Penholders .....	@	3.00
6	"	658 " .....	@	1.40
12	"	6/379 6/383 Penholders ....	@	3.60
1	"	410 Eras. Pencils .....	@	3.00
$\frac{1}{2}$	"	428 " " .....	@	3.00
6	"	3/F, 3/H.V.D. Pencils .....	@	6.50
1	"	F.W./Prot. V.D. " .....	@	8.10
6	"	Steno. " .....	@	3.00

Duty  $27\frac{1}{2}\%$ . Show invoice and customs paper.

# AVERAGE AND ITS APPLICATIONS

**An Average** is a medial sum or quantity made out of unequal sums or quantities.

To illustrate: A grocer sells a certain tea at 50c a pound and another grade at 70c a pound. He is getting an average of 60c a pound for his tea. One pound sold at 50c and one at 70c returns him exactly \$1.20. Two pounds at 60c a pound gives the same return. Note that in selling a 50c tea at 60c a pound there is a gain of 10c on a pound, while in selling a 70c tea at 60c a pound there is a loss of 10c a pound. A true average should be such that the gains just counterbalance the losses.

Again, a partner in a business invests \$5,000 for one month, and for the next month he invests \$7,000. On an average he is giving the business the use of \$6,000 a month. Note, again, that, while \$6,000 is \$1,000 more than he invests the first month, it is \$1,000 less than he invests the second month. The average is greater than the investment for one month by exactly the figure that it is less than the investment for the other month.

Again, a man stores 500 bushels of grain in a storehouse for one month, and for the next month he stores 700 bushels. On an average he has in store 600 bushels a month. This is 100 bushels more than he had in the first month, but 100 bushels less than he had in the second month. The surplus in one case exactly balances the deficiency in the other.

## QUESTIONS OF THE FIRST ASPECT

**Given the items, to find the average.**

**ILLUSTRATION 1.**—A grocer mixed 25 lbs. tea at 30c a pound, 20 lbs. at 50c a pound, 40 lbs. at 75c a pound, and 15 lbs. at a dollar a pound. What was the average value of the mixture?

### SOLUTION

25 lbs. at 30c .....	\$7.50
20 lbs. at 50c .....	10.00
40 lbs. at 75c .....	30.00
15 lbs. at \$1.00 .....	15.00

100	\$62.50
-----	---------

$$\$62.50 \div 100 = \$.62\frac{1}{2}$$

The tea is worth on an average 62½c a pound.



ILLUSTRATION 2.—On Jan. 1, 1908, J. Watson commences business, investing \$2,000. On April 1 he invests \$5,000 more, and on Sept. 1st \$2,000 more. His investment then remained unaltered for the balance of the year. What is his average investment per month throughout the year ?

## SOLUTION

From Jan. 1st to April 1st = 3 months.

\$2,000 invested for 3 months = \$ 6,000 for 1 month.

April 1st to Sept. 1st = 5 months.

\$7,000 invested for 5 months = \$35,000 for 1 month.

Sept. 1st to end of year = 4 months.

\$9,000 invested for 4 months = \$36,000 for 1 month.

Total investment,	\$77,000 for 1 month.
-------------------	-----------------------

\$77,000 ÷ 12 = \$6,416 $\frac{2}{3}$ —average investment per month.

ILLUSTRATION 3.—A commission merchant places in store the following items :

Sept. 9th, 200 barrels of pork.

Oct. 17th, 500     "     of beef.

Nov. 10th, 250     "     of pork.

Dec. 1st, 150     "     of beef.

On Dec. 8th he desires to make a settlement. Suppose he is charged at the rate of 4c a barrel for a period of 30 days, what does he owe ?

## SOLUTION

The storage of 200 bbls. for 90 days = the storage of 18,000 bbls. for 1 day.

The storage of 500 bbls. for 52 days = the storage of 26,000 bbls. for 1 day.

The storage of 250 bbls. for 28 days = the storage of 7,000 bbls. for 1 day.

The storage of 150 bbls. for 7 days = the storage of 1,050 bbls. for 1 day.

---

All = the storage of 52,050 bbls. for 1 day.

The storage of 52,050 bbls. for 1 day = the storage of

$\frac{52050}{30}$  or 1,735 bbls. for 30 days.

The storage bill will be 4c × 1735 = \$69.40.

SERIES 99

1. Eight men weigh respectively 145 lbs., 187 lbs., 164 lbs., 138 lbs., 197 lbs., 205 lbs, 179 lbs., and 169 lbs. What is their average weight ?

2. A grocer mixed 40 lbs. of tea costing 45c a pound, 48 lbs. costing 47c a pound, and 64 lbs. costing 53c a pound. What was the value of the mixture per pound ?

3. Find the average value of a mixture of 17 lbs. coffee at 32c, 45 lbs. at 26c, 37 lbs. at 29c, 42 lbs. at 37c. and 24 lbs. at 30c.

4. The following summary is taken from a book of cash sales : Aug. 7, sold 310 at \$1.09 ; Aug. 8, sold 470 at \$1.25 ; Aug. 9, sold 640 at \$.95 ; Aug. 10, sold 430 at \$1.07 ; Aug. 11, sold 580 at \$.99 ; Aug. 12, sold 360 at \$1.16. What was (a) the average number sold daily, (b) the average daily cash business, and (c) the average selling price ?

5. Alex. Brown invested in business on Jan. 1, 1908, \$4,000 ; on Mar. 1, \$1,000 more ; July 1, \$2,000 more ; and Nov. 1, \$1,000 more. From Nov. 1 to the end of the year his investment remained unchanged. What was his average investment for the year ? (Calculate time in months.)

6. Weir and Thayer formed a partnership. On April 1 Weir invested \$3,500, and on Aug. 17 \$1,200 more. On April 1 Thayer invested \$3,200, and on January 10 \$2,000 more. Find the average investment of each for the year.

7. Liddle and Tufford became partners in a business, agreeing to share gains and losses according to average investments. On May 10 Liddle invested \$1,600 ; on June 17, \$500 more ; and on Aug. 9, \$700 more. On May 10 Tufford invested \$1,300 ; on July 3, \$600 more ; and on Sept. 20. \$400 more. The partnership was dissolved on Oct. 17. Find the average investment of each.

8. A warehouseman received the following quantities of wheat at the rate of 1 cent per bushel for a period of 30 days of average storage : June 18, 500 bu. ; June 28, 400 bu. ; July 9, 300 bu. ; and Aug. 2, 800 bu. What was the amount of storage due Aug. 15 ?

9. At 3 cents a barrel per 30 days of average storage, what is the storage due May 18 on the following receipts : Jan. 20, 125 bbls. ; Jan. 31, 75 bbls. ; Feb. 12, 140 bbls ; Feb. 29, 80 bbls. ; Mar. 15, 150 bbls. ; Mar. 24, 95 bbls. ; April 4, 475 bbls. ; April 9, 38 bbls ; April 15, 100 bbls. ; April 28, 50 bbls. ; May 10, 150 bbls ?

10. A farmer received for pasture: April 30, 12 head of cattle; May 15, 14 head of cattle; May 23, 27 head of cattle; June 9, 5 head of cattle; June 30, 8 head of cattle; July 16, 40 head of cattle. If the charges were 75 cents per head for each week of 7 days of average pasture, what would be owing for the pasture on July 25?

ILLUSTRATION 4.—A warehouseman received and delivered the following:

RECEIVED.			DELIVERED.		
June	11,	150 bbls.	June	26,	120 bbls.
"	30,	200 "	July	15,	150 "
July	18,	90 "	"	25,	160 "
"	28,	180 "	Aug.	4,	190 "

What was paid for storage at 3 cents per barrel for a period of 30 days' average storage, a settlement having been made Aug. 4?

			SOLUTION		
DATES.	DAYS.		BARRELS.		
June 11 to June 26	= 15	×	150 (received)	=	2,250 bbls. stored for 1 day.
" "			120 (delivered)		
<hr/>					
" 26 to "	30	= 4	×	30 (in store)	= 120 bbls. stored for 1 day.
" "				200 (received)	
<hr/>					
" 30 to July 15	= 15	×	230 (in store)	=	3,450 bbls. stored for 1 day.
" "			150 (delivered)		
<hr/>					
July 15 to "	18	= 3	×	80 (in store)	= 240 bbls. stored for 1 day.
" "				90 (received)	
<hr/>					
" 18 to "	25	= 7	×	170 (in store)	= 1,190 bbls. stored for 1 day.
" "				160 (delivered)	
<hr/>					
" 25 to "	28	= 3	×	10 (in store)	= 30 bbls. stored for 1 day.
" "				180 (received)	
<hr/>					
" 28 to Aug. 4	= 7	×	190 (in store)	=	1,330 bbls. stored for 1 day.

Total storage = 8,610 bbls. stored for 1 day.

Total storage =  $\left(\frac{8610}{30} = 287\right)$  bbls. stored for 30 days.

$\$.03 \times 287 = \$8.61$ , required amount of storage.

ILLUSTRATION 5.—The following accounts show the investments and withdrawals of two partners during a year. Find the average investment of each.

<i>Dr.</i>	FRANK SMITH.		<i>Cr.</i>
1908.	WITHDRAWALS.	1908.	INVESTMENTS.
Apr. 23 .....	\$1,000	Jan. 1 .....	\$16,000
Aug. 17 .....	900	Oct. 20 .....	3,000

<i>Dr.</i>	CHAS. ROBERTSON.		<i>Cr.</i>
1908.	WITHDRAWALS.	1908.	INVESTMENTS.
July 28 .....	\$600	Jan. 1 .....	\$12,000
Dec. 4 .....	800	May 17 .....	600

# SOLUTION

## SMITH'S ACCOUNT

From Jan. 1 to Apr. 23 = 113 dys. Hence \$16,000 for 113 dys. = \$1,808,000 for 1 day.  
 Withdrawal 1,000

From Apr. 23 to Aug. 17 = 116 dys. Hence \$15,000 for 116 dys. = \$1,740,000 for 1 day.  
 Withdrawal 900

From Aug. 17 to Oct. 20 = 64 dys. Hence \$14,100 for 64 dys. = \$902,400 for 1 day.  
 Investment 3,000

From Oct. 20 to Jan. 1 = 73 dys. Hence \$17,100 for 73 dys. = \$1,248,300 for 1 day.

Smith's total investment, \$5,698,700 for 1 day.

Dividing \$5,698,700 by 365, we get \$15,612 $\frac{4}{5}$  as the average investment for the year.

## ROBERTSON'S ACCOUNT

From Jan. 1 to May 17 = 137 dys. Hence \$12,000 for 137 dys. = \$1,644,000 for 1 day.  
 Investment 600

From May 17 to July 28 = 72 dys. Hence \$12,600 for 72 dys. = \$907,200 for 1 day.  
 Withdrawal 600

From July 28 to Dec. 4 = 129 dys. Hence \$12,000 for 129 dys. = \$1,548,000 for 1 day.  
 Withdrawal 800

From Dec. 4 to Jan. 1 = 28 dys. Hence \$11,200 for 28 dys. = \$313,600 for 1 day.

Robertson's total investment, \$4,412,800 for 1 day:

Dividing \$4,412,800 by 365, we get \$12,089 $\frac{8}{13}$  as the average investment for the year.

QUESTION.—Suppose the gain for the year were \$3,000, and the partners shared this gain in proportion to their average net investments, how much is each entitled to ?

## SOLUTION

Smith's average net investment is \$15,612 $\frac{64}{73}$

Robertson's average net investment is 12,089 $\frac{63}{73}$

Total average net investment is \$27,702 $\frac{54}{73}$

Smith's share of the gain is  $\frac{15612\frac{64}{73}}{27702\frac{54}{73}}$  of \$3,000 = \$1,690.76

Robertson's share of the gain is  $\frac{12089\frac{63}{73}}{27702\frac{54}{73}}$  of \$3,000 = \$1,309.24

## SERIES 100

1. Compute the storage on the following account at 1 cent per bushel for a period of 30 days of average storage :

## RECEIVED.

Sept. 7, 130 bu.  
 " 23, 90 "  
 Oct. 3, 180 "  
 " 29, 160 "

## DELIVERED.

Oct. 1, 200 bu.  
 " 10, 145 "  
 " 30, 75 "  
 Nov. 11, 140 "

2. At 4 cents per barrel for a period of 30 days of average storage, what is the storage due Aug. 15 on the following produce, received and delivered as stated ?

## RECEIVED.

June 9, 160 bbls. apples.  
 " 22, 140 " potatoes.  
 July 14, 70 " turnips.  
 Aug. 5, 100 " onions.

## DELIVERED.

June 21, 85 bbls. apples.  
 " 26, 110 " potatoes.  
 " 29, 75 " apples.  
 July 16, 30 " turnips.  
 " 30, 30 " potatoes.  
 Aug. 10, 75 " onions.  
 " 12, 40 " turnips.  
 " 15, 25 " onions.

3. Find the pasturage, at 65 cents per head, for a period of 30 days of average pasturage on the following account :

## CATTLE RECEIVED.

June 33, 16 cattle.  
 " 16, 27 "  
 July 7, 11 "  
 " 15, 26 "  
 " 27, 7 "

## CATTLE WITHDRAWN.

July 2, 18 cattle.  
 " 18, 20 "  
 Aug. 4, 9 "  
 " 19, 13 "  
 Sept. 5, 27 "



4. A and B formed a partnership Mar. 1, and agreed to divide gains and losses according to their average net investments. At first A put in \$3,000 and B \$2,500. On June 1 A put in \$1,000 more and B \$2,000. On Sept. 1 A withdrew \$1,500, and on Nov. 1 B withdrew \$1,000. For the remainder of the year their investments remained unchanged. The profit for the year being \$1,200, how much should each receive ?

5. Jan. 1, 1890, Hess, Dunn, and Norris engaged in business, Hess investing \$8,000, Dunn \$7,000, and Norris \$6,000. June 1 Hess invested \$2,000 additional, but on Sept. 1 he withdrew \$1,500. May 1 Dunn withdrew \$2,000, but July 1 invested \$1,500 additional. Apr. 1 Norris invested \$2,000 additional, but on Aug. 1 withdrew \$1,000. Divide their gain, \$5,320, in proportion to their average investments.

6. A, B, and C began business Jan. 1st, when A put in \$7,500, and July 1st he put in \$2,500 more ; B put in Jan. 1st \$12,000, and May 1st withdrew \$4,000 ; C put in Jan. 1st \$10,000, Aug. 1st he added \$3,000, and Oct. 1st he withdrew \$7,000. At the close of the year the profit was \$8,500. How much ought each to have, the gains being divided according to their average investment ? (Calculate time in months.)

## QUESTIONS OF THE SECOND ASPECT

Given the average, to find the items.

ILLUSTRATION.—A grocer wishes to mix four kinds of coffee, worth respectively 17, 18, 24, and 25c a pound, in such a way that the mixture will be worth 20c a pound. How many pounds of each will be used ?

### SOLUTION

1st Step : Find the gain or loss on each kind, considering it at the average price.

17c coffee gains 3c on 1 lb., or 1c on  $\frac{1}{3}$  lb.

18c coffee gains 2c on 1 lb., or 1c on  $\frac{1}{2}$  lb.

24c coffee loses 4c on 1 lb., or 1c on  $\frac{1}{4}$  lb.

25c coffee loses 5c on 1 lb., or 1c on  $\frac{1}{5}$  lb.

2nd Step : Mix quantities so that a cent of gain will be offset by a cent of loss, which must be the case if a true average is to be maintained.

On  $\frac{1}{3}$  lb. of 17c coffee there is gain of 1c.

On  $\frac{1}{4}$  lb. of 24c coffee there is loss of 1c.

On  $\frac{1}{2}$  lb. of 18c coffee there is gain of 1c.

On  $\frac{1}{5}$  lb. of 25c coffee there is loss of 1c.

Therefore to get an average price of 20c a pound from these four coffees, the mixture should be :

$\frac{1}{3}$  lb. at 17c.  
 $\frac{1}{4}$  lb. at 24c.  
 $\frac{1}{2}$  lb. at 18c.  
 $\frac{1}{5}$  lb. at 25c.

NOTE.—At this stage it may be pointed out that there is no limit to the number of correct answers that may be given to such a question. If the quantities we have given will produce the required average, so will any number of times these quantities. We can therefore get as many correct answers as we can find figures by which to multiply or divide the obtained result.

Similarly we may obtain a different, but correct, list of answers by multiplying or dividing any pair of quantities by any number, or by multiplying or dividing each pair by different numbers.

To clear our answer, as thus far obtained, of fractions, multiply through by the L.C.M. of the denominators 60. This gives us.

20 lbs. at 17c.  
 15 lbs. at 24c.  
 30 lbs. at 18c.  
 12 lbs. at 25c.

This will prove to be an average of 20c a pound.

VARIATION 1.—Suppose that it is desired to have twice as much 24c coffee as there is 18c coffee in the mixture, how can it be arranged ?

SOLUTION.—Looking at the answer as we have it, there are 15 lbs. of 24c coffee and 30 lbs. of the 18c coffee. To get the quantities as required, take four times the quantities in the first pair and leave the second pair untouched. The quantities will be :

80 lbs. at 18c.  
 60 lbs. at 24c.  
 30 lbs. at 18c.  
 12 lbs. at 25c.

VARIATION 2.—Suppose that there is but a limited quantity of one kind of coffee—say, 40 lbs. of the 17c variety—how can we arrange to include just this quantity in the mixture ?

SOLUTION.—In the answer as stated there are 20 lbs. at 17c. To raise this to 40, multiply first pair through by 2. The result reads :

40 lbs. at 17c.  
 30 lbs. at 24c.  
 30 lbs. at 18c.  
 12 lbs. at 25c.

**VARIATION 3.**—Suppose that it is desired to have the whole mixture within a certain limit—say, 154 lbs. —how can it be arranged ?

**SOLUTION.**—As the mixture stands, it contains 77 lbs. Therefore double the quantities all round.

40 lbs. at 17c.

30 lbs. at 24c.

60 lbs. at 18c.

24 lbs. at 25c.

---

154 lbs.

**SERIES 101**

1. In what proportion must two kinds of coffee, which cost 50c and 65c per pound respectively, be mixed to give a mixture worth 54c per pound ?

2. In what proportion must teas worth 25c, 35c, and 40c per pound respectively be mixed so that a mixture may be formed which will be worth 34c per pound ?

3. A liquor dealer has wines worth 30c, 40c, 80c, and  $83\frac{1}{3}$ c per quart. He wishes to make a mixture of 80 quarts so that he may sell at 70c per quart, and make 20% profit. How much of each kind must he use ?

4. A mixture of 60 lbs. of two kinds of tea cost \$24.60 ; the cheaper is worth 35c per pound and the dearer 45c. Find the number of pounds of each in the mixture.

5. A mixture of 7 lbs. of black tea and 8 lbs. of green is worth \$5.28, while a mixture of 12 lbs. of black and 3 lbs. of green is worth \$5.73. Find the value per pound of each.

6. 6 geese and 5 turkeys are worth \$5.95, and 7 geese and 8 turkeys are worth \$8.35. Find the price of each.

7. The mean height of six mountains is 10,357 feet. What must be the height of a seventh mountain to make the mean height of the seven 10,643 feet ?

8. A and B enter into partnership for a year, and A receives  $\frac{3}{8}$  of the profits and B the remainder. A invests \$2,000 additional the second year, and at the end of the year receives  $\frac{4}{9}$  of the profits. How much did each invest the first year ?

9. The average of seven numbers is 26.98. The average of the first two is 34.5, and of the next three 19.3. Find the average of the remaining two.

10. A dishonest milk-dealer buys 135 gallons of pure milk at 23c a gallon, and, after mixing it with water, sells the mixture at  $4\frac{1}{2}$ c a quart, thereby gaining \$2.43. How much water has he used ?

11. A, B, and C engage in manufacturing shoes. A puts in \$1,920 for 6 months, B a sum not specified for 12 months, and C \$1,280 for a time not specified. A received \$2,400 for his stock and profits, B \$4,800 for his, and C \$2,080 for his. Required, B's stock and C's time.

12. Dodd and Brown became partners June 1, 1907, Dodd investing \$10,000 and Brown \$8,000. On Oct. 1. Dodd invested \$2,000 additional, and Brown withdrew \$2,000. How much should Brown invest on Jan. 1, 1908, to entitle him to half the year's profits ?

13. How much tea worth, respectively, 55c and 75c per pound must be mixed with 30 lbs., worth 90c per pound, in order that the mixture may be sold for 70c per pound ?

14. How much water will it require to dilute 60 gallons of alcohol, worth \$1.50 per gallon, so that the mixture may be worth only \$1.20 per gallon ?

15. How many gallons of kerosene oil, worth 60c per gallon, must be mixed with 12 gallons of coal oil, worth 36c, and 8 gallons of Aurora oil, worth 56c, so that the mixture may be sold for 50c. per gallon ?

16. A farmer has 16 bushels of corn, worth 48c per bushel, and 12 bushels of oats, worth 34c per bushel, which he wishes to mix with rye, at 60c, and barley, at 80c, in order to sell the mixture at 56c per bushel. How many bushels of rye and barley will be required ?

17. A farmer has three different qualities of wool, worth 33c, 37c, and 45c per pound respectively. He wishes to make up a package amounting to 120 lbs., which he can afford to sell at 39c per pound. How many pounds of each kind must he take ?

18. A confectioner mixes three different qualities of candy, worth 14c, 18c, and 30c per pound respectively, so as to make a box of 84 lbs. How many pounds of each sort must he take so as to sell the mixture at an average price of 24c per pound ?

19. How much sugar at 10c, 13c, 15c, 17c, and 18c per pound must be taken to make a mixture worth 16c ?

20. A dealer mixed two kinds of wines, worth respectively \$2.40 and \$3.20 per gallon, in such proportion that by selling

the mixture at \$2.80 per gallon he made a profit of 10%. Find the proportion in which the wines were mixed.

21. When wheat is worth 90c per bushel, 17 bushels of a mixture of wheat and oats are worth \$12.55; but if the proportions in the mixture were interchanged, its value would be \$8.70. Find the price of oats per bushel.

22. A cask contains 7 parts of brandy and 5 parts of water;  $\frac{1}{7}$  of the mixture is drawn off, and the cask filled with water. What is the strength of the mixture then?

23. A mixture of 50 gallons of alcohol and water contains 80% alcohol. (a) How much water must be added to reduce the strength to  $62\frac{1}{2}\%$ ? (b) How much alcohol must be added to increase its strength to  $87\frac{1}{2}\%$ ?

24. How much water must be added to a mixture of 16 quarts of vinegar at 13c and 10 quarts at 10c that the whole mixture may be worth 11c the quart?



# PARTNERSHIP

A **Partnership** or **Co-Partnership** is the contract relation subsisting between persons who have combined their property, labor, or skill in an enterprise or business as principals for the purpose of joint profit.

The contracting parties are called partners or co-partners. Collectively they are called a firm, a house, or a company.

There are two kinds of partnership, general and limited.

A **General Partnership** is composed of two or more general partners. That is, partners who are liable not only as partners for the debts of the firm, but also personally liable if the resources of the firm are not sufficient to pay its debts in full.

A **Limited Partnership** is composed of one or more general partners, who are liable as general partners always are, and one or more limited partners, who are not liable for the debts of the firm beyond the amount of their investment. A limited partner must take no active interest in the business.

The **Liabilities** of a firm are its entire debts. These are of two kinds : first, its debts to the public ; second, its debts to the partners.

The **Resources** of a firm are the available means it has for the payment of its debts.

A perfectly solvent partnership should be able not only to pay its debts to the public, but to return to the partners the amount of their investment.

The **Investment** of a partner is the aggregate of the sums contributed by him for the purpose of carrying on the business. The contributions need not be in money, but may be in goods, real estate, trade marks, trade secrets, patent rights, or any other resources.

It will be understood that a partner in entering a business may invest resources, and he may also bring with him certain liabilities, which are assumed by the firm. Then, again, he may, during the course of the business, withdraw a part of his investment, or, on the other hand, he may add additional amounts to his original

investment. In other words, the investment is not necessarily a fixed figure. •

The **Net Investment** or **Net Credit** of a partner is the difference between his investment on the one hand and the liabilities assumed for him and withdrawals on the other hand.

As the object of any business is to make a profit for those concerned in it, it is clear that the results of a year's trading or manufacturing will bring about a change in the way of adding a profit to the investment or subtracting a loss therefrom.

A **Balance Sheet** is a statement of the resources and liabilities of a business.

The **Net Capital** or **Present Worth** of a firm is the amount by which the resources exceed the liabilities to the public.

The **Net Gain** of a firm is the amount by which the net capital exceeds the net investment. In double-entry bookkeeping this figure should be the same as the difference between the two sides of the loss and gain account, when the credit side is the larger, or, as it may be more broadly called, the revenue account, or income and expenditure account.

The **Net Loss** is the amount by which the net investment exceeds the net capital. In double-entry bookkeeping this is the same as the amount by which the debit of the loss and gain account exceeds the credit.

The **Net Insolvency** of a firm is the amount by which the liabilities to the public exceed the resources.

## PARTNERSHIP ADJUSTMENTS

Partnership adjustments involve generally the adjusting between the partners of such matters as the distribution of the net profit or of the net loss, the payment of salaries according to agreement, or the payment of interest according to agreement.

It will be understood that, as a partnership is the outcome of a contract, the parties to that contract are at liberty to make any agreement that they may see fit. No matter how absurd

that agreement may appear, it must be followed in the adjustment of the affairs. There is, therefore, no set rule for the division of gains and losses, for instance, between partners, for the simple reason that partners may agree to divide gains and losses as they choose. The same thing is true concerning allowances for salary or allowances for interest. If, however, the agreement is silent on such points, it may be implied that gains and losses are to be divided proportionately to the number of partners. Unless there is a specific agreement, it could not be inferred that interest or salary was to be allowed.

Partners may also be brought into contact with the public through failure of their resources to cancel the debts to the public. In such cases general partners must expect to contribute from their private means. Here, again, partners must make adjustments between themselves.

We recommend a careful study of the following principles which must guide one in making partnership adjustments, and which will be understood to apply in all cases unless modified by agreement :

(a) Losses, including losses and deficiencies of capital, shall be paid, first, out of profits, next out of capital, and, lastly, if necessary, by the partners individually in the proportion in which they were entitled to share in the profits.

(b) The assets of the firm, including the sums, if any, contributed by the partners to make up losses or deficiencies of capital, shall be applied in the following manner and order :

1. In paying the debts and liabilities of the firm to persons who are not partners therein.

2. In paying to each partner rateably what is due from the firm to him for advances as distinguished from capital.

3. In paying to each partner rateably what is due from the firm to him in respect to capital.

4. The ultimate residue, if any, shall be divided among the partners in the proportion in which profits are divisible.

ILLUSTRATION 1.—J. Coleman and R. Dawson formed a partnership on Jan. 1st, 1907, agreeing to invest equally and to share equally in gains and losses. To provide for any possible inequality in investment, 8% interest was to be allowed each partner, on investments, and interest at the same rate was to be charged on all withdrawals. It is, further, agreed that Coleman is to be paid a salary of \$2,000 a year and Dawson a salary of \$1,000 a year. The adjustment of both interest and salary to be made after the adjustment of the ordinary net profit or loss of the year has been made. The following is the statement of each partner's account :

<i>Dr.</i>		J. COLEMAN.		<i>Cr.</i>	
1907.		1907.			
Apr. 3	Withdrawal..	\$3,000.00	Jan. 1	Investment ..	\$6,000.0
June 7	" ..	1,500.00	Mar. 14	" ..	4,000.00
Oct. 15	" ..	800.00	July 3	" ..	3,000.00
1908.			Sept. 18	" ..	5,000.00
Jan. 1	$\frac{1}{2}$ loss from		1908.		
	interest ad-		Jan. 1	$\frac{1}{2}$ net gain of	
	justment ..	601.75		\$5,800 ...	2,900.00
" 1	$\frac{1}{2}$ loss from		" 1	Net credit int.	710.05
	salary ad-		" 1	Salary .....	2,000.00
	justment ..	1,500.00			
" 1	Present worth	16,208.30			
		\$23,610.05			\$23,610.05
<i>Dr.</i>		R. DAWSON.		<i>Cr.</i>	
1907.		1907.			
Mar. 8	Withdrawal..	\$2,000.00	Jan. 1	Investment .	\$4,000.00
Aug. 15	" ..	1,000.00	May 12	" ..	5,000.00
Sept. 30	" ..	1,200.00	July 16	" ..	2,000.00
1908.			Oct. 5	" ..	1,500.00
Jan. 1	$\frac{1}{2}$ loss from		1908.		
	interest ad-		Jan. 1	$\frac{1}{2}$ net gain of	
	justment .	601.76		\$5,800 ...	2,900.00
" 1	$\frac{1}{2}$ loss from		" 1	Net credit int.	493.46
	salary ad-		" 1	Salary .....	1,000.00
	justment .	1,500.00			
" 1	Present worth	10,591.70			
		\$16,893.46			\$16,893.46



## SOLUTION

NOTE.—It is presumed that the detail work of figuring interest on investments and withdrawals is already familiar to the student from that section of the text which deals with interest on partners' accounts.

It might not be out of place to mention here that a close acquaintance with the work on sharing, and also on average as it applies to partners' accounts, will help the student in work on partnership adjustments.

1st Step: The gain of \$5,800, according to agreement, is to be divided equally. Therefore each partner is credited with \$2,900 as his share of this gain.

2nd Step: Having found the net interest due to Coleman to be \$710.05 and the net interest due to Dawson to be \$493.46, these amounts are credited to their respective accounts. It is here that the principle of double-entry bookkeeping must be kept in mind. We cannot credit the partners with these interest amounts without taking into consideration the fact that some other account would have to be debited with these amounts. If we followed the bookkeeping for the transactions right through, we would find these amounts are debited to interest account. This, being a loss and gain account, would, in due course, be closed into loss and gain account. Loss and gain account, in turn, would be closed into the partners' accounts. Thus it is that the partners, who are credited with their net interests, are, in turn, debited, each for his share of the complete loss (\$1,203.51) it would occasion to the business by paying these interests.

3rd Step: Each partner is credited for the amount of salary due to him as per agreement—namely, \$2,000 to Coleman and \$1,000 to Dawson. As in the case of the interest, the loss to the business occasioned by paying this salary (\$3,000) will be divided equally between the partners, and each partner debited with his share, \$1,500.

4th Step: With all the entries thus made in the accounts, a comparison of the debits with the credits will give us the present worth of each of the partners.

ILLUSTRATION 2. — Adams, Wilson, and Green enter into partnership as grain dealers. Green is a special or limited partner, putting \$10,000 into the business. Wilson contributes \$15,000, and Adams nothing. Wilson and Adams are each allowed \$1,500 a year salary. Profits are divided equally. Partnership deed duly drawn and registered. In the second year Green, being on a holiday trip, sees a chance of buying a quantity of grain, which he



buys at a profitable figure, with the consent of his partners. He sells part of the lot on his way home, but, after his return, the market takes a disastrous turn, and the firm is obliged to assign. Their statement of affairs is as follows :

LIABILITIES.		ASSETS.	
Bills Payable to Bank.....	\$75,000	Office Furniture.....	\$ 500
(Secured by warehouse receipts on 100,000 bushels of wheat at \$1.10. Wheat realizes \$80,000.)		Grain in Elevator, hypothecated to Bank..	110,000
			<hr/>
			\$110,500
Trade Creditors .....	25,000	Deficiency .....	14,500
Capital Account :			<hr/>
Green .....	\$10,000		
Wilson .....	15,000		
	<hr/>		
	25,000		
	<hr/>		
	\$125,000		<hr/>
			\$125,000

Partners have sufficient personal assets to pay all creditors. Furniture sells for \$500. Adjust the loss and show the partners' accounts, including final adjustment with one another, it being understood that no payments have been made on account of salary.

## SOLUTION

<i>Dr.</i>	GREEN (Capital).	<i>Cr.</i>	
To $\frac{1}{3}$ Loss .....	\$15,833 $\frac{1}{3}$	By Investment.....	\$10,000
		„ Cash to Settle.....	5,833 $\frac{1}{3}$
	<hr/>		<hr/>
	\$15,833 $\frac{1}{3}$		\$15,833 $\frac{1}{3}$
<i>Dr.</i>	WILSON (Capital).	<i>Cr.</i>	
To $\frac{1}{3}$ Loss .....	\$15,833 $\frac{1}{3}$	By Investment.....	\$15,000
„ Cash to Settle .....	666 $\frac{2}{3}$	“ Salary .....	1,500
	<hr/>		<hr/>
	\$16,500		\$16,500

<i>Dr.</i>	ADAMS (Capital).		<i>Cr.</i>
To $\frac{1}{3}$ Loss .....	\$15,833 $\frac{1}{3}$	By Salary .....	\$ 1,500
		“ Cash to Settle.....	14,333 $\frac{1}{3}$
	<hr/> \$15,833 $\frac{1}{3}$		<hr/> \$15,833 $\frac{1}{3}$

<i>Dr.</i>	Cash.	<i>Cr.</i>	
Surplus from Wheat . . . . .	\$5,000	Paid Trade creditors . . . .	\$25,000
Received for Furniture . . . . .	500	“ Wilson . . . . .	666 $\frac{2}{3}$
“ from Green . . . . .	5,833 $\frac{1}{3}$		
“ “ Adams.. . . .	14,333 $\frac{1}{3}$		
	<hr/>		<hr/>
	\$25,666 $\frac{2}{3}$		\$25,666 $\frac{2}{3}$

EXPLANATION.—Green, by purchasing and selling grain for the firm, violated one of the most important provisions of the Act governing limited partnerships, and so made himself liable as a general partner. This being the case, his loss is not limited by his investment, but he may be charged with one-third of the firm's losses, no matter how large they may be. The grain in the elevator has been sold for \$80,000; \$75,000 of this amount is retained by the bank to settle the bills payable due them. The \$5,000 surplus from this source, together with the \$500 received for the office furniture, makes a total of \$5,500 cash, which represents the entire assets of the firm. Against this there are liabilities to trade creditors of \$25,000, thus showing a shortage or deficiency of \$19,500, which has to be made up by the partners. The loss of the partnership is the sum of the investments shown on the balance-sheet, together with the \$19,500 needed to settle the accounts, and \$3,000 for salary, making a total of \$47,500, of which each has to bear one-third, or \$15,833 $\frac{1}{3}$ . When these amounts have been charged, the capital accounts will show Green with a debit balance of \$5,833 $\frac{1}{3}$ , Adams with a debit balance of \$14,333 $\frac{1}{3}$ , and Wilson with a credit balance of \$666 $\frac{2}{3}$ . Green and Adams must pay sufficient cash into the firm to square off their respective accounts. When this is done, the firm has \$25,666 $\frac{2}{3}$  available cash, of which \$25,000 is paid over to the trade creditors and the remaining \$666 $\frac{2}{3}$  to Wilson. All of the accounts will then balance, as shown.

#### SERIES 102

1. A, B, and C form a partnership, investing respectively \$9,600, \$8,400, and \$7,300. They agree to share gains and losses equally, and to allow interest on all investments and charge interest on all withdrawals at the rate of 6%. At the end of four months

A invested \$3,000 more, B \$1,000, and C \$1,200. At the end of the year no settlement of the interest is made until after the net gain, \$12,800, has been divided and entered up. The interest is then adjusted. Show partners' accounts after such adjustment, properly closed, with balances brought down for next year's business. (Calculate time in months.)

2. H. Mann and J. Ross enter into partnership on Jan. 1st, 1898, each investing \$8,000, under agreement to share gains and losses according to average net investment. On April 1st Mann withdrew \$3,200 and Ross \$1,600. On Sept. 1st each took out \$2,400, and on Nov. 1st Mann withdrew \$900 and Ross \$700. At the end of the year the capital is \$6,000. Show the partners' accounts as they stand on Jan. 1st, 1899. (Calculate time in months.)

3. A, B, C are partners. On January 1st, 1898, their respective capital accounts showed the following credit balances : A, \$75,000 ; B, \$25,000 ; C, \$10,000. From that date each partner was to get 6% per annum on his capital. A was to share  $\frac{5}{9}$  of the gains or losses ; B,  $\frac{1}{3}$  ; and C the remainder. At the end of the year their assets and liabilities were as follows : Premises, \$50,000 ; machinery, \$21,000 ; tools, \$4,000 ; sundry debtors, \$40,000, less 5% allowance for bad debts ; bills receivable, \$20,000, less 5% allowance for bad debts ; cash in bank, \$10,000 ; cash on hand, \$200 ; sundry creditors, \$30,000 ; bills payable, \$12,000. A was charged up with \$4,000 withdrawn, B with \$1,500, and C with \$700. Write up the capital accounts of the three partners in the ledger.

4. A, B, and C form a partnership, investing respectively \$9,700, \$8,400, and \$7,200. They agree to share gains and losses according to average net investment, and to allow interest on all investments and charge interest on all withdrawals at the rate of 6%. At the end of four months A invests \$4,000 more, B \$1,000, and C \$1,200. At the end of the year no settlement of the interest is made until after the net gain, \$12,800, has been divided and entered up. The interest is then adjusted. Show the partners' accounts after such adjustment, properly closed, with the balances brought down for next year's business. (Calculate time in months.)

5. A and B are partners in business, sharing gains and losses in the following manner : A,  $\frac{3}{4}$  ; B,  $\frac{1}{4}$ . A is to receive a salary of \$1,200 per annum for special services. How much money is he practically clearing in salary ? Why ?

6. A and B are partners, with respective investments of \$18,000 and \$30,000, and sharing equally losses and gains. At the end of the first year the interest on excess of capital at the rate of 7% is adjusted, without entry in books, by private payments of A to B. How much is paid?

7. A, B, and C are partners, with equal investments, and share gains and losses equally. A is allowed a salary of \$900 per year; B, \$1,100; and C, \$1,200. The net gain, irrespective of salaries, is \$3,000. They have exactly this amount of cash on hand. As it is the intention of the firm to leave their original capital unchanged, this amount of money is divided among them. Prepare a statement showing clearly how much each partner is entitled to, no salaries having yet been paid.

8. Thompson and Wilson went into partnership Jan. 1, 1907, Thompson investing \$22,000 and Wilson \$4,000. During the year they lost \$5,900, without reckoning interest on capital, which, by agreement, is to be taken at 8% per annum. During the year the partners have withdrawn the following amounts on which no interest is to be charged: Thompson, \$2,800; and Wilson, \$2,000. They share gains and losses as follows: Thompson,  $\frac{5}{8}$ ; and Wilson,  $\frac{3}{8}$ . Draw up a statement showing the position of both partners at the beginning of 1908.

9. G, H, and N formed a co-partnership on Jan. 1, 1907, and agreed to invest equal sums, and share equally gains and losses, interest at 10% per annum being allowed on the excess and charged on the deficiency of each partner's required investment. G invested \$8,000 on Jan. 1, \$6,400 on Apr. 15, \$5,600 on May 20; and withdrew \$1,200 on May 5, \$1,800 on August 13, and \$2,400 on Nov. 16. H invested \$7,000 on Jan. 1, \$9,000 on July 24, \$1,600 on Oct. 12; and withdrew \$6,000 on Mar. 26, \$8,000 on May 5, and \$6,700 on June 14. N invested \$7,500 on Jan. 1, \$6,000 on Jan. 25, \$6,800 on Feb. 14; and withdrew \$2,000 on April 27, \$3,600 on June 4, and \$800 on Aug. 2. If the net gain during the year was \$9,842.19, what was G's capital in the business on Jan. 1, 1908? What interest did H owe the remaining partners? How much interest did H owe N?

10. H and G entered into co-partnership, equally insolvent. The resources of the firm at closing are: Cash, \$2,440.60; bills receivable, \$1,644; personal accounts, \$4,234.73. The liabilities are: Bills payable, \$2,149; due S, \$3,241.67; due J, \$1,340.33. The firm paid private debts of \$260 for H, and G paid sundry expenses for the firm amounting to \$375. There was a net gain of \$2,136.72. What was each partner's insolvency at commencing? What was each partner's net capital at closing?



11. S, G, and H are partners in business, and on the date of settlement the books of the concern show that S drew out for private use \$590, that the firm assumed a private debt of \$930 for G, and that H's account was overdrawn \$125. On the date of the last settlement each partner had an equal sum standing to his credit. The resources closing were: Cash, \$8,294; personal accounts, \$1,560; bills receivable, \$4,400; merchandise, \$18,220. The liabilities at closing were: Bills payable, \$7,278; personal accounts due \$13,651; interest on notes unpaid, \$190. What was each partner's net capital at closing, there being a net gain of \$2,100?

12. L and W are equal partners, but, having become financially involved, they arrange a compromise with their creditors at 50c on the dollar, and wind up the business. Their balance-sheet at time of compromise stood as follows:

LIABILITIES.		ASSETS.	
L (Capital Acct.)	.....\$ 190	Cash	..... \$ 50
Sundry Creditors	.....3,910	Sundry Debtors	..... 535
		Goods	.....1,800
		Furniture	..... 380
		Machinery and Plant	... 1,000
		W (Capital Acct.)	..... 335
	\$4,100		\$4,100

The assets realize as follows: Cash, as stated, \$50; sundry debtors, \$410; goods, \$1,250; furniture, \$150; machinery and plant, \$500. W's private estate being insolvent, he is unable to contribute towards the debt. The creditors are paid as arranged. How should the partners settle with each other? Show ledger accounts dealt with in handling winding-up proceedings.

13. C, G, M, and F are partners, sharing profits or losses in the proportions of  $\frac{5}{12}$ ,  $\frac{1}{6}$ ,  $\frac{1}{3}$ , and  $\frac{1}{12}$ . The balance-sheet, after realizing on the estate and settling with the creditors, is as follows:

LIABILITIES.		ASSETS.	
C (Capital Acct.)	..... \$280	M (Capital Acct.)	..... \$400
G (Capital Acct.)	..... 800	F (Capital Acct.)	..... 223
		Cash	..... 457
	\$1,080		\$1,080

As none of the partners are able to contribute anything further to the business, explain clearly the settlement, and show the disposition of the \$457 cash.



14. H and J purchased a hotel for \$40,000, the former investing \$16,000, the latter investing the remainder. They agree to share profits and losses in these proportions. The business has two distinct branches, one of which is taken charge of by H and the other by J. Each keeps separate accounts of his receipts and disbursements for the term of partnership. H's total receipts were \$40,910.15, and J's \$37,201.19. H's disbursements were \$50,912.24, and J's \$26,870.15. At the end of the period the business was sold for \$53,700.00, of which \$30,000 was received in cash, and banked to their joint credit. A note of \$23,700 was taken for the balance and then afterwards transferred to H at a discount of 10% on its face. A final settlement is then made, and each received a cheque for his share of the \$30,000. Find the amount of the respective cheques.

15. A is the owner of a business, the balance-sheet of which stands as follows :

LIABILITIES.		ASSETS.	
Creditors .....	\$10,000	Stock-in-trade .....	\$50,000
A (Capital) .....	70,000	Cash .....	12,000
		Building .....	15,000
		Book Debts .....	3,000
	<hr/>		<hr/>
	\$80,000		\$80,000

B and C are admitted as equal partners with A, upon each putting into the business \$30,000 cash, and each paying A personally \$18,000 in cash. Show the balance-sheet of the new firm on commencing business : (a) supposing no goodwill account to be opened ; (b) supposing goodwill account opened.

16. A and B are partners, sharing profits and losses equally. Their respective capital accounts are : A, \$6,000 ; B, \$1,500. Their liabilities amount to \$15,000, which includes \$3,000 due to A on loan account and \$1,500 due to B on loan account. The assets of the firm were sold for \$18,000 cash. How do the partners settle with each other ? Show balance-sheet at start and accounts to explain your adjustment.

# APPENDIX

## THE METRIC SYSTEM OF MEASUREMENT

The **Metric System of Measurement** gets its name from the metre, which is the unit of length of the system.

All the units of measurement are derived in a simple manner from the metre.

This system of measurement is used in all countries for scientific purposes on account of its exactness, and in many countries it is used for ordinary purposes.

### Measures of Length

The **Standard Metre** is a bar of platinum, which is supposed to be the length of one ten-millionth ( $\frac{1}{10000000}$  or .0000001) of a quarter of the earth's circumference, measured by a line passing through Paris, France, from the equator to the pole.

TABLE

10 millimetres, marked mm.,	are 1 centimetre, marked cm. = $\frac{1}{100}$ or .01	metre
10 centimetres, " cm.,	" 1 decimetre, " dm. = $\frac{1}{10}$ or .1	"
10 decimetres, " dm.,	" 1 metre, " m.	
10 metres, " m.,	" 1 dekametre, " Dm. = 10	metres
10 dekametres, " Dm.,	" 1 hektometre, " Hm. = 100	"
10 hektometres, " Hm.,	" 1 Kilometre, " Km. = 1000	"
10 Kilometres, " Km.,	" 1 Myriametre, " Mm. = 10000	"

**Comparative Lengths** are as follows :

	INCHES.	FEET.	YARDS.
1 Metre =	39.37079	3.2808992	1.0936331
1 Decimetre =	3.93708	.3280899	.1093633
1 Centimetre =	.39371	.032809	.0109363
1 Millimetre =	.03937	.0032809	.0010936

The **Metre** is used in measuring cloth and short distances ; the **Kilometre** is used in measuring long distances.

It will be noted that the system is a decimal one, since each unit is 10 times as large, or  $\frac{1}{10}$  as small, as some other unit, so that each unit is 10, 100, 1,000, etc., times the standard, or  $\frac{1}{10}$ ,  $\frac{1}{100}$ ,  $\frac{1}{1000}$ , etc., of the standard. This relation among the units exists in every table of the metric system. So that, if the standard is known, all the other units can be formed by prefixing the same syllables to the standards.

The prefix milli is derived from the Latin	<i>mille</i> , meaning	thousand.
" " centi " "	" <i>centum</i> , " "	hundred.
" " deci " "	" <i>decem</i> , " "	ten.
" " {deca or } " "	Greek <i>deka</i> , " "	ten.
" " {hecto or } " "	" <i>hekaton</i> " "	hundred.
" " {hekto } " "	" " " "	
" " kilo " "	" <i>kilioi</i> , " "	thousand.
" " myria " "	" <i>myria</i> , " "	ten "

In abbreviating the names of the units, those in each table that are larger than the standard are capitalized, while the others are not.

Since, in the Metric System, 10, 100, 1000, etc., units of a lower denomination make a unit of a higher denomination, it follows that any one of the metric measures may be expressed in terms of another measure by simply moving the decimal point to the right or left.

### Land or Surface Measure

The **Are** is the unit of land measure (or area). It is a square whose side is 10 metres.

#### TABLE

100 centiares, marked	ca.,	are 1 <b>Are</b> , marked	a.
100 ares	a.,	1 hektar, marked	Ha.

For measuring other surfaces, squares of the metre and its subdivisions are used.

An **Are** is 100 square metres, marked m<sup>2</sup>. It is about 119.6 square yards. The **Hektar** is nearly 2½ acres (2.47).

The **Are** is the principal unit of surface of small plots of land. The area of a farm is expressed in hektars; of a country, in square kilometres.

### Measures of Capacity

The **Litre** is the unit of capacity. It is a cubic decimetre; that is, a cube whose edge is a decimetre long.

#### TABLE

10 centilitres, marked	cl.,	are 1 decilitre, marked	dl.
10 decilitres,	dl.,	1 <b>litre</b> ,	l.
10 litres,	l.,	1 dekalitre,	Dl.
10 dekalitres,	Dl.,	1 hektolitre,	Hl.

The measures commonly used are the **litre** and the hektolitre. The litre is very nearly a quart; it is used in measuring milk, wine, etc., in modern quantities. The hektolitre is about 2 bu. 3½ pk.; it is used in measuring grain, fruit, roots, etc., in large quantities.

For measuring wood the **Stere** is used; it is a cubic metre (= 35.316 cub. ft.).

### Measures of Weight

The **Gram** is the unit of **Weight**. It is the weight of a cubic centimetre of water.

#### TABLE

10 milligrams, marked	mg.,	are 1 centigram, marked	cg.
10 centigrams,	cg.,	1 decigram,	dg.
10 decigrams,	dg.,	1 <b>gram</b> ,	g.
10 grams,	g.,	1 dekagram,	Dg.
10 dekagrams,	Dg.,	1 hektogram,	Hg.
10 hektograms,	Hg.,	1 kilogram,	Kg.
10 kilograms,	Kg.,	1 myriagram,	Mg.
10 myriagrams,	Mg.,	1 quintal,	Q.
10 quintals or 1000 kilograms are 1 Metric ton, marked M.T.			

The weights commonly used are the **Gram**, **Kilogram**, and **Metric Ton**. The **Gram** is used in mixing medicines, in weighing the precious metals, and in all cases where great exactness is required. The **kilogram** (commonly called the "Kilo") is the usual weight for groceries and coarse articles generally; it is very nearly 2½ lbs. Avoir. The **metric ton** is used for weighing hay and other heavy articles; it is about 204 lbs. more than our ton.

Legal and Approximate Values are as follows:

DENOMINATION.	LEGAL VALUE.	APPROXIMATE VALUE.
Metre .....	39.37 inches .....	3ft. $3\frac{3}{8}$ inches.
Centimetre .....	39371 " .....	$\frac{2}{5}$ inch.
Kilometre .....	62137 mile .....	$\frac{5}{8}$ mile.
Square Metre .....	1.196 sq. yards .....	$10\frac{3}{4}$ sq. feet.
Are .....	119.6 sq. yards .....	4 sq. rods.
Hektar .....	2.471 acres .....	$2\frac{1}{2}$ acres.
Cubic Metre .....	1.308 cub. yds. ....	$35\frac{1}{8}$ cub. feet.
Stere .....	2759 cord .....	$\frac{1}{4}$ cord.
Litre .....	1.0567 quarts .....	$\left\{ \begin{array}{l} 1\frac{1}{20} \text{ liquid quart.} \\ \frac{9}{10} \text{ dry quart.} \end{array} \right.$
Hektolitre .....	2.8375 bushels .....	2 bush. $3\frac{1}{3}$ pk.
Gram .....	15.432 gr. Troy .....	$15\frac{1}{2}$ grains.
Kilogram .....	2.2046 lb. Avoir .....	$2\frac{1}{5}$ pounds.
Metric Ton (or tonneau) .....	2204.6 lb. " .....	1 T. 204 lbs.
Stere .....	0.27590 cord .....	$\frac{1}{4}$ cord.







Levee Huerfano

January 1914

Leve. Hupken & Co. Exporters

Genl. Wm. H. B. C.

Genl. Wm. H. B. C.  
Genl. Wm. H. B. C.



